Momentum transport from tearing modes

Fatima Ebrahimi

S. C. Prager, V. V. Mirnov and C. R. Sovinec

University of Wisconsin-Madison

Center for Magnetic Self Organization in Laboratory and Astrophysical Plasmas

Motivations

- Both in laboratory and astrophysical plasmas, there are situations in which momentum transport and flow phenomena can not be explained by classical processes.
- Magnetic fluctuations arising from MHD instabilities are believed to have an important role in transporting momentum.
- In astrophysical plasmas, momentum is believed to be transported due to flow-driven instabilities (MRI).
- Here, we investigate whether current-driven reconnection can transport momentum.
- Is there correlation between the anomalous momentum transport and the tearing fluctuations?
- Whether spontaneous and/or driven reconnection can cause momentum transport?

Outline

- Momentum transport from a single tearing mode
  - Analytical calculations of quasilinear $< \widetilde{J} \times \widetilde{B} >$ torque in the presence of mean flow
  - Computation of $< \widetilde{J} \times \widetilde{B} >$ torque
- Momentum transport from multiple tearing modes – role of coupled spontaneous and driven reconnection

Summary

Single tearing mode

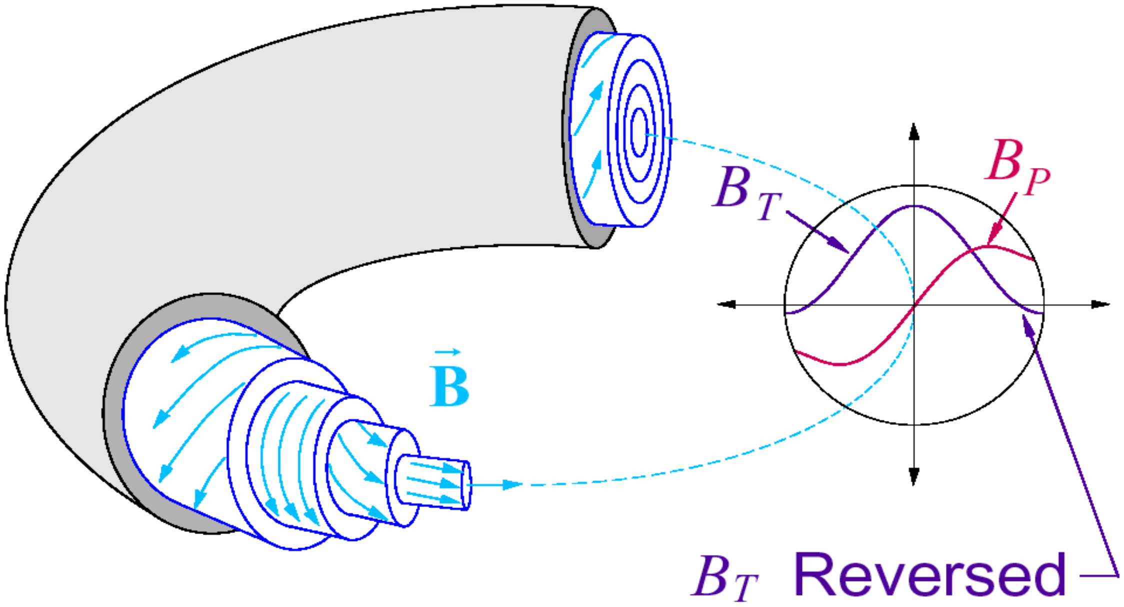
- In the absence of mean flow
  - $< \widetilde{J} \times \widetilde{B} > = 0$
- In the presence of mean flow
  - In the outer region, $< \widetilde{J} \times \widetilde{B} > \approx 0$
  - In the inner layer, $< \widetilde{J} \times \widetilde{B} > \neq 0$

Multiple tearing modes

- $< \widetilde{J} \times \widetilde{B} > \neq 0$

  nonlinear mode coupling enhances momentum transport.

Plasma flow is affected by the MHD turbulence in lab plasmas.



