Magnetic reconnection with anomalous resistivity in 2.5 dimensions

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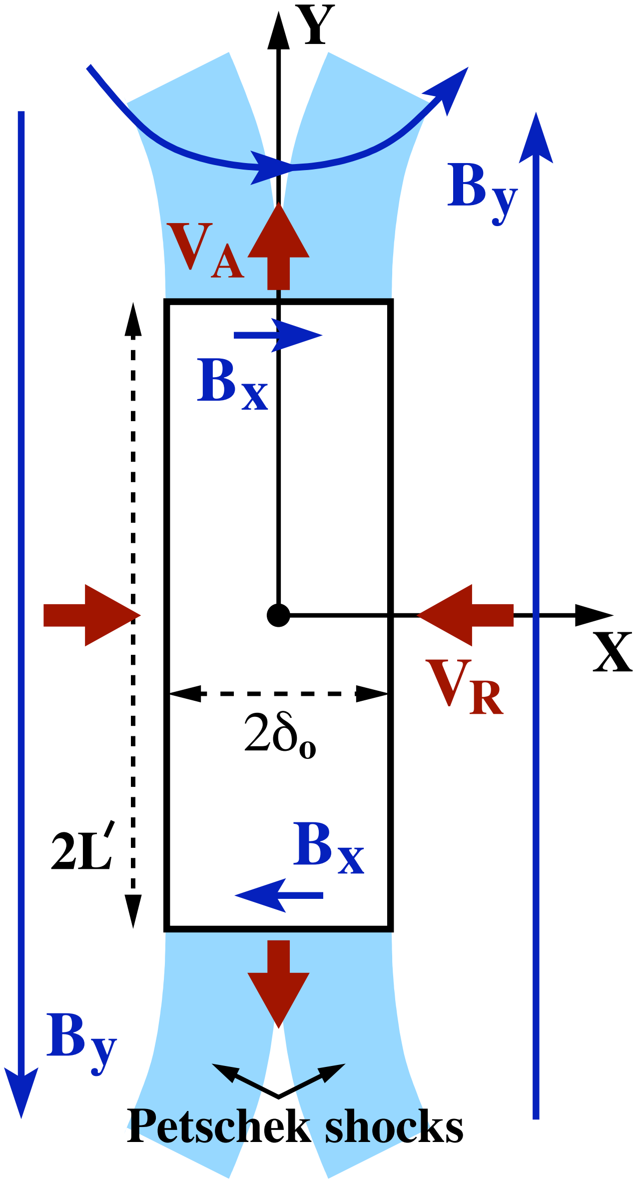
Outline

- Introduction to the problem and our assumptions
- The Sweet-Parker classical model
- A new, local-equations approach to the Sweet-Parker reconnection
- Reconnection with anomalous resistivity
- Summary and major results

Introduction to the problem & assumptions



In units $4\pi = 1$ and $c = 1$ the MHD equations are

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E} = \nabla\times([\mathbf{V}\times\mathbf{B}] - \eta\mathbf{j}),$$

$$\operatorname{div}\mathbf{B} = 0,$$

$$\rho(\mathbf{V}\cdot\nabla)\mathbf{V} = -\nabla\left(P + B^2/2\right) + (\mathbf{B}\cdot\nabla)\mathbf{B} + \rho\nu\nabla^2\mathbf{V}.$$

We assume

- resistive MHD applies;
- two-and-a-half-dimensional geometry, $\partial/\partial z \equiv 0$;
- symmetries of reconnection layer with respect to the x- and y-axes;
- large Lundquist number $\Longleftrightarrow$ negligible resistivity outside the layer;
- plasma incompressibility div $\mathbf{V} = 0$;
- quasi-stationarity $\Longleftrightarrow$ small reconnection rate $V_R/V_A \ll 1$,
  no plasma instabilities;
- thin reconnection layer $\delta_o/L' \ll 1$, equivalent to small reconnection rate $\delta_o/L' \approx V_R/V_A \ll 1$ in case $\nu \lesssim \eta$, otherwise a stronger assumption.

