Properties of Magnetic Reconnection in MHD Turbululence

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Overview

- Laminar vs. turbulent reconnection
- 2D MHD Turbulence Simulations
- Analyze reconnection regions
  - Sweet-Parker scaling organizes data
  - Intemittency/coherence plays critical role
- Is reconnection important for dissipation?
  - Relative dissipation of X-line current sheets
- Conclusions

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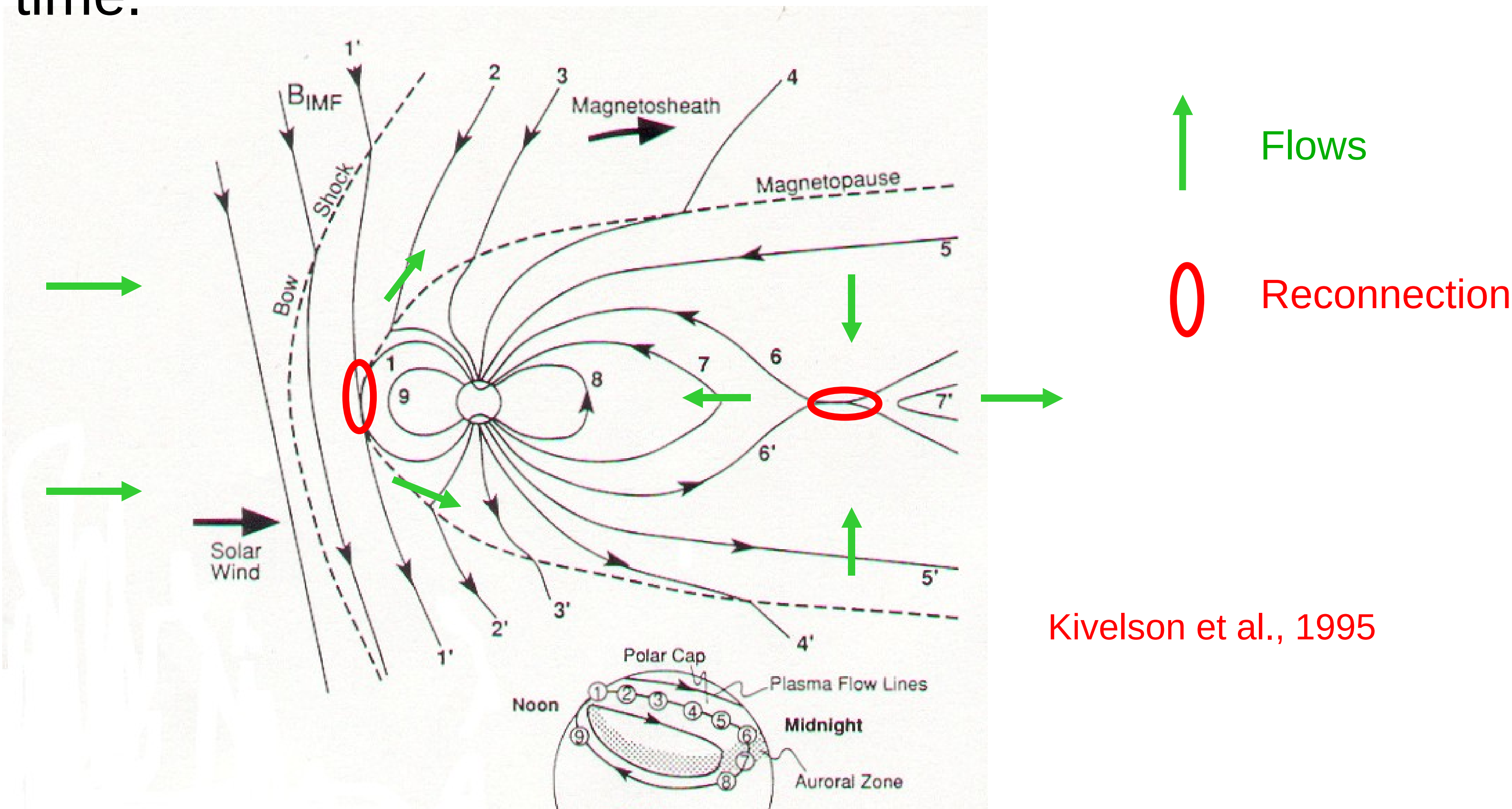
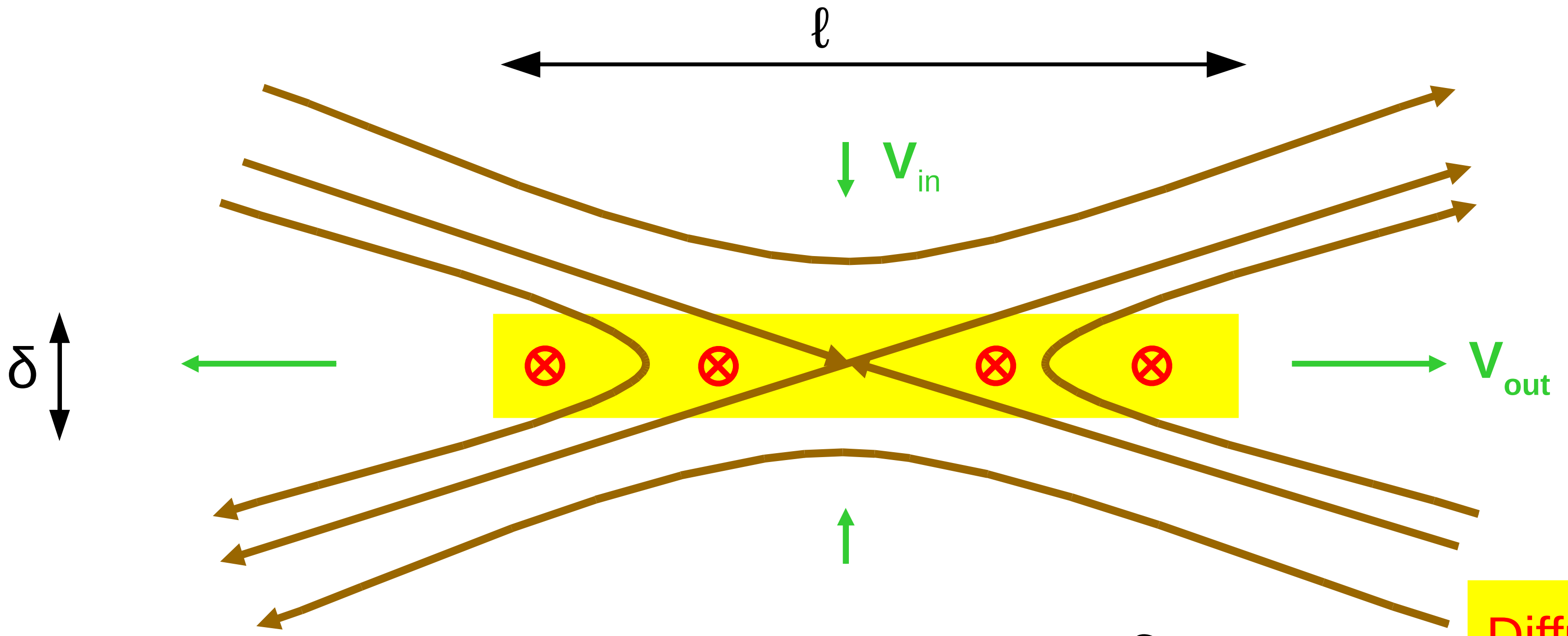
S. Servidio et al., **Magnetic Reconnection in Two-Dimensional Magnetohydrodynamic Turbulence**, PRL, 102, 115003, 2009

Laminar Magnetic Reconnection Picture

- Global dynamics presses topologically distinct regions together
  - Magnetic energy stored in global current sheets
- Energy released on timescales much faster than loading time.



Laminar Magnetic Reconnection



Diffusion Region
MHD not valid

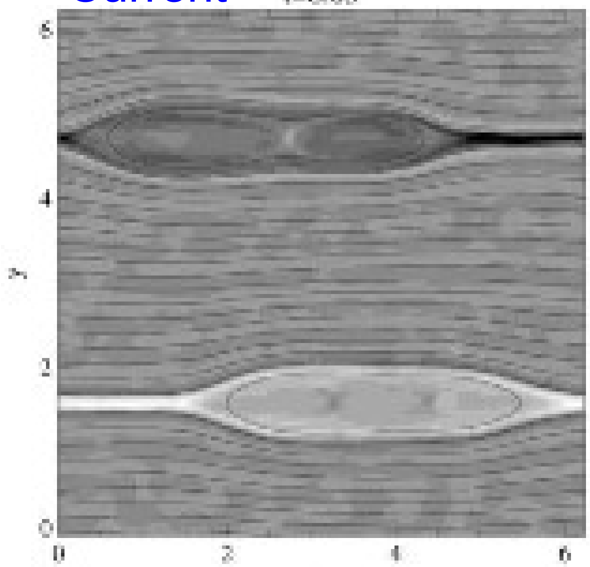
- Conservation of mass: $V_{in} \sim \frac{\delta}{\ell} V_{out}$
- Conservation of energy: $V_{out} \sim \frac{B_{up}}{\sqrt{4\pi m_p n}}$
- Ohm's law (steady state): $V_{in} \sim \frac{\eta c^2}{4\pi\delta}$
- R. Rate
  - $E_{\times} \sim \frac{c_A B_{up}}{c} S^{-1/2}$ $\quad \frac{\delta}{\ell} \sim \sqrt{\frac{\eta c^2}{4\pi c_A \ell}} \sim S^{-1/2}$