- Tearing fluctuations (about a few % of mean field) play an important role in sustaining the RFP configuration (e.g. current transport, sawteeth oscillations).

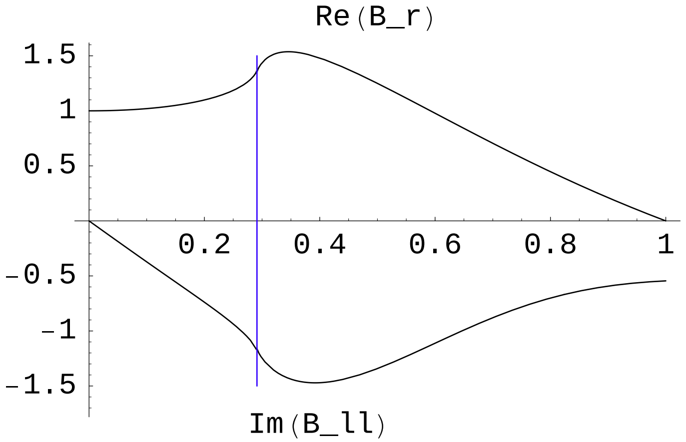
Experimental observations

- There is large mean flow (about a few percent of V_A) in RFP.
- The flow decelerations in RFP are coherent with the relaxation events (sawteeth oscillations).

Analytical calculations of quasilinear torque

The outer solutions are modified by the mean flow profile.

- In the absence of mean flow
  - The linearized outer solutions are obtained from Newcomb equation.



  - $(\widetilde{J} \times \widetilde{B})_{||} = \frac{1}{r}\frac{\partial}{\partial r} < r\widetilde{B}_{||}\widetilde{B}_r > = 0$
  - $\widetilde{B}_{||}$ and $\widetilde{B}_r$ are out of phase.

- In the presence of mean flow

- We solve linearized outer equations with mean flow;

Q(r, θ,z,t)=Q(r) exp(i ω t + i (m θ + k z))

$\mathbf{B} = B_\theta(r)\mathbf{e}_\theta + B_z(r)\mathbf{e}_z$ $\mathbf{V}_0 = V_z(r)\mathbf{e}_z$

$$\boxed{(r\widetilde{V}_r)'' + c1(r\widetilde{V}_r)' + c2(r\widetilde{V}_r) = 0}$$

The outer solutions are modified by the mean flow profile.

$$c1 = \frac{1}{g}\frac{dg}{dr} - \frac{kV_z'}{\Omega}$$

$$c2 = \frac{1}{g}[\frac{d}{dr}(\frac{kV_z'g}{\Omega}) + \frac{2k^2B_\theta V_z'G}{\Omega^2 H} - \frac{d}{dr}(\frac{2mB_\theta F}{r\Omega H})$$
$$+ \frac{2B_\theta}{r\Omega}(\frac{B_\theta}{r})' + \frac{4k^2B_\theta^2}{r\Omega(1-M^2)H} - \frac{F^2(1-M^2)}{r\Omega} - \frac{2B_\theta^2 kV_z'}{r^2\Omega^2}]$$

$$g = \frac{rF^2(1-M^2)}{\Omega H}, \qquad H = m^2 + k^2r^2$$

$$F = \frac{mB_\theta}{r} + kB_z, \qquad G = -\frac{mB_z}{r} + kB_\theta$$

Earlier studies by Gatto, Terry and Hegna (2002)

The single mode Lorentz force is small in the outer region.

$$\widetilde{B}_{||} = i(1 - M^2)\widetilde{B}_{||}^{(0)}$$

$$\widetilde{B}_{||}^{(0)} = \left[\frac{Fr(r\widetilde{B}_r)' - rG\lambda(r\widetilde{B}_r)}{B(m^2 + k_z^2 r^2)}\right] = \text{solution without flow}$$

$$M = \sqrt{\rho}\frac{(\omega + kV_z)}{K \cdot B} \rightarrow \text{Machnumber}$$

$$\omega = \omega_r + i\gamma, \qquad \omega_r = -kV_z|_{(r=r_s)}$$

The single mode Lorentz term is small due to the small growth rate in the outer region.

