Evidence for the Nonlinear Transport of Galactic Cosmic Rays

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Abstract

The measured decrease of the abundance ratio of secondary to primary Galactic cosmic ray nuclei is explained by the increase of the cosmic ray parallel mean free path with magnetic rigidity $\lambda_\parallel \propto R^{0.6}$ that results from the weakly nonlinear transport theory of cosmic rays in the turbulent Galactic magnetic fields. Because the ratio of fluctuating to ordered magnetic fields in the Galaxy $\delta B/B_0$ is large, this nonlinear transport theory has to be favored over the traditional quasilinear theory that requires very small values $\delta B/B_0 \ll 1$. Our explanation provides an alternative to Galactic transport theories of cosmic rays with significant distributed stochastic acceleration.

Supported by the DFG through SFB 591.

Subject headings: cosmic rays – diffusion – turbulence

Introduction

The observed ratio of secondary to primary cosmic ray nuclei indicates that primary cosmic rays at relativistic energies penetrate a total column density of matter $X = n_0 \tau v$ during their residence time τ in the Galaxy, where n_0 is the average density of the interstellar gas and v is the cosmic ray velocity. For diffusive propagation the mean residence time τ can be expressed by the system size (thickness of Galactic disk) L and the parallel spatial diffusion coefficient $\kappa_\parallel$ as $\tau = L^2/\kappa_\parallel$, so that

$$X(R) = \frac{n_0 v L^2}{\kappa_\parallel(R)} = \frac{3 n_0 L^2}{\lambda_\parallel(R)} \propto \lambda_\parallel^{-1} \tag{1}$$

where we used the parallel mean free path $\lambda_\parallel = 3\kappa_\parallel/v$. The measured decrease of the abundance ratio of secondary to primary cosmic ray nuclei as B/C and N/O at kinetic energies above 1 GeV/nucleon, implies a variation of the total column density as a function of rigidity R as (Swordy et al. 1990)

$$X(R) = 6.9 \left(R/[20GV/nucleon]\right)^{-a} \text{ g cm}^{-2}, \quad a = 0.6 \pm 0.1. \tag{2}$$

The rigidity dependence and therefore the parameter a is controlled by the rigidity dependence of the inverse parallel mean free path $X \propto \lambda_\parallel^{-1}$.

⇒ A theoretical explanation of

$$\lambda_\parallel \propto R^{0.6 \pm 0.1} \tag{3}$$

must be found. In plain diffusion or leaky-box transport models without distributed stochastic acceleration the implied value of $a = 0.6$ is not accord with the prediction of the quasilinear theory (e.g. Schlickeiser & Miller 1998, Schlickeiser 2002) with a Kolmogorov power spectrum of magnetic fluctuations $g(k) \propto k^{-5/3}$:

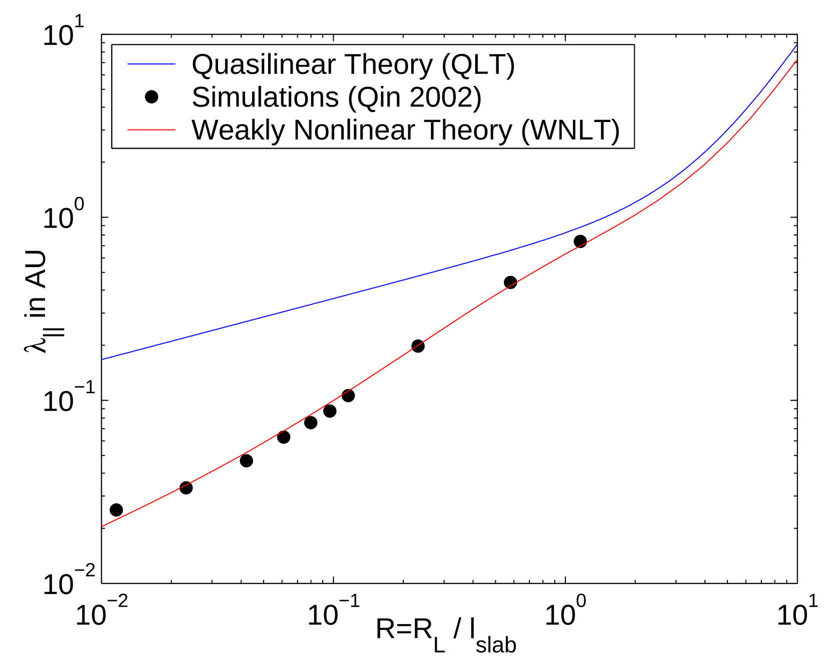
$$\lambda_\parallel^{QLT} \propto R^{1/3} \tag{4}$$

For this reason models with distributed stochastic acceleration have been favored (e.g. Jones et al. 2001) despite the disagreement of the implied weak energy dependence of the secondary-to-primary ratios at energies ≥ 20 GeV/nucleon with the high-energy HEAO-3 data by Binns et al. (1981) on the sub-Fe/Fe ratio.

It is the purpose of this poster to offer an alternative explanation of the rigidity dependence of the secondary-to-primary ratio. We demonstrate that the increase of the cosmic ray spatial diffusion coefficient at relativistic rigidities $\lambda_\parallel \propto R^{0.6}$ results from the improved weakly nonlinear theory (WNLT) of Galactic cosmic rays in the partially turbulent Galactic magnetic field with a Kolmogorov power spectrum of magnetic fluctuations.

The weakly nonlinear theory

Recently proposed numerical test particle simulations (Qin 2002, for a review see Minnie 2002) have shown that nonlinear effects are essential if the parallel mean free path is calculated (see Fig. 1).