δ, ℓ, B_{up}

Turbulent Reconnection

- System-sized current sheets with turbulence generated or added.
  - Matthaeus et al., 1986, 2003.
  - Malara et al., 1992
  - Kliem, 1995
  - Lazarian and Vishniac, 1999, 2009
  - Lapenta, 2008
  - Daughton et al., 2009
  - Loureiro, 2009

Current



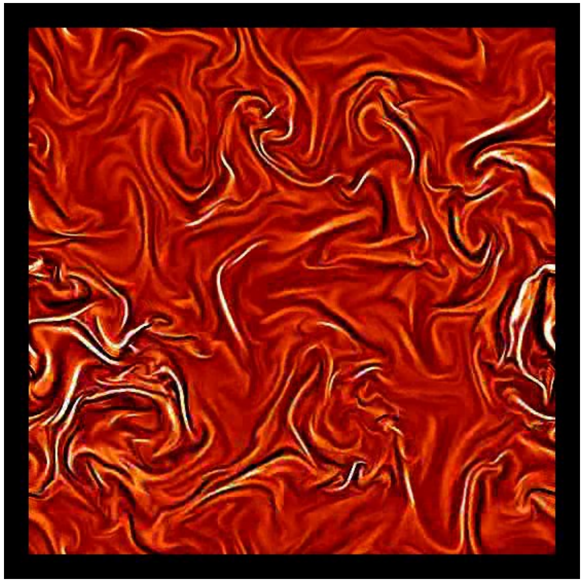
Dmitruk et al., 2003

- Typically see increases in reconnection rate due to turbulence (externally imposed and generated locally)
- Question: What are the properties of magnetic reconnection in a fully turbulent system?

Reconnection in Turbulent Systems

- Not well understood
  - How does the reconnection rate change in an unbounded system, without any large scale equilibrium magnetic shear?
  - What is the reconnection rate in turbulence?
  - Is reconnection fast in turbulence?
  - What is its nature?
  - How is it described?

Current



Dmitruk et al.

2D MHD Turbulence Simulations

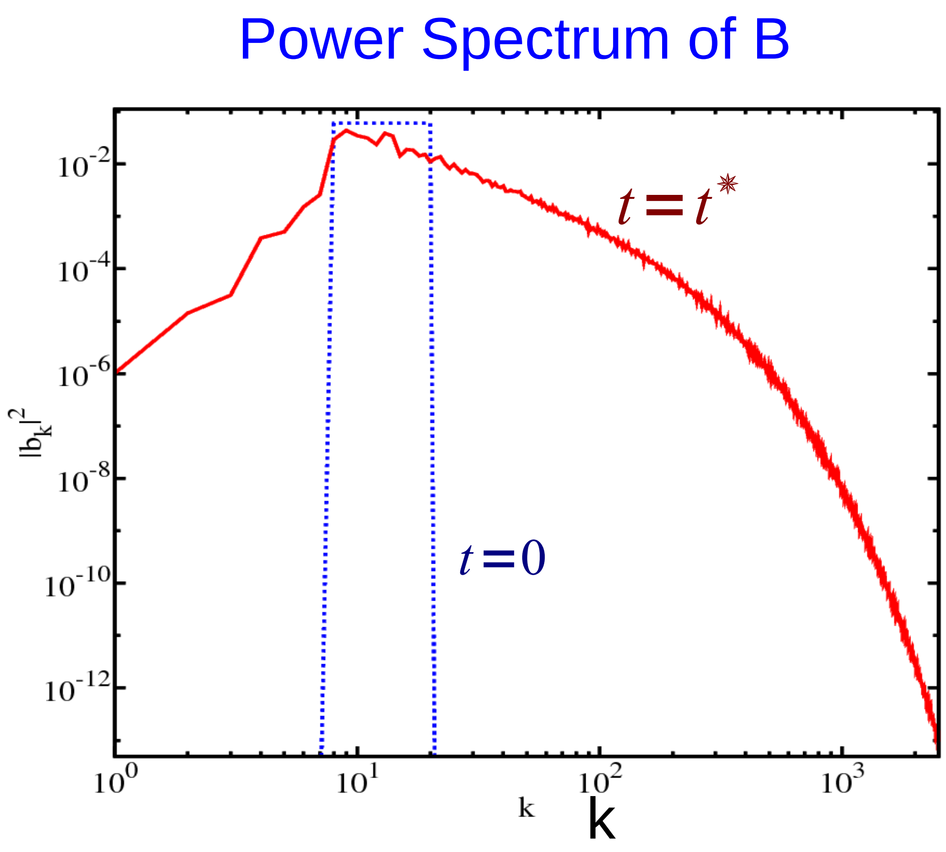
$$\frac{\partial \omega}{\partial t} = -(\boldsymbol{v}\cdot\boldsymbol{\nabla})\,\omega + (\boldsymbol{b}\cdot\boldsymbol{\nabla})\, j + R_\nu^{-1}\nabla^2\omega$$

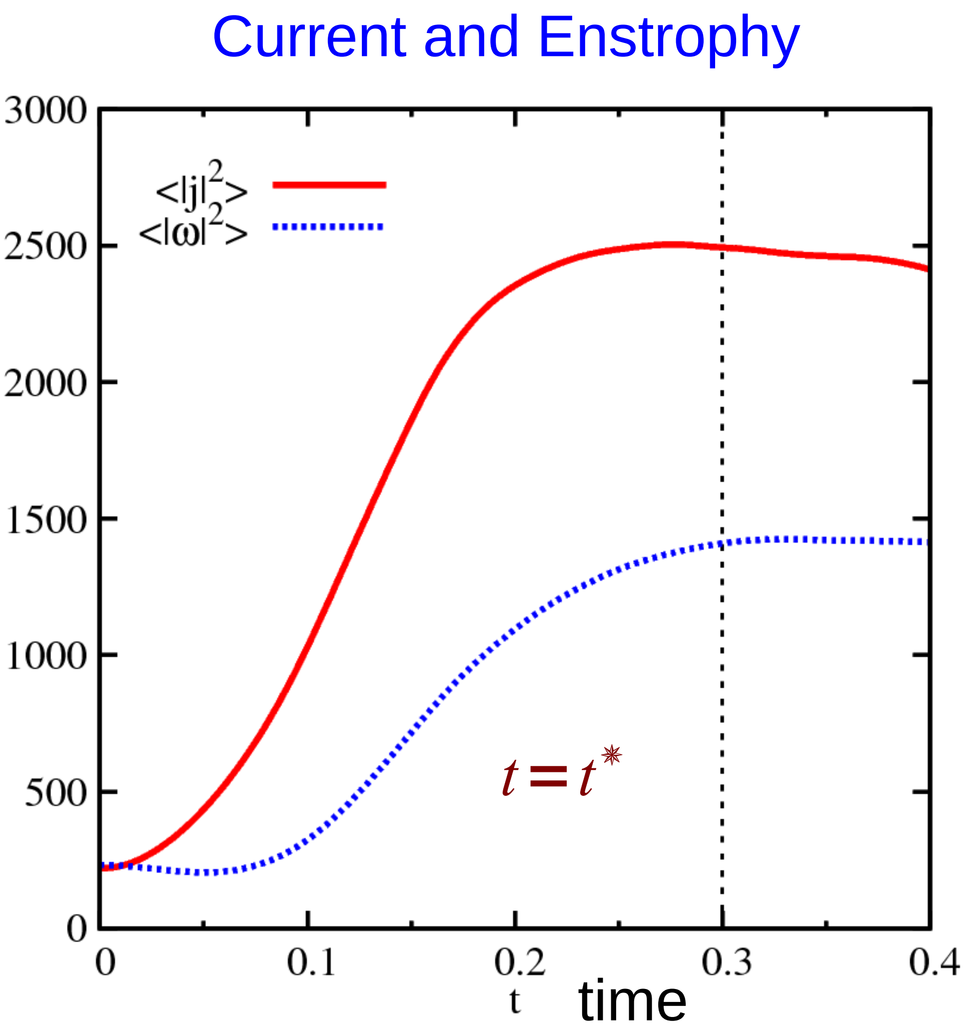
$$\frac{\partial a}{\partial t} = -(\boldsymbol{v}\cdot\boldsymbol{\nabla})\, a + R_\eta^{-1}\nabla^2 a$$

where: $\boldsymbol{b} = \boldsymbol{\nabla} a \times \hat{z},\ \ \boldsymbol{v} = \boldsymbol{\nabla}\phi\times\hat{z},\ \ j = -\nabla^2 a,\ \ \omega = -\nabla^2\phi$

- Dealiased (2/3 rule) pseudo-spectral code.
- Resolution up to 8192^2 grid points
- $R_\eta = R_\nu = 5000$.
- Total Energy: $E = (½) \langle v^2+b^2 \rangle \sim 1$

2D MHD: Direct Numerical Simulations





- Energy is initially in $5 \leq k \leq 30$ (k in units of $1/L_0$)