$$\boxed{(\widetilde{J} \times \widetilde{B})_{||} = \frac{4\gamma}{r}\left[\rho\frac{(\omega_r + kV_z)}{(K \cdot B)^2} r\widetilde{B}_{||}^{(0)}\widetilde{B}_r\right]'}$$

The inner layer equations are solved in cylinder with mean flow

The set of linearized MHD momentum, adiabatic pressure, and induction equations in the presence of a mean flow velocity are;

$$\rho\omega^2\xi + \rho\omega(\mathbf{v}_0 \cdot \nabla)\xi + \rho\omega(\xi \cdot \nabla)\mathbf{v}_0 = (\nabla \times \widetilde{\mathbf{B}}) \times \mathbf{B} + \mathbf{J} \times \widetilde{\mathbf{B}} - \nabla\widetilde{p}$$

$$\tilde{p} = -(\xi \cdot \nabla p) - \gamma p \nabla \cdot \xi$$

$$\widetilde{\mathbf{B}} - \eta/\omega\nabla^2\widetilde{\mathbf{B}} = \nabla \times (\xi \times \mathbf{B}) + 1/\omega\nabla \times (\mathbf{v}_0 \times \widetilde{\mathbf{B}})$$

1- adopt CGJ tearing ordering ($\omega \propto \eta^{3/5}$)

2- consider small flow shear

- $\omega \rightarrow \epsilon^3$, $\eta \rightarrow \epsilon^5$, $(r - r_s) \rightarrow \epsilon^2 x$, $v_0 \approx \eta^{1/5}$ (the inertial regime)
- $g = (k \cdot \mathbf{v}_0)'_{(r_s)} x \rightarrow \epsilon^3$, $\boxed{(k \cdot \mathbf{v}_0)'_{(r_s)}} \longrightarrow$ flow shear

The inner layer equations are solved in cylinder with mean flow

The inner resistive layer equations in the presence of mean flow :

$$\Psi_2^{''} = \Omega[X\Xi + (1 + i\frac{G'X}{\Omega})\Psi_0]$$

$$(1 + i\frac{G'X}{\Omega})\Xi^{''} = \frac{X^2\Xi}{\Omega} - \frac{\Upsilon}{\Omega^2} + [\frac{X}{\Omega}(1 + i\frac{G'X}{\Omega}) - \frac{J_p}{\Omega^2}]\Psi_0$$

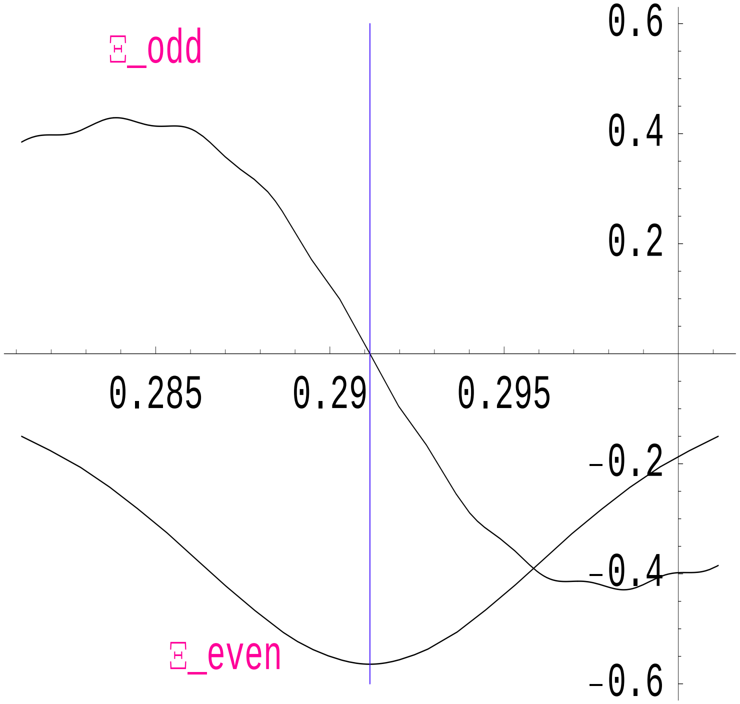
$$(1 + i\frac{G'Z}{\Omega^{3/4}})\Upsilon^{''} - \left(Z^2 + \frac{2l^6}{\gamma\beta\Omega^{-3/2}}(1 + i\frac{G'Z}{\Omega^{3/4}})\right)\Upsilon - \left(\frac{S}{\Omega^{-3/2}}(1 + i\frac{G'Z}{\Omega^{3/4}}) + i\frac{G'G_pZ}{\Omega^{-3/4}}\right)\Xi + \left(JZ^2 + \frac{2Jl^6}{\gamma\beta\Omega^{-3/2}}(1 + i\frac{G'Z}{\Omega^{3/4}}) - i\frac{G'G_p}{\Omega^{-1/2}}(1 + i\frac{G'Z}{\Omega^{3/4}})\right)\Psi_0 = 0$$