We do NOT make assumptions about

- functional form of resistivity, consider an arbitrary $\eta = \eta(j_z, x, y)$;
- value of the guide field B_z;
- value of plasma viscosity (except assumptions already made above).

2.5D assumption, $\partial/\partial z \equiv 0 \quad \Longrightarrow$

$$\partial A/\partial t = -E_z = V_x B_y - V_y B_x - \eta j_z .$$

Quasi-stationarity assumption, $\partial/\partial t \equiv 0 \quad \Longrightarrow$

$$\frac{\partial B_\alpha}{\partial t} = -\left[\mathrm{rot}\,\mathbf{E}\right]_\alpha \quad \Longrightarrow \quad \begin{aligned} \partial_x E_z &= \ \ \partial_t B_y \approx 0, \\ \partial_y E_z &= -\partial_t B_x \approx 0 \end{aligned} \quad \Longrightarrow \quad E_z \approx \text{const in space}.$$

Incompressibility assumption $\quad \Longrightarrow$

$$\partial_x V_x + \partial_y V_y = 0.$$

Unknowns are $j_0, \delta_0, V_R, (\partial_y V_y)_0$, and $(\partial_y B_x)_0$. Equations for them are:

$$j_0 = \frac{B_m}{\delta}; \quad \text{Ampere's law} \tag{1}$$

$$\eta_0 j_0 = \frac{B_m}{\delta}; \quad E_z = \text{constant} \tag{2}$$

$$\left(\frac{\partial V_y}{\partial y}\right)_0 = \frac{V_R}{\delta}; \quad \text{Incompressibility} \tag{3}$$

$$\rho \left(\frac{\partial V_y}{\partial y}\right)_0^2 = \left(\frac{\partial B_y}{\partial y}\right)_0 j_0; \quad \text{Bernoulli} \tag{4}$$

$$\frac{\partial^2}{\partial y^2}\left(V_x B_y - B_y B_x - \eta j_z\right) = 0; \quad \text{constant } E_z \tag{5}$$

$$\frac{\partial^2}{\partial y^2}(V_x B_y - B_y B_x - \eta j_z) = 0; \quad \text{constant } E_z \quad (6)$$

Or, for η constant, this becomes

$$-2\left(\frac{\partial V_y}{\partial y}\right)_0 \left(\frac{\partial B_x}{\partial y}\right)_0 + \eta_0 \left(\frac{\partial^2 j_z}{\partial y^2}\right)_0 = 0 \quad (7)$$

Then with

$$\left(\frac{\partial^2 j_z}{\partial y^2}\right)_0 = \frac{j_0}{L^2} \quad (8)$$

we get the Sweet Parker result.

$$V_A \sqrt{\frac{\eta_0}{V_A L}} \quad (9)$$

Sweet-Parker classical model

Constant resistivity $\eta = \text{const} = \eta_o$

We have the following equations:

(1) $j_z = (\nabla \times \mathbf{B})_z \;\Rightarrow\; j_o = (j_z)_o \approx B_m/\delta_o$