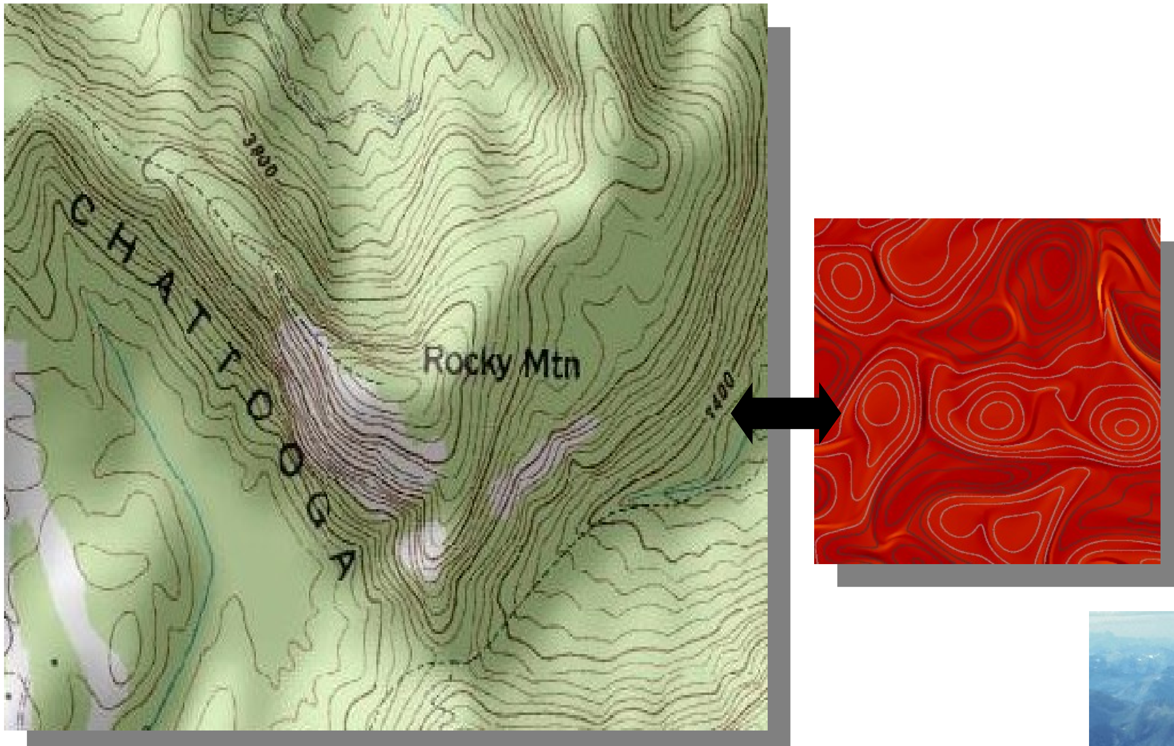
Appearance of Coherent Structures

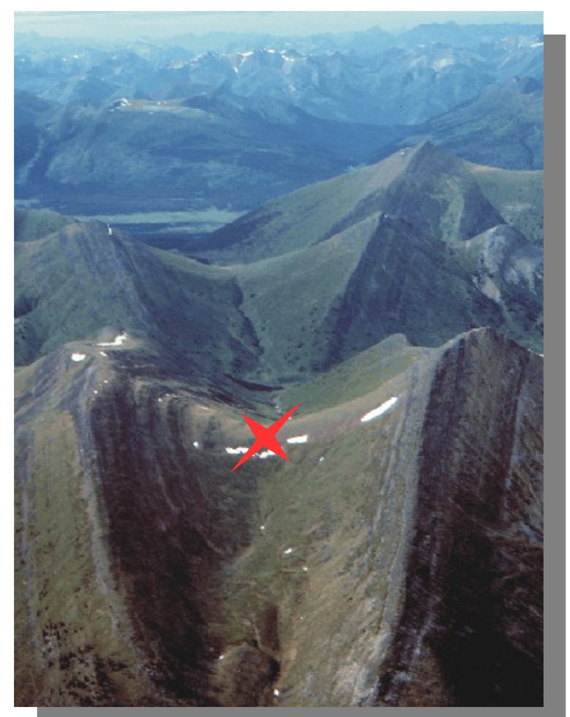
Color: J
Contours: a (gray: a>0, black: a<0)

(Only 1/40 of the box is shown)

- Intermittent J structures
- Magnetic dissipation
  - Plain Diffusion
  - Magnetic Reconnection
    - Notice x-lines!
- Magnetic Topology
  - Extremum (critical points) of a
  - O-lines:
    - Minimum and Maximum
  - X-lines:
    - Saddle points

Some Topography





We are looking
for saddle points...
in turbulence!

Hessian Theory

- Study the Hessian of a to find extremum(critical points)
- Hessian: $$H^a_{i,j}(\boldsymbol{x}) = \frac{\partial^2 a}{\partial x_i \partial x_j}$$
- Critical points: $$\boldsymbol{x}^* \rightarrow \nabla a = 0$$