We solve for Ξ^0, Ξ^1, Υ^0 and Υ^1 to calculate quasilinear $< \widetilde{J} \times \widetilde{B} >$ torque.

The inner layer zeroth order solutions are used to calculate the Lorentz term.

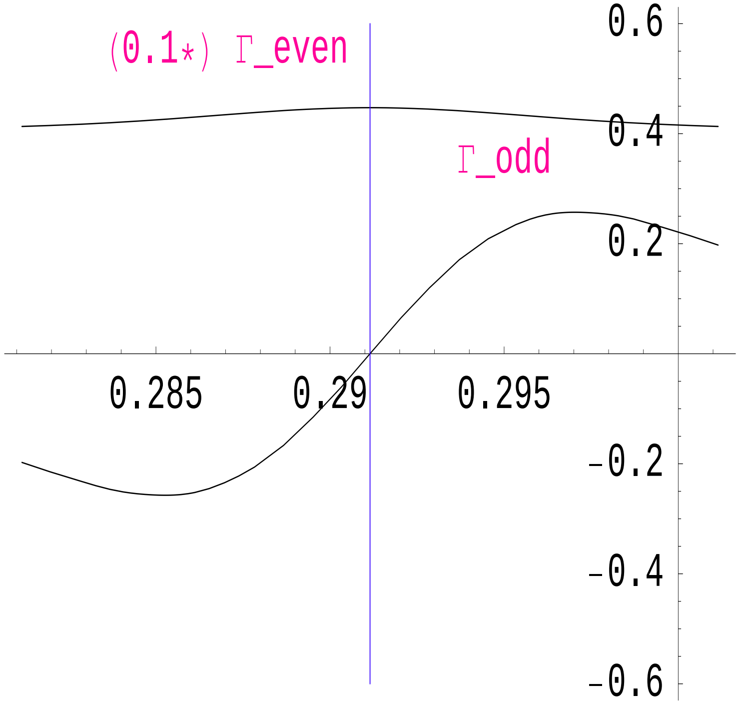
ξ_r

radial displacement



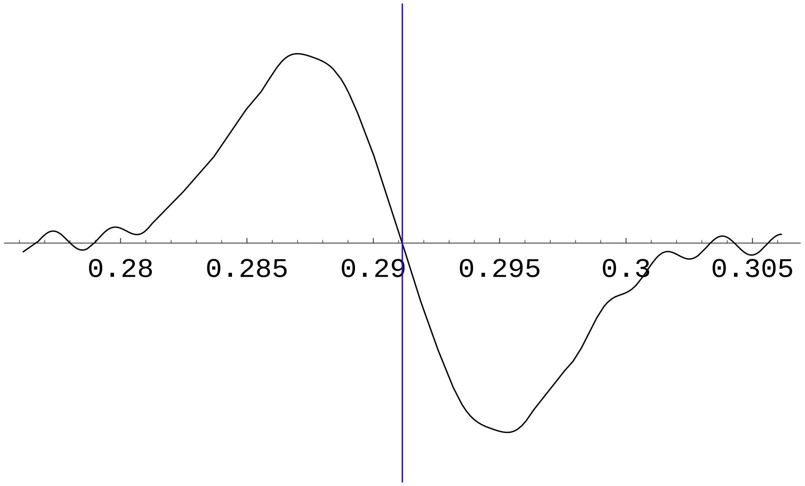
$\widetilde{B}_{||}$

parallel magnetic field perturbation

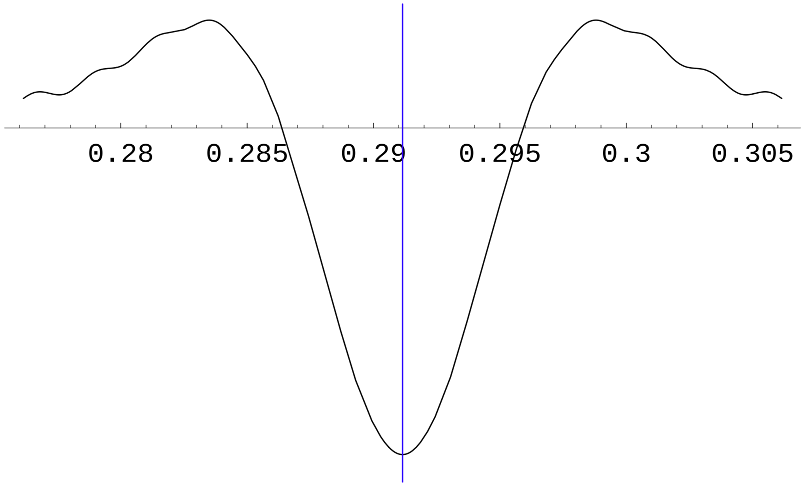