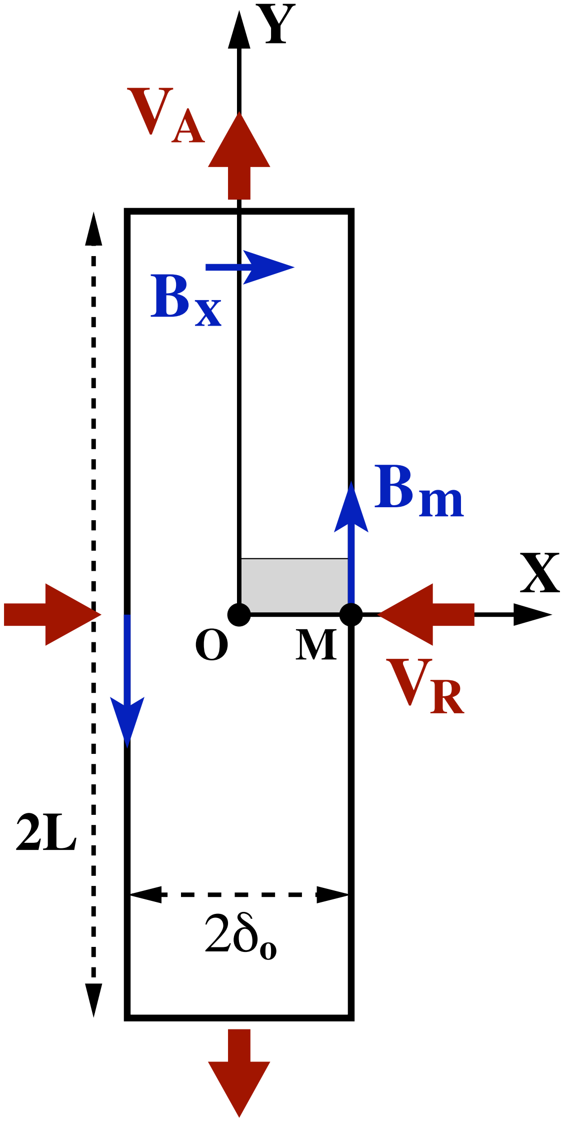
(2) $E_z = \text{const along x-axis} \;\Rightarrow\; \eta_o j_o \approx V_R B_m$

(3) incompressibility $\Rightarrow\; V_R L \approx V_{out}\delta_o$

(4) outflow acceleration $\Rightarrow$

$$\frac{\rho}{2}V_{out}^2 \approx B_x j_o L \approx \frac{\delta_o B_m}{2L}\frac{B_m}{\delta_o}L \approx \frac{B_m^2}{2} \approx \Delta P$$

$$\Rightarrow\; V_{out} \approx V_A \equiv B_m/\sqrt{\rho}$$

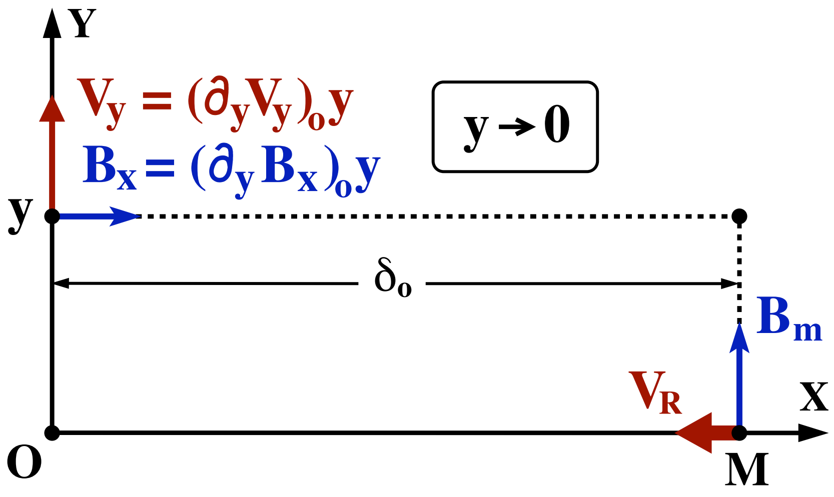


4 equations and 4 parameters $(j_o,\, \delta_o,\, V_R,\, V_{out})$

$\Rightarrow\; V_R \approx \sqrt{\eta_o/V_A L}$

Next consider the shaded infinitesimal region.