Connection to Diffusion region: $$\sqrt{\lambda_R} \simeq \frac{\ell}{\delta}$$

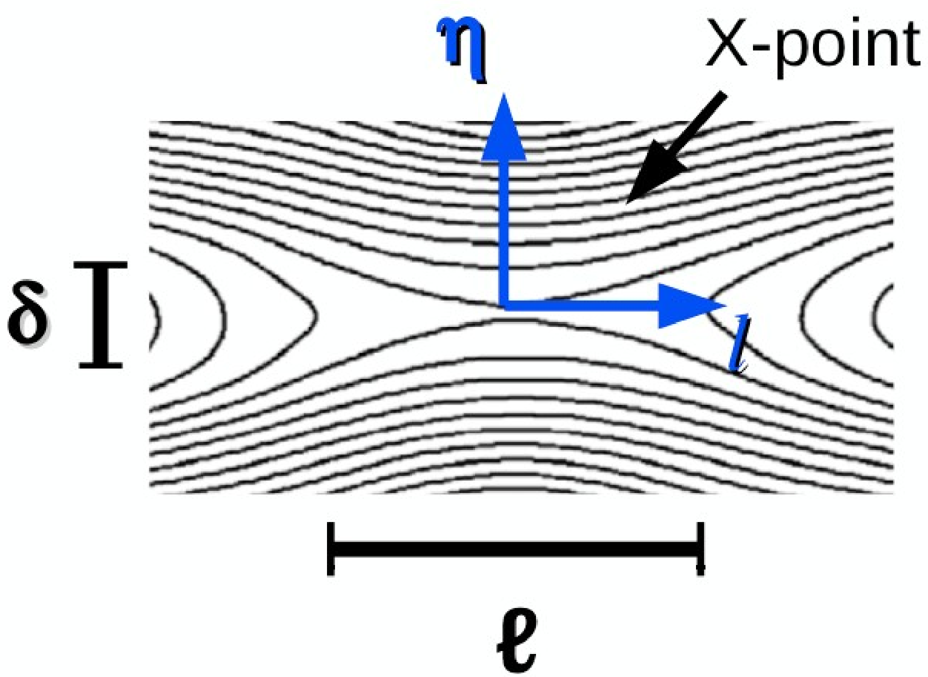
eigenvalues

$$\lambda_{max} = \frac{\partial^2 a}{\partial \eta^2}$$

$$\lambda_{min} = \frac{\partial^2 a}{\partial l^2}$$

eigenvectors

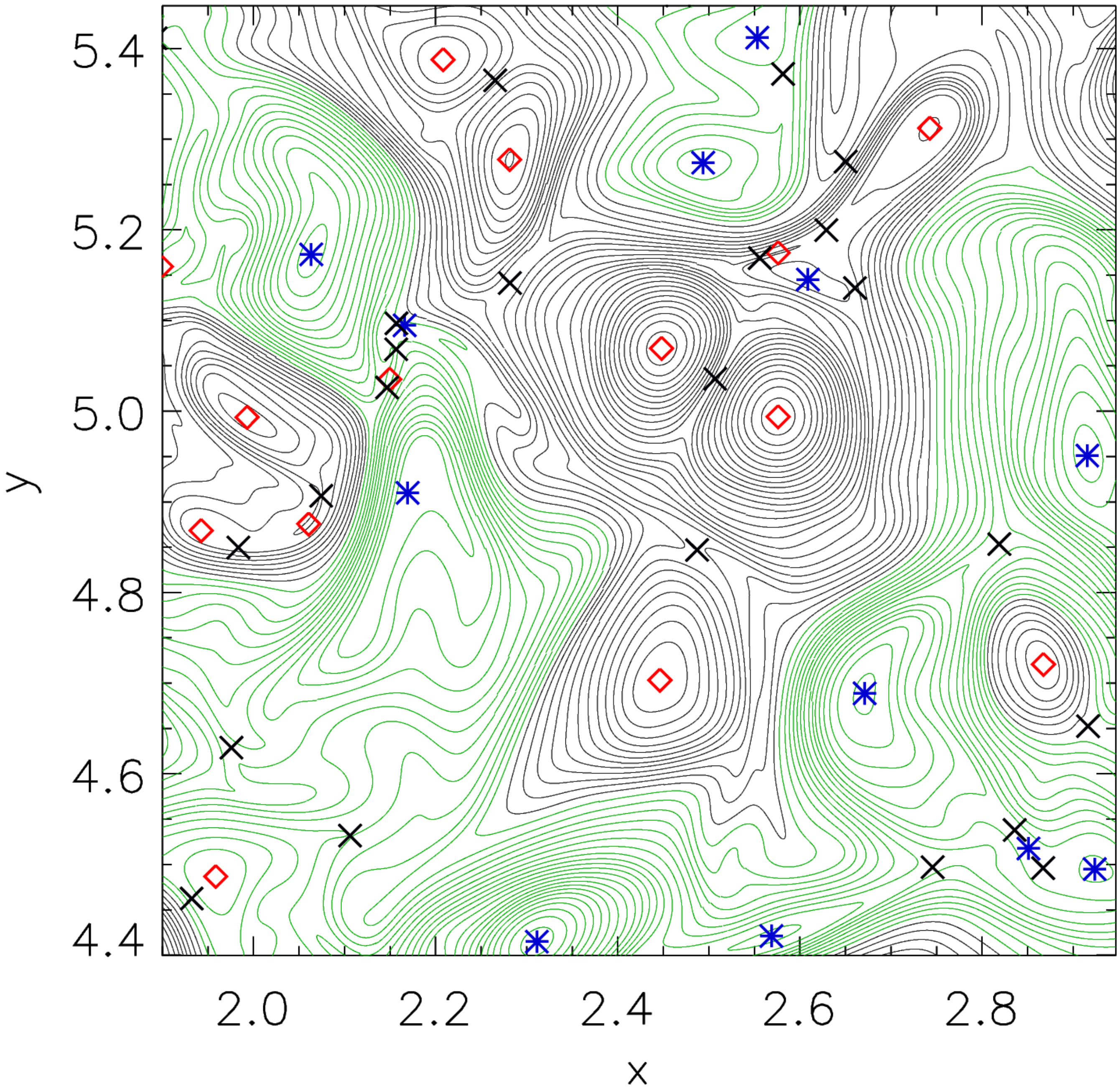
$$\{\vec{s}_\eta, \vec{s}_l\}$$



D. Biskamp, *Magnetic Reconnection in Plasmas* (Cambridge Univ. Press, Cambridge, 2000).

S. Rana, Surface *Topological Data Structures* (John Wiley & Sons, Chichester, England, 2004).

Critical Points in Turbulence



Magnetic potential *a* and critical points

a > 0

a < 0

maximum ✳

minimum ◇

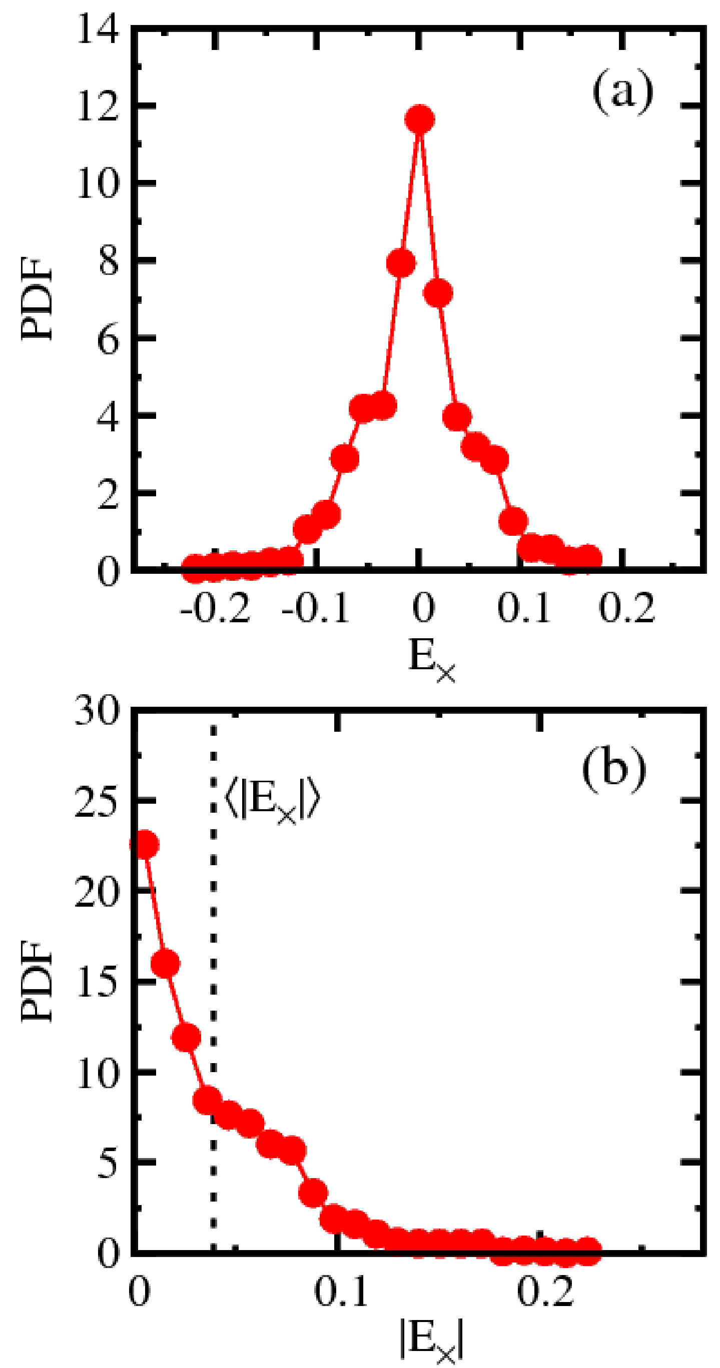
X-point x

Distribution of Reconnection Rates

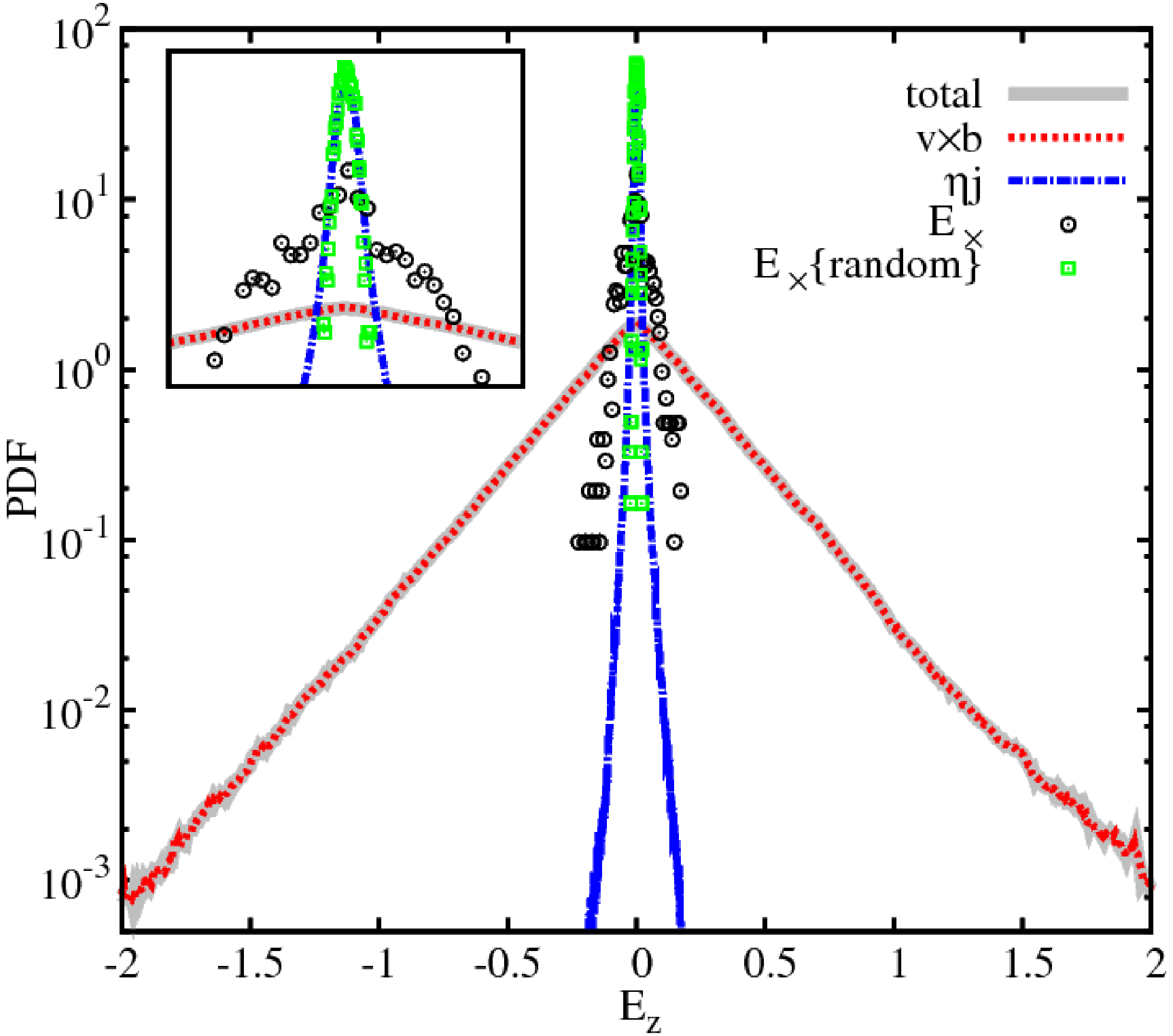
$$\dot{a} = R_{\mu}^{-1} \nabla^2 a |_{\times - point} = -E_{\times}$$

(normalized to the root mean-square magnetic fluctuation δb_{RMS}^2*)*

- Reconnection rates are broadly distributed
- Turbulence can be viewed as a sea of reconnecting islands with different reconnection rates.



