Null-point reconnection in fluid pair plasma without anomalous resistivity

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Abstract

We demonstrate fast rates of magnetic reconnection near a magnetic null point in a fluid model of collisionless pair plasma without the use of resistivity. In particular, we demonstrate a reconnection rate roughly half the rate demonstrated in particle simulations in an anisotropic adiabatic two-fluid model of collisionless pair plasma with relaxation toward isotropy, for a broad range of isotropization rates. For very rapid isotropization we see fast reconnection, but instabilities eventually arise that cause numerical error and cast doubt on the simulated behavior. We give evidence that numerical methods that use isotropic pressure require either physical or numerical anomalous resistivity to sustain fast rates of reconnection.

GEM challenge problem

The GEM problem[a] initiates reconnection by pinching adjacent oppositely directed field lines from their equilibrium state. The original study identified the Hall effect as critical for fast reconnection, prompting study of reconnection in pair plasma (for which the Hall term vanishes).

BIRN ET AL.: GEM RECONNECTION CHALLENGE

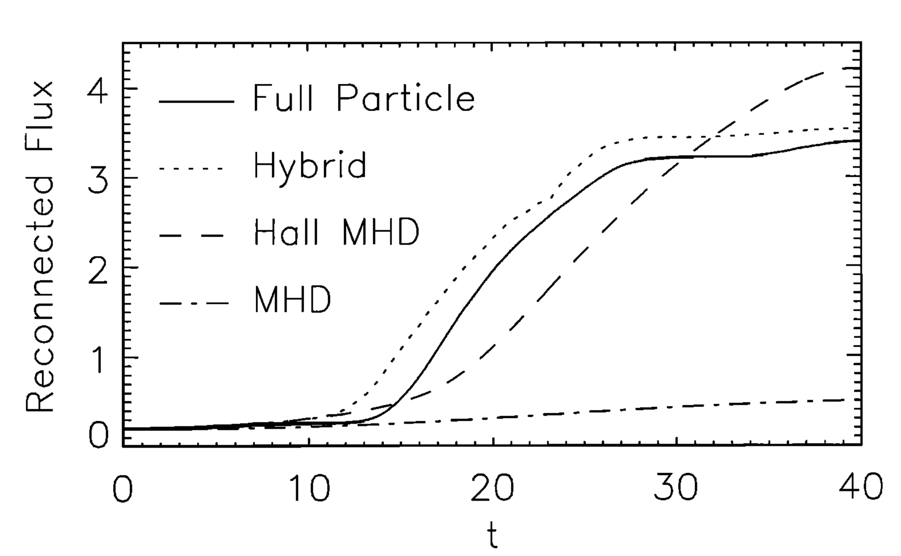
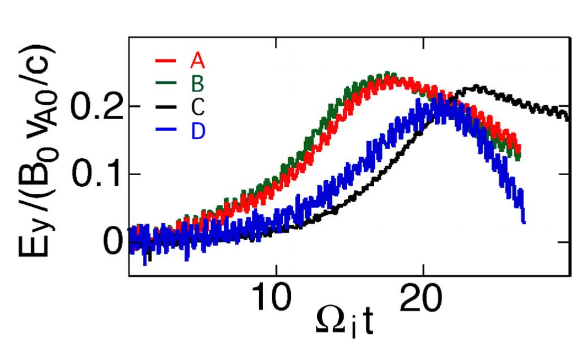


Figure 1. The reconnected magnetic flux versus time from a variety of simulation models: full particle, hybrid, Hall MHD, and MHD (for resistivity $\eta = 0.005$).

In 2005 Bessho and Bhattacharjee simulated fast reconnection in a pair plasma version of the GEM problem using a kinetic model.[b]



(Red curve is rate of reconnection for GEM-like pair plasma problem.)

[a]Birn et al, *Geospace environmental modeling (GEM) magnetic reconnection challenge*, Journal of Geophysical Research—Space Physics, 106:3715–3719, 2001.

[b]N. Bessho and A. Bhattacharjee, *Collisionless reconnection in an electron-positron plasma*, Phys. Rev. Letters, 95:245001, December 2005.

Theory for the GEM problem

At the X-point the momentum equation for one species says that the out-of-plane component of the electric field (i.e. the rate of reconnection) is the sum of a *resistive term*, a (nongyrotropic[a]) *pressure term*, and an *inertial term*:

$$\mathbf{E}_3(0) = \left[\frac{-\mathbf{R}_i}{en_i} + \frac{\nabla \cdot \mathbb{P}_i}{en_i} + \frac{m_i}{e}\partial_t \mathbf{u}_i \right]_3 \bigg|_{\text{null-point}} .$$

So reconnection must be provided by:

1. the *pressure* term for *steady-state* reconnection without resistivity,
2. the *resistive* term for *steady-state* reconnection in *gyrotropic* plasma, and
3. the *inertial* term for a *gyrotropic* plasma without resistivity (i.e. each species velocity at the origin should track exactly with reconnected flux).

[a]M. Hesse, M. Kuznetsova, and J. Birn, *The role of electron heat flux in guide-field magnetic reconnection*, Physics of Plasmas, 11(12):5387–5397, 2004.