Local-equations approach to Sweet-Parker reconnection



Constant resistivity $\eta = \text{const} = \eta_o$

Local equations in region of size $\delta_o \times y$:

(1) $j_z = (\nabla \times \mathbf{B})_z \;\Rightarrow\; j_o \approx B_m/\delta_o$

(2) $E_z = \text{const along x} \;\Rightarrow\; \eta_o j_o \approx V_R B_m$

(3) local incompressibility $\Rightarrow$

$$V_R y \approx V_y \delta_o = (\partial_y V_y)_o y \delta_o \;\Rightarrow\; -(\partial_x V_x)_o = (\partial_y V_y)_o = V_R/\delta_o$$

(4) local outflow acceleration $\Rightarrow$ $\frac{\rho}{2}V_y^2 = \frac{\rho}{2}[(\partial_y V_y)_o y]^2 \approx \int_0^y B_x j_z dy =$

$$= \int_0^y (\partial_y B_x)_o y j_o dy = (\partial_y B_x)_o j_o y^2/2 \;\Rightarrow\; \rho(\partial_y V_y)_o^2 = (\partial_y B_x)_o j_o$$

(5) new local $E_z = \text{const along y-axis}$ $\Rightarrow$ $\partial_y^2(V_x B_y - V_y B_x - \eta j_z) = 0 \;\Rightarrow$

$$-2(\partial_y V_y)_o(\partial_y B_x)_o - \eta_o(\partial_y^2 j_z)_o \approx -2(\partial_y V_y)_o(\partial_y B_x)_o + 2\eta_o j_o/L^2 = 0$$

5 eqs. & 5 parameters $[j_o,\, \delta_o,\, V_R,\, (\partial_y V_y)_o,\, (\partial_y B_x)_o] \;\Rightarrow\; V_R \approx V_A\sqrt{\eta_o/V_A L}$

Reconnection with anomalous resistivity

Anomalous resistivity $\eta = \eta(j_z, x, y)$:

- outflow acceleration equation is modified to include $P + B_z^2/2$ and ν;
- $E_z =$const along y-axis equation is modified to include $\eta \neq$ const.
- final equations are

$$3 + \frac{j_o(\partial_{j_z}\eta)_o}{\eta_o} + \frac{B_m(\partial_y^2\eta)_o}{\eta_o(\partial_y^2 B_y)_m} \approx -\left(1 + \frac{\nu}{\eta_o}\right) \frac{\eta_o^2 j_o^4}{V_A^2 B_m^4} \frac{2B_m}{(\partial_y^2 B_y)_m},$$

$$V_R \approx \eta_o j_o/B_m \ll V_A, \quad \delta_o \approx B_m/j_o, \quad L^2 \equiv -2B_m/(\partial_y^2 B_y)_m.$$

Special cases ($\boldsymbol{\nu = 0}$):

- $\eta = \text{const} = \eta_o$ ⇒ Sweet-Parker reconnection $V_R/V_A \approx \sqrt{\eta_o/V_A L}$;
- $\eta = \eta(j_z)$ and $(d\eta/dj_z)_o \gg \eta_o/j_o$ ⇒ Petschek-Kulsrud reconnection
  ⇒ $V_R/V_A \approx \left[(B_m/V_A L^2)(d\eta/dj_z)_o\right]^{1/3}$;
- $l_\eta^2 = -2\eta_o/(\partial_y^2\eta)_o \ll L^2$ ⇒ $V_R/V_A \approx \sqrt{\eta_o/V_A l_\eta}$

Summary and major results

- A new theoretical approach for calculation of the reconnection rate is based on local analytical derivations in a thin reconnection layer.
- Able to calculate approximate reconnection rate for an arbitrary anomalous resistivity and with few assumptions made.
- A quasi-stationary reconnection rate is fully determined by a particular functional form of the anomalous resistivity and by the local configuration of the magnetic field just outside the reconnection layer.
- Unforced (free) and forced reconnection is similar & are treated equally:
  - ▷ unforced $\longrightarrow$ given outside field B_m, find reconnection velocity V_R;
  - ▷ forced $\longrightarrow$ given V_R, field B_m gets piled up until equilibrium.
- In case of constant resistivity reconnection is Sweet-Parker, not Petschek.
- When resistivity is anomalous, then reconnection can be much faster than the Sweet-Parker reconnection (simulations supported by CMRS).