Statistics of the Electric Field



How much of electric field *contributes* to reconnection?

$$E = -\frac{v}{c} \times b + \eta j$$

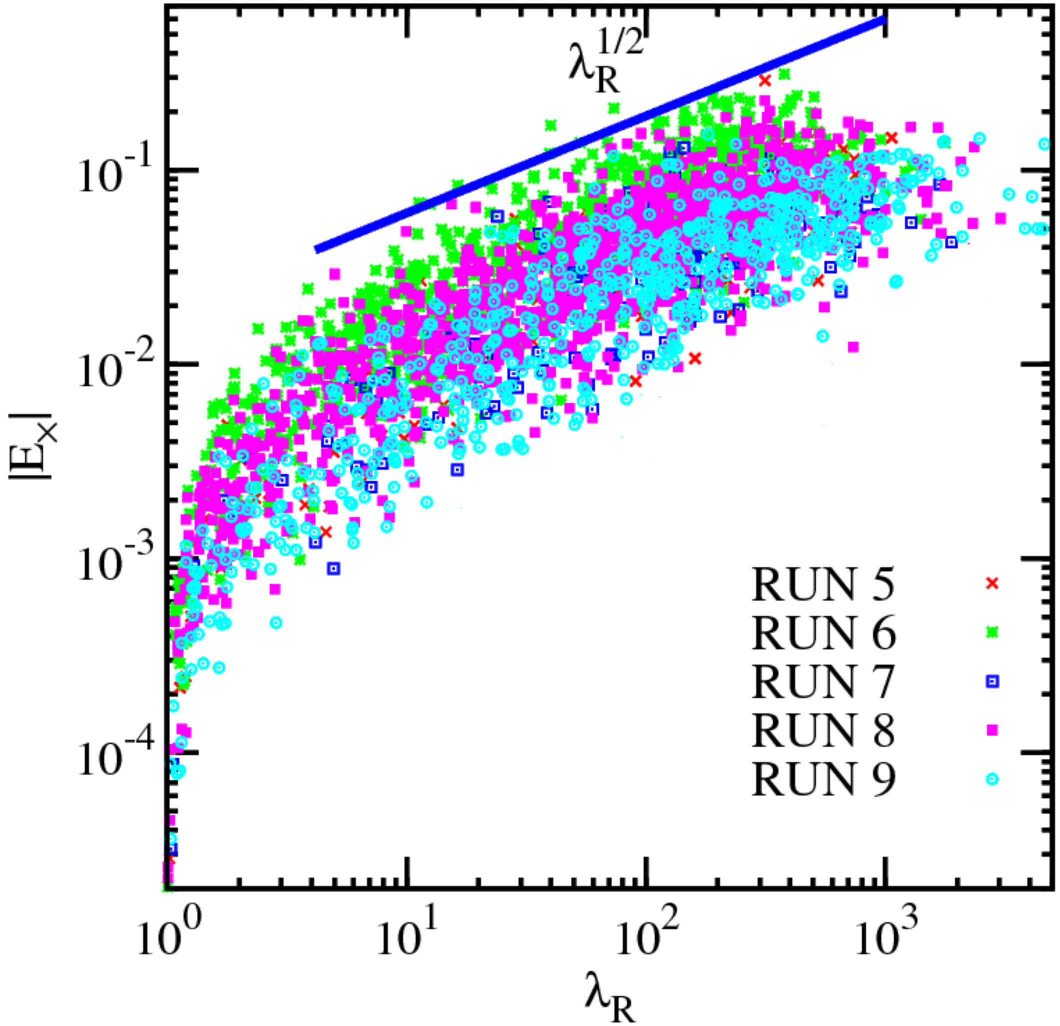
PDF($E_{v\times b}$) is typical of solar wind plasmas

Milano *et al. PRE* (2002), Breech *et al.* JGR(2003)

$$E_{v\times b} > E_{\times} > E_{\eta j}$$

- In General: Convection > Reconnection > Diffusion

R. Rate Geometry Dependence



- Highest R. Rates scale with diffusion region geometry

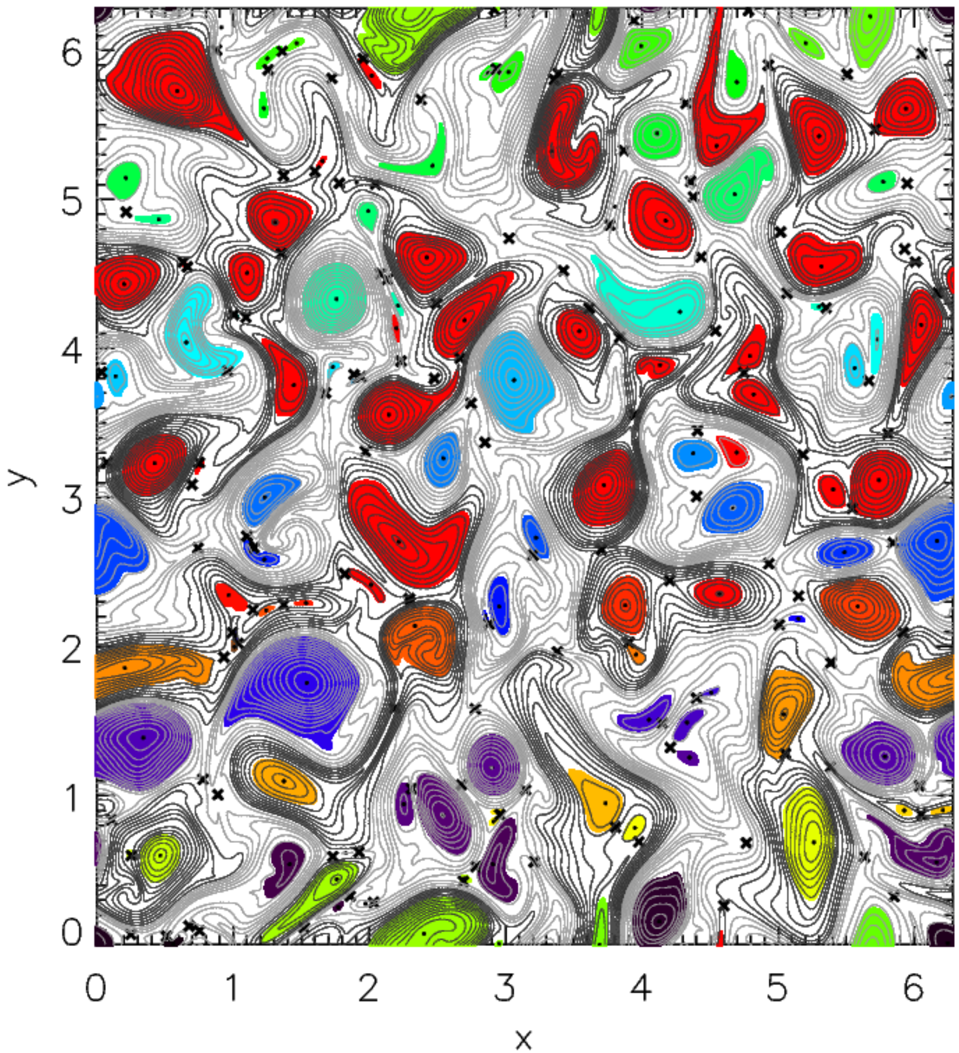
$$\lambda_R \equiv \frac{\lambda_{max}}{\lambda_{min}}$$

Since $\sqrt{\lambda_R} \simeq \frac{\ell}{\delta}$

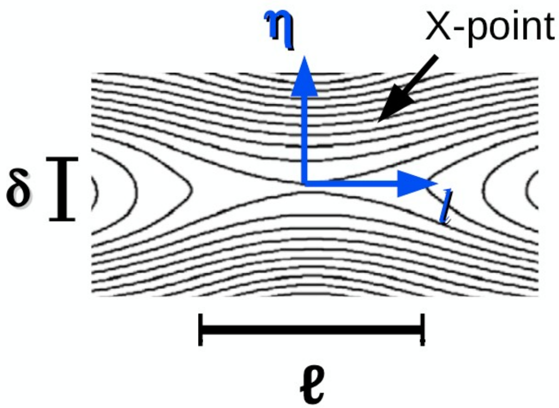
$$E_\times \sim \frac{\ell}{\delta}$$

- At first glance:
  - Opposite of Sweet-Parker prediction!

Complexity of Reconnection



- How do we characterize this complex reconnection?
  - Hessian Eigenvalues



  - Examine each x-line
    - Determine δ,ℓ, and B
    - E_x(th.) vs. E_x (exp.)