Fluid models of pair plasma

Magnetized pair plasma. Chacon et al.[a] obtained an analytical fluid model for fast reconnection in magnetized pair plasma (no null point). Viscosity provides the needed pressure anisotropy.

Unmagnetized pair plasma. One can "cook up" the reconnection desired by defining the resistive term appropriately. One seeks the simplest formula for this *anomalous resistivity* that matches the broadest range of conditions. One-fluid models (i.e. MHD) make no assumption about mass ratio and can give fast reconnection when equipped with an anomalous resisivity. Zenitani et al.[b] have simulated fast reconnection in the vicinity of a null point with two-fluid five-moment models of collisionless pair plasma using anomalous resistivity. We seek a parsimonious fluid model based on simple physical assumptions rather than problem-specific simulation results.

[a]L. Chacón, Andrei N. Simakov, V.S. Lukin, and A. Zocco, *Fast reconnection in nonrelativistic 2D electron-positron plasmas* Phys. Rev. Letters, 101:025003, July 2008.

[b]S. Zenitani, M. Hesse, and A. Klimas. Two-fluid magnetohydrodynamic simulations of relativistic magnetic reconnection. *The Astrophysical Journal*, 696:1385–1401, May 2009.

Five- and ten-moment models

Hakim et al. simulated the GEM problem using two-fluid adiabatic models with five moments for the electron fluid and five[a] or ten[b] moments for the ion fluid.

The isotropic five-moment model cannot reconnect without (numerical, anomalous) resistivity. The ten-moment model fails to (reliably) reconnect due to undamped oscillatory exchange between the inertial and pressure terms of Ohm's law. The five-moment model is the ten-moment model instantaneously relaxed to isotropy. We get an intermediate model which avoids both problems by slowing down the rate of isotropization.

[a]A. Hakim, J. Loverich, and U. Shumlak. *A high-resolution wave propagation scheme for ideal two-fluid plasma equations*, J. Comp. Phys., 219:418–442, 2006.

[b]A.H. Hakim, *Extended MHD modelling with the ten-moment equations*, J. Fusion Energy, 27(1–2):36–43, June 2007.

Ten-moment two-fluid model

Generic physical equations for the ten-moment two-fluid model are: (1) conservation of mass and momentum and pressure tensor evolution for each species:

$$\partial_t \rho_s + \nabla \cdot (\rho_s \mathbf{u}_s) = 0,$$

$$\partial_t(\rho_s \mathbf{u}_s) + \nabla \cdot (\rho_s \mathbf{u}_s \otimes \mathbf{u}_s + \mathbb{P}_s) = \frac{q_s}{m_s}\rho_s(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + \mathbf{R}_s,$$

$$\partial_t \mathbb{P}_s + 3\nabla \cdot (\mathbf{u}_s \vee \mathbb{P}_s) + \nabla \cdot \mathbb{Q}_s = 2\,\mathrm{Sym}\,(\frac{q_s}{m_s}\mathbb{P}_s \times \mathbf{B}) + \mathbb{R}_s,$$

and (2) Maxwell's equations for evolution of electromagnetic field:

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0, \qquad \nabla \cdot \mathbf{B} = 0,$$

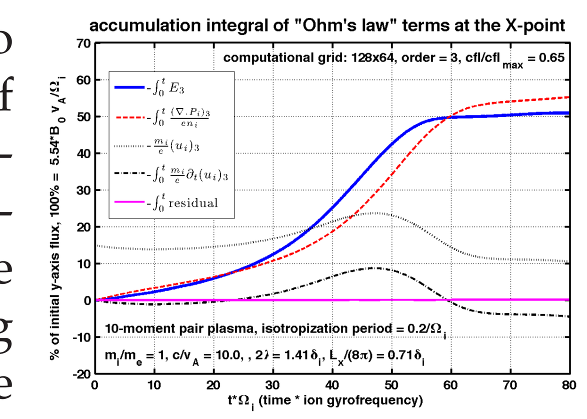
$$\partial_t \mathbf{E} - c^2 \nabla \times \mathbf{B} = -\mathbf{J}/\epsilon, \qquad \nabla \cdot \mathbf{E} = \sigma/\epsilon.$$

We assume that $\mathbf{R}_s = 0$, and to provide for isotropization we let $\mathbb{R}_s = \frac{1}{\tau_s}\left(\frac{1}{3}(\mathrm{tr}\,\mathbb{P}_s)\mathbb{I} - \mathbb{P}_s\right)$, which respects the entropy $s := \log(\det(\mathbb{P}^{-1})/\rho^5)$.[a] In this work we set $\mathbb{Q}_s = 0$. For conservation and shock-capturing purposes we evolve the *energy* tensor $\mathbb{E}_s := \mathbb{P}_s + \rho_s \mathbf{u}_s \mathbf{u}_s$ rather than the pressure tensor. We implemented an explicit third-order discontinuous Galerkin two-fluid solver.

[a]C. David Levermore, *Kinetic theory, Gaussian moment closures, and fluid approximations*, talk presented at IPAM KT2009 Culminating Retreat, Lake Arrowhead, California, June 2009.