WNLT-results (red line) in comparison with QLT-results (blue line) and simulations (Qin 2002, dots) for the parallel mean free path. Results are for 20% slab/80% 2D geometry and heliospheric parameters.
⇒ WNLT can reproduce test particle simulations!

Therefore a nonlinear theory for cosmic ray transport was derived to describe particle transport in agreement with simulations. The WNLT (weakly nonlinear theory) of Shalchi et al. 2004 is based on the quasilinear formulation but nonlinear effects were included. By the formal replacement

$$\pi\delta\left(k_\parallel v_\parallel + n\Omega\right) \to \frac{D_\perp k_\perp^2 + \omega}{\left(D_\perp k_\perp^2 + \omega\right)^2 + \left(k_\parallel v_\parallel + n\Omega\right)^2} \tag{5}$$

with

$$\omega = \begin{cases} \frac{2D_{\mu\mu}}{1-\mu^2} & \text{for perpendicular diffusion} \\ 0 & \text{for pitch-angle diffusion.} \end{cases} \tag{6}$$

we can substitute the sharp delta function of quasilinear theory (QLT, Jokipii 1966) by the Breit-Wigner resonance function of WNLT. Resonance-broadening in WNLT arises from pitch-angle diffusion, described by the Fokker-Planck coefficient $D_{\mu\mu}$ and perpendicular diffusion, described by the Fokker-Planck coefficient $D_\perp$. Thus, we have a coupled system of nonlinear Fokker-Planck coefficients within WNLT:

$$D_{\mu\mu} = D_{\mu\mu}(D_{\mu\mu}, D_\perp) \qquad D_\perp = D_\perp(D_{\mu\mu}, D_\perp) \tag{7}$$

The parallel mean free path can be calculated by applying the standard relation (see e.g. Schlickeiser 2002)

$$\frac{\lambda_\parallel}{l_{slab}} = \frac{3}{4}\int_0^1 d\mu \, \frac{(1-\mu^2)^2}{\tilde{D}_{\mu\mu}}. \tag{8}$$

To calculate diffusion coefficients theoretically the properties of the interstellar medium have to be specified.

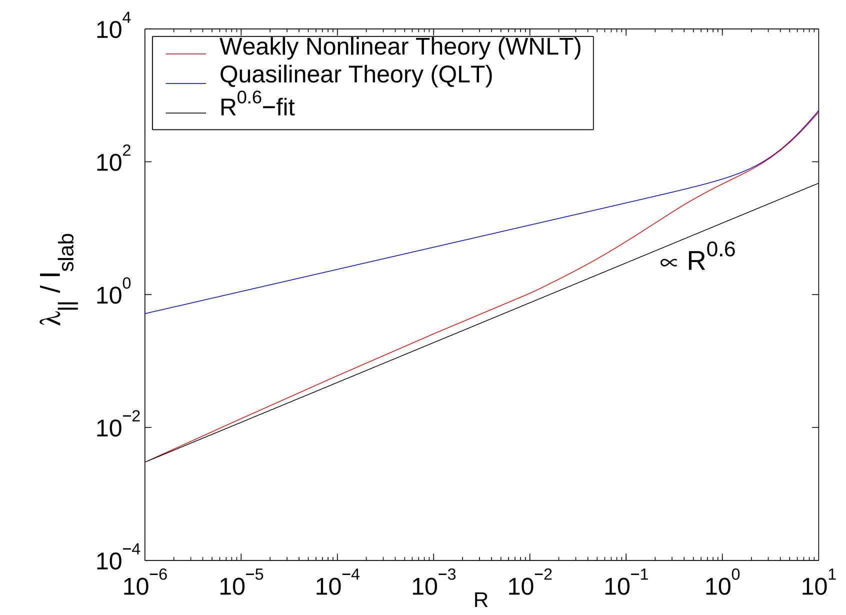
Our turbulence model:

- *Turbulence geometry:* slab/2D composite model
- *Wavespectrum:* $g(k) \sim (1 + k^2 l^2)^{-\nu}$ where we used the bendover scale l and the spectral index of the inertial range $2\nu = 5/3$
- *Time-dependence:* magnetostatic model where we neglect electric fields, plasma wave effects and dynamical turbulence effects

With WNLT it is straightforward to calculate the parallel mean free path (and the perpendicular mean free path) numerically.

Explanation of the secondary-to-primary ratio

Fig. 2 shows the parallel mean free path $\lambda_\parallel/l_{slab}$ as a function of the dimensionless rigidity $R = R_L/l_{slab}$ calculated with WNLT in comparison with QLT results.



The parallel mean free path devided by the slab-bendover-scale $\lambda_\parallel/l_{slab}$. We have shown WNLT-results (red line), QLT-results (blue line) and a $R^{0.6}$-fit (black line).

For our calculations we used parameters which should be appropriate for the interstellar medium:

Parameter	Symbol	Value
Intertial range spectral index	2ν	$5/3$
Slab-bendover-scale	l_{slab}	$2 \cdot 10^{18} cm$
2D-bendover-scale	l_{2D}	$0.1 \cdot l_{slab}$
Magnetic background field	B_0	$0.4nT$
Slab / 2D ratio	$\delta B_{slab}^2/\delta B_{2D}^2$	0.25
Turbulence strength	$\delta B^2/B_0^2$	0.5

(9)

⇒ The value $a = 0.6 \pm 0.1$ in Eq. (2) can by explained theoretically by the WNLT!

Therefore we come to the conclusion that within a nonlinear transport theory the abundance ratio of secondary to primary cosmic ray nuclei can be explained, which thus provides evidence for the presence of nonlinear transport of Galactic cosmic rays.

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The content of this poster is published in:
Shalchi, A. & Schlickeiser, R., 2005, ApJ 626, L97