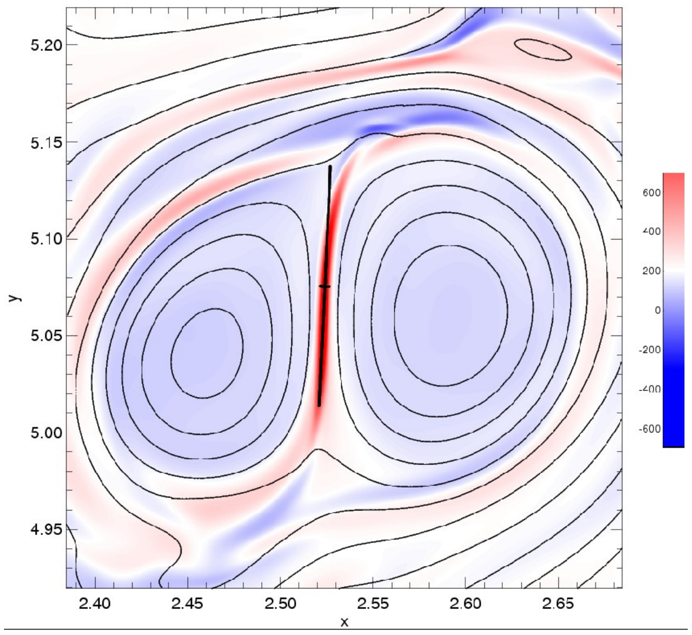
Dimensions of the Diffusion Region

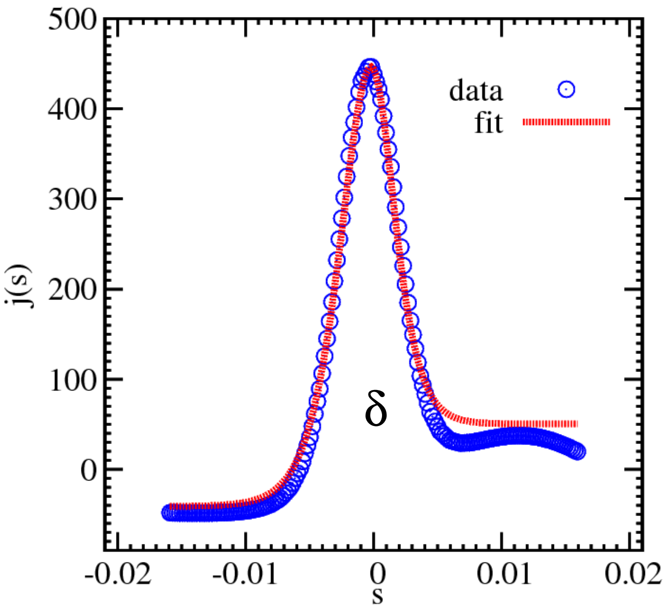
- Hessian eigenvectors
  - Fit determines δ, B_{up1}, B_{up2}

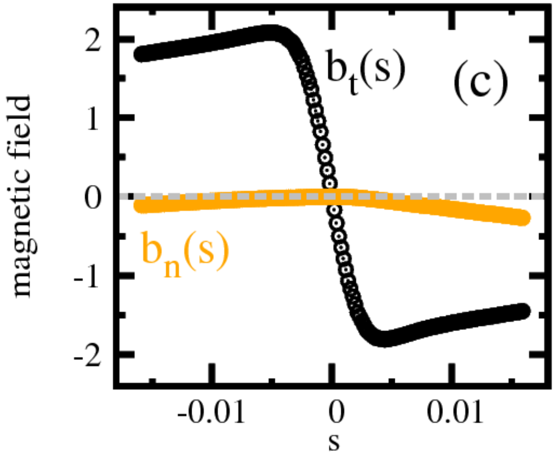
$$\ell \simeq \delta \sqrt{\frac{\lambda_{max}}{\lambda_{min}}}$$

$$J_z = A_1 \, sech^2\left(\frac{s - s_0}{\delta_1}\right) \quad \{s \geqslant s_0\}$$

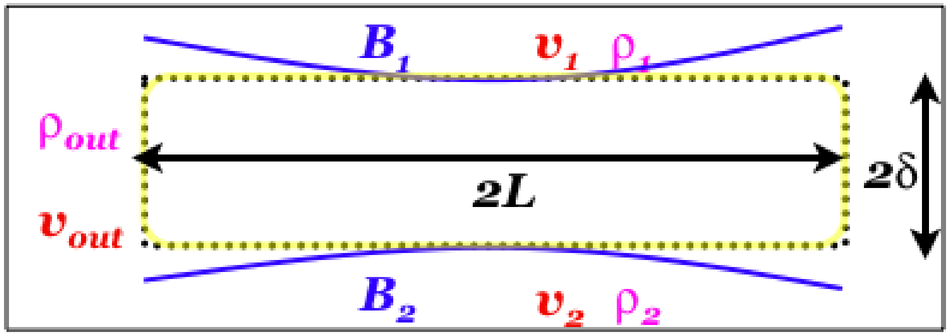
$$J_z = A_2 \, sech^2\left(\frac{s - s_0}{\delta_2}\right) \quad \{s < s_0\}$$







Asymmetric Reconnection



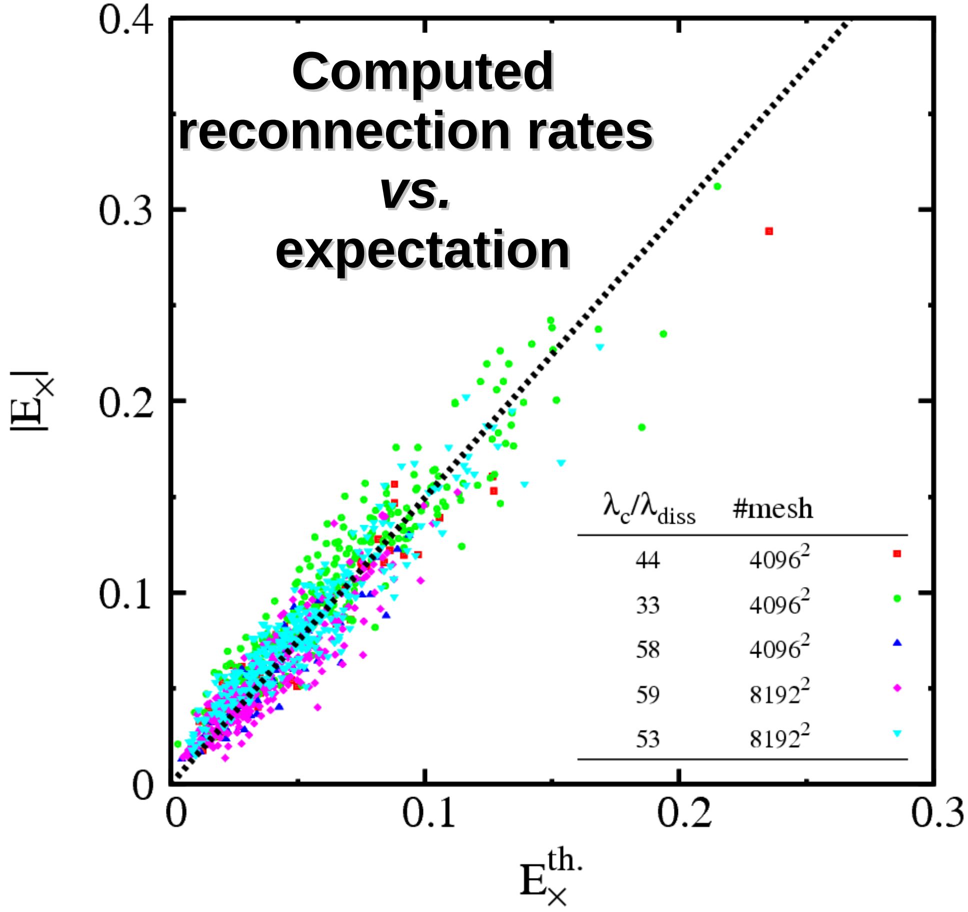
- Examine conservation laws
  - Mass density, Energy

Outflow speed $$v_{out}^2 \sim \frac{B_1 B_2}{4\pi} \frac{B_1 + B_2}{\rho_1 B_2 + \rho_2 B_1}$$

Reconnection Rate $$E \sim \frac{1}{c}\left(\frac{\rho_{out} B_1 B_2}{\rho_1 B_2 + \rho_2 B_1}\right) v_{out} \frac{2\delta}{L}$$

- Borovosky et al., 2007, Cassak and Shay, 2007, 2008, Swisdak et al., 2007, Pritchett et al., 2008.

Reconnection Rate in Turbulence



$$E_\times^{th.} = \sqrt{\frac{b_1^{3/2}\, b_2^{3/2}}{R_\mu\, \ell}}$$

- Asymmetric Sweet-Parker model organizes data
  - Limited to coherent current sheets
    - Faster R. Rates
  - Turbulence determines SP parameters
    - B_{up}
    - ℓ