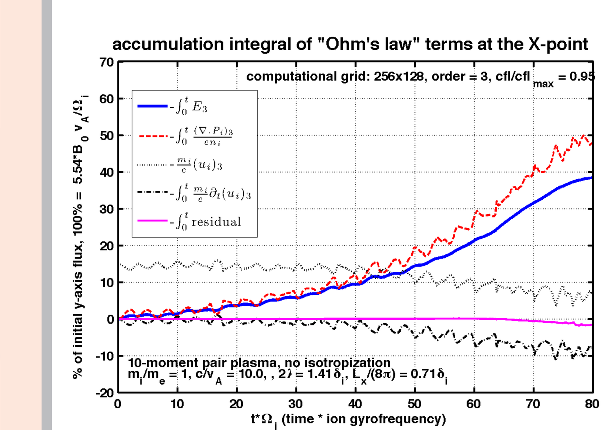
GEM pair plasma simulations

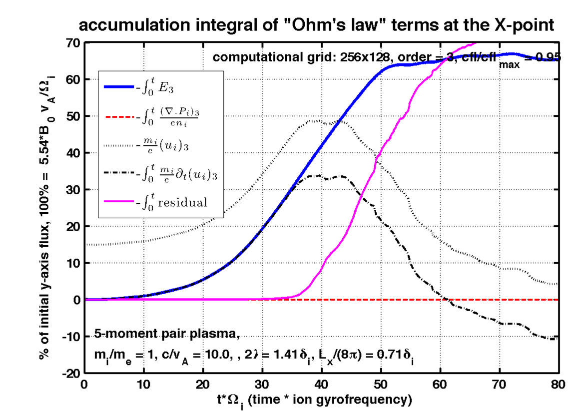
Rescaled problem. To avoid the formation of magnetic islands, we modified (Bessho and Bhattacharjee's version of) the GEM problem, shrinking the domain and the particle mass to half their original values.



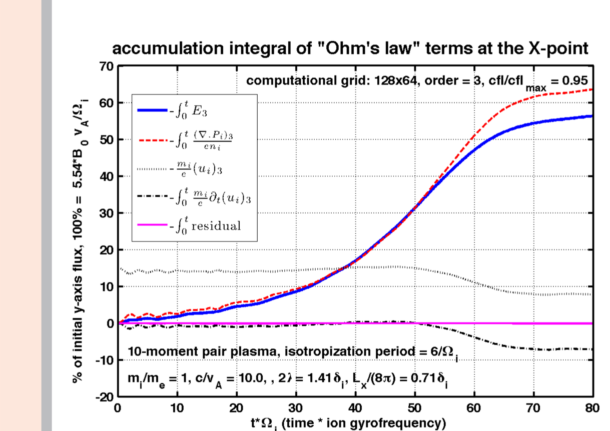
In the five-moment model the inertial term initially tracks with reconnected flux, but eventually instability kicks in and numerical anomalous resistivity instead supplies reconnection:

The hyperbolic ten-moment model exhibits undamped oscillatory exchange between the pressure and inertial terms and requires high resolution for convergence:

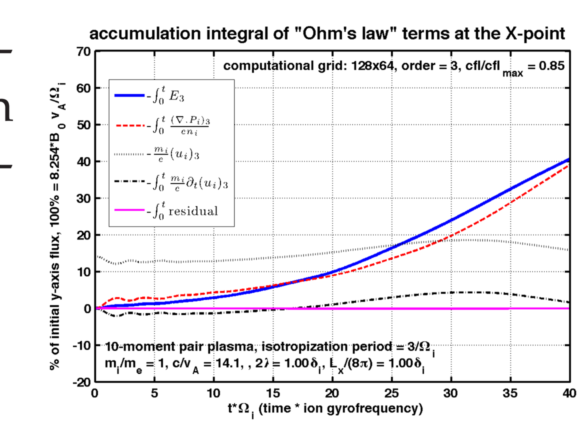


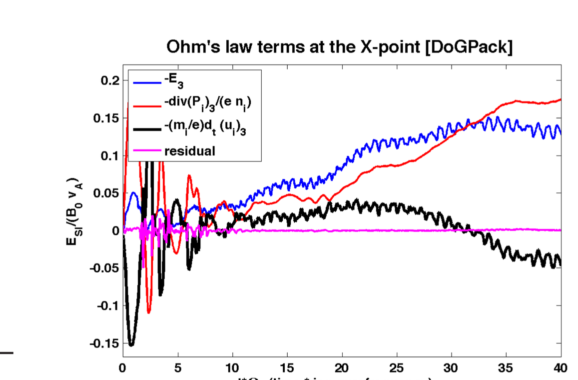


Isotropization dampens the oscillatory exchange between the pressure and inertial terms. The pressure term supplies reconnection, in agreement with theory and PIC simulations:



Original settings. To compare with Bessho and Bhattacharjee, we also used their settings. Our peak rate of reconnection was about 60% of theirs:





For fast isotropization the inertial term begins to supply some initial reconnection, although the pressure term is the ultimate supplier:

We are currently investigating nonzero heat flux closures to try to increase the reconnection rate.