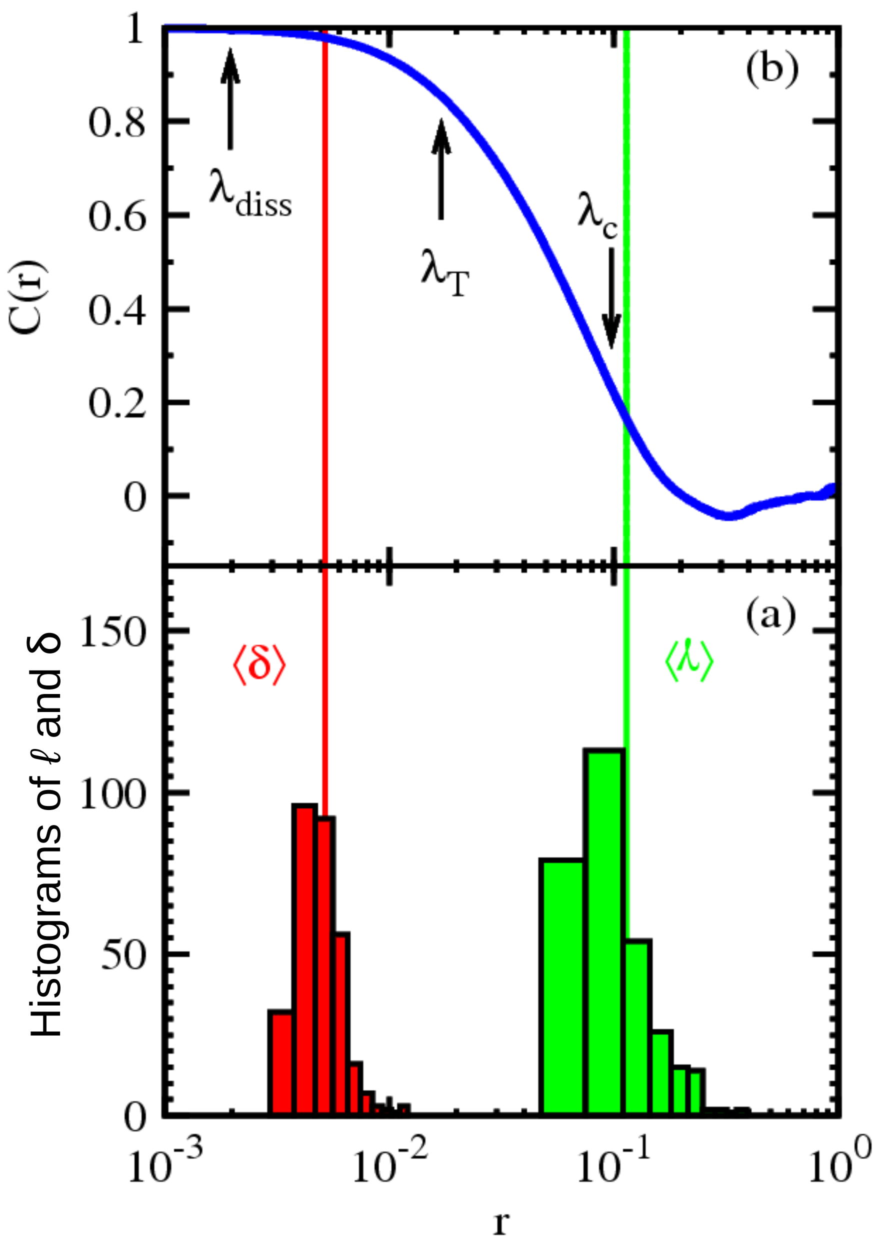
Servidio *et al.*, Phys Rev. Lett. (2009).

- Very Surprising:
  - Remember: $-\frac{\boldsymbol{v}}{c}\times\boldsymbol{B} > E_{\times}$
  - MHD smashing islands together much faster than reconnection.
  - Sweet-Parker assumes Steady-State!
    - Yet, somehow it works.

How can quasi-steady theory be valid?

- We have limited ourselves to very actively reconnecting islands.
- What does it take to get “fast” reconnection?
  - Continuous pushing for a “long” time.
  - A quick bounce between islands won't do it.
  - Continuous pushing => “Quasi-steady” reconnection.
  -
- Examine time dependence of reconnection rates.

Link Between Reconnection and Turbulence



Autocorrelation function:

$$C(\boldsymbol{r}) = \frac{\langle \boldsymbol{b}(\boldsymbol{x}+\boldsymbol{r})\cdot\boldsymbol{b}(\boldsymbol{x})\rangle}{\langle b^2\rangle}$$

Characteristic lengths in turbulence and in reconnection:

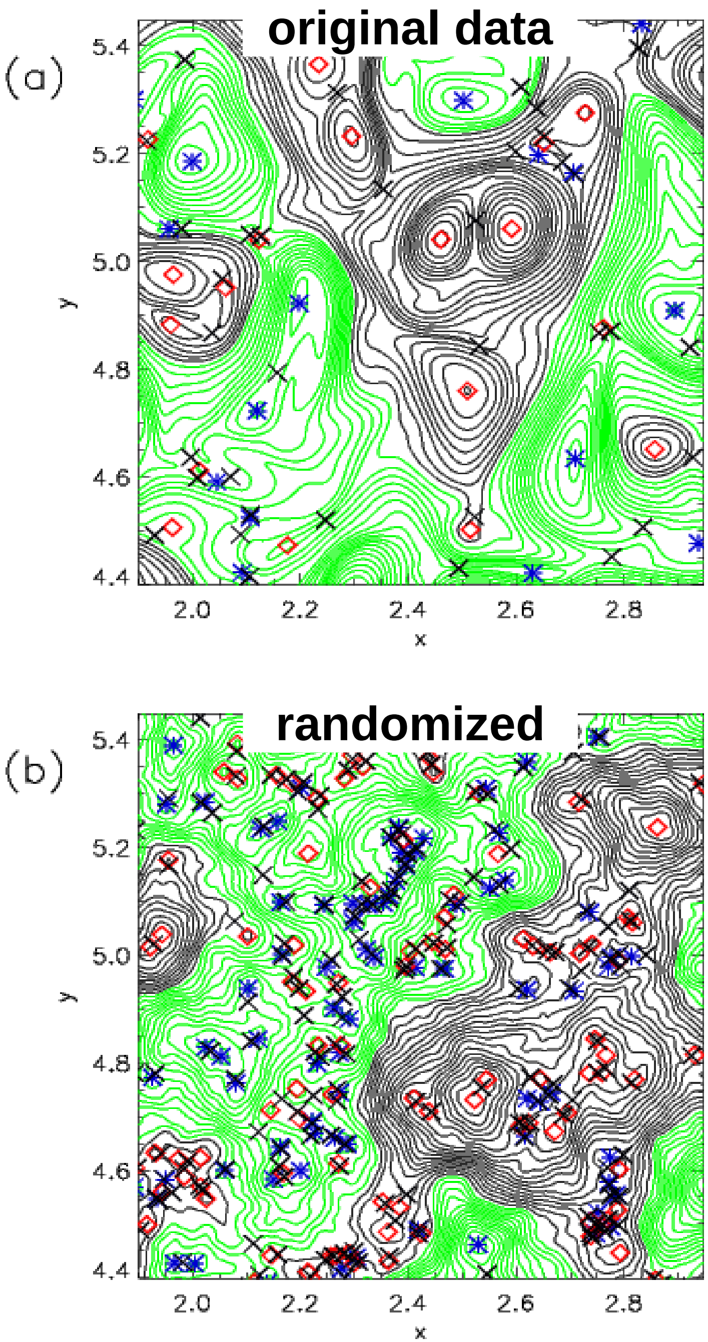
$$\lambda_C = \int C(r)\,dr$$

$$\lambda_T = \sqrt{\frac{\langle b^2\rangle}{\langle j^2\rangle}} \quad \Longleftrightarrow \quad \ell, \delta$$

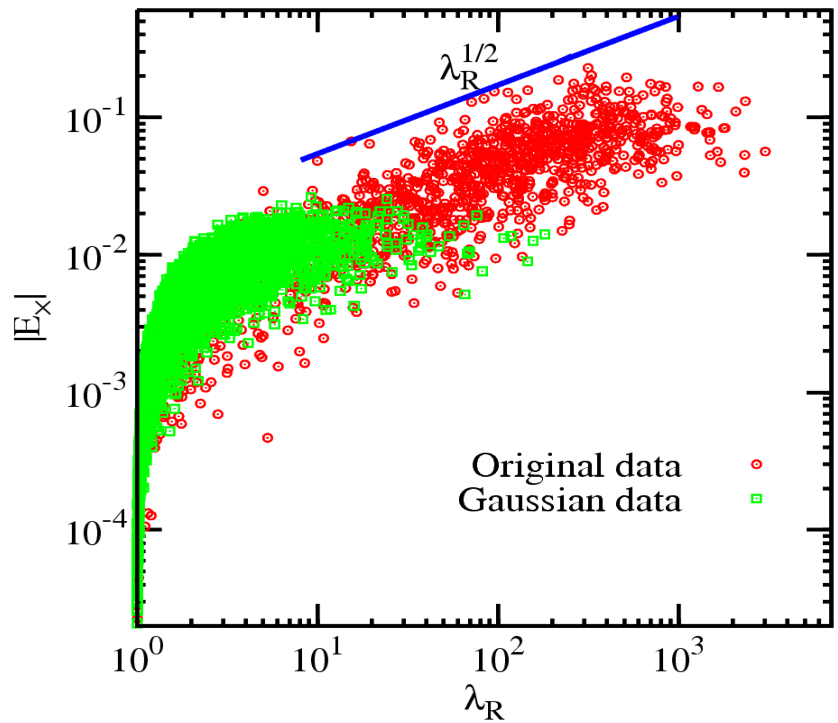
$$\lambda_{diss} = R_\mu^{-\frac{1}{2}} \langle j^2\rangle^{-\frac{1}{4}}$$

- δ scales with the dissipation scale
- ℓ is on the order of the correlation length

Coherence Key for Reconnection



- Randomize a → slower reconnection



- Reconnection in turbulence not described by random phases.

Coherence/Randomness?

- What does it mean?
  - Current Sheets are critical
  - Randomizing phases destroys current sheets

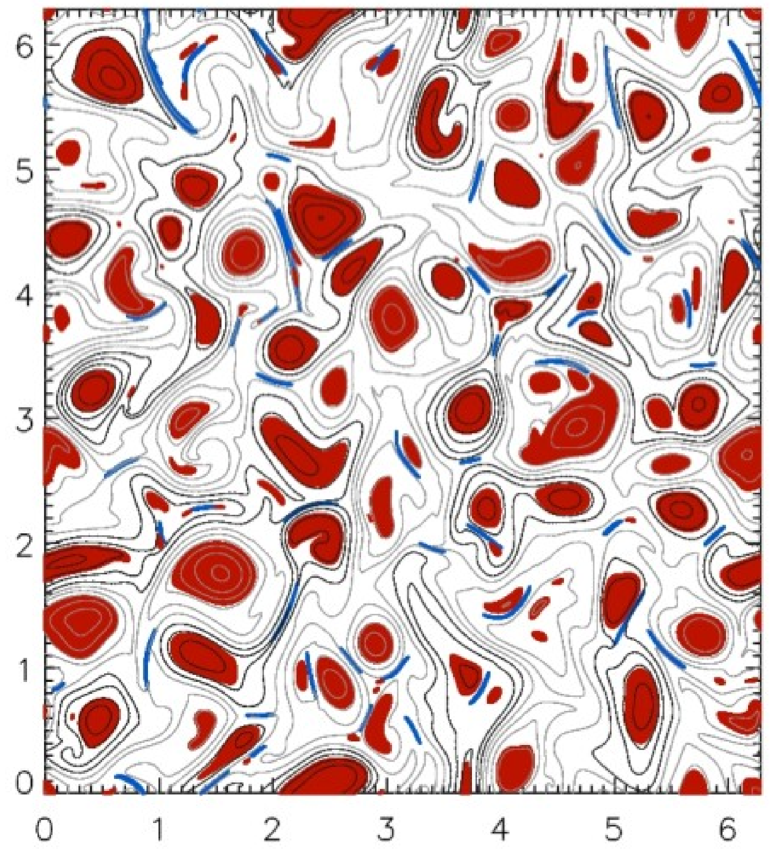
Color: J
Contours: a (gray: a>0, black: a<0)

Do We Care?

- So we need current sheets for the reconnection.
  - So?
- Key Question:
  - Are current sheets and magnetic reconnection critical to understand the dissipation in turbulence?
    - Duuuh. Yes (the reconnection guy says)
    - However
      - Dissipation in turbulence often characterized through wave analysis (random phases).

Dissipation in Current Sheets

- Find Current Sheets and Magnetic Islands
  - Cellular Automata techniques
- How much dissipation?



$\Gamma_i = Current\ Sheets$

$\chi_i = Magnetic\ Islands$

$\Omega = Everything\ else$

Determine Relative Dissipation

- Integrate resistive damping over the three regions.

$$W_{tot} = \int_0^{2\pi} \int_0^{2\pi} \eta\, j^2\, dx\, dy \qquad \langle W_{tot} \rangle = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \eta\, j^2\, dx\, dy$$

$$W_\Gamma = \sum_i \Gamma_i = \int_\Gamma \eta\, j^2\, dA \qquad \langle W_\Gamma \rangle = \frac{W_\Gamma}{A_\Gamma}$$

$$W_\chi = \sum_i \chi_i = \int_\chi \eta\, j^2\, d\Sigma \qquad \langle W_\chi \rangle = \frac{W_\chi}{A_\chi}$$

$$W_\Omega = \int_\Omega \eta\, j^2\, dA \qquad \langle W_\Omega \rangle = \frac{W_\Omega}{A_\Omega}$$

$A_\Gamma = Area\ of\ islands$

$A_\chi = Area\ of\ current\ sheets$

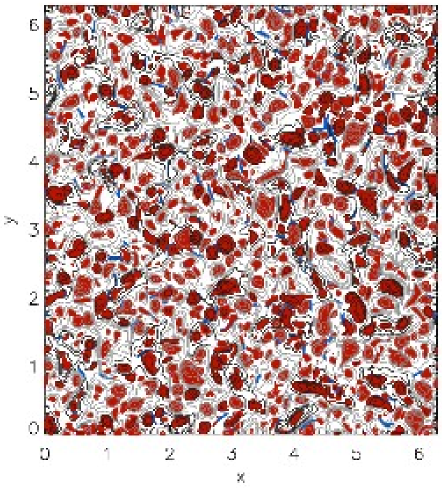
$A_\Omega = Everything\ else$

Key Quantities: W_χ / W_{tot}, W_Γ / W_{tot}, W_Ω / W_{tot}

One Case

RUN VI

- Resolution 4096^2
- Initial spectrum 5<=k<=30
- Viscosity (resistivity) ~ 0.0004
- Current peak=0.148
- Statistics -> t=0.2
- Input data-> type 1, REAL
- D_CURRENT_MAX=0.007, D_CURRENT_WDT=0.03
- Ez- limit = 0.02
- dlamblim = 150.d0
- Kfilt -> nofilt
- Cellular automata AZ-> eps=0.3 ; eps2 = 0.01; eps_RIP = 0.1
- Cellular automata JZ-> epsj=0.3



Results

$$W_{tot}=35.85, \quad \langle W_{tot}\rangle=0.91,$$
$$W_{\Gamma}=6.35, \quad \langle W_{\Gamma}\rangle=0.73,$$
$$W_{\chi}=4.86, \quad \langle W_{\chi}\rangle=22.16,$$
$$W_{\Omega}=24.65, \quad \langle W_{\Omega}\rangle=0.81$$

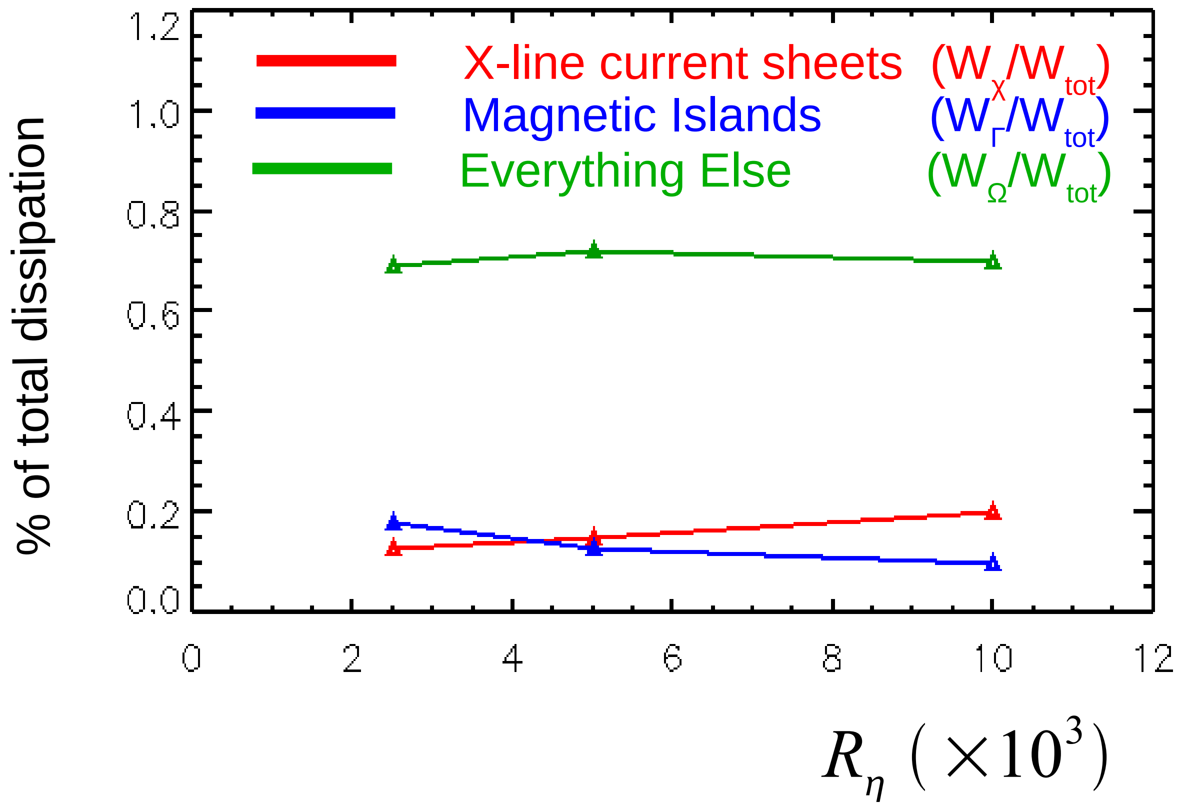
$$W_{\Gamma}/W_{tot}=0.18$$
$$W_{\chi}/W_{tot}=0.13$$
$$W_{\Omega}/W_{tot}=0.69$$

- Only 13% of damping in current sheets!
  - Wait! How does it scale?

Tentative Scaling

- Resolution (4096^2, 8192^2, 16384^2)
- Initial spectrum: 5 <= k <= 30
- Viscocity/Resistivity = (4e-4, 2e-4, 1e-4)

- X-line current dissipation increases with R_η
- Realistic system
  - R_η much larger
- Problem?
  - Will "Everything Else" decrease?



Only Scratched the Surface

- Velocities
  - Properties
  - Viscous damping?
- Dynamic time behavior of x-lines?
  - Typical time scale for reconnection?
    - Onset, fast, decay
    - Quasi-steady assumption okay?
- Collisionless plasma
  - Hall term
- Three-dimensional simulations

Conclusions

- Self-organization processes in turbulence produce coherent current sheets
- Hessian analysis of extrema
  - Broad range of reconnection rates
- Asymmetric Sweet-Parker analysis
  - Organizes coherent x-line current sheets
  - Very surprising (Quasi-steady theory works!)
- Robustly Reconnecting current sheets are strongly coherent
  - Random phase approximation not necessarily valid.
- Role of X-line current sheet dissipation
  - % dissipation increases with Lundquist