The analytical quasilinear $< \widetilde{J} \times \widetilde{B} >$ term in the presence of flow shear.

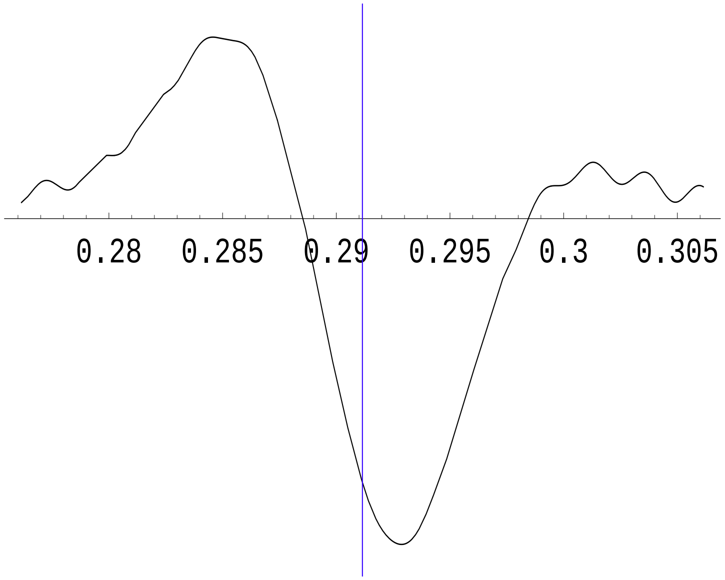
odd $< \widetilde{J} \times \widetilde{B} >$



even $< \widetilde{J} \times \widetilde{B} >$



total (JxB)ll w/ flow



<==== total $< \widetilde{J} \times \widetilde{B} >$

$$T = \int r < \widetilde{J} \times \widetilde{B} >_z dr = 0$$

Single mode computations

3-D resistive MHD code

- The 3-D resistive nonlinear MHD code DEBS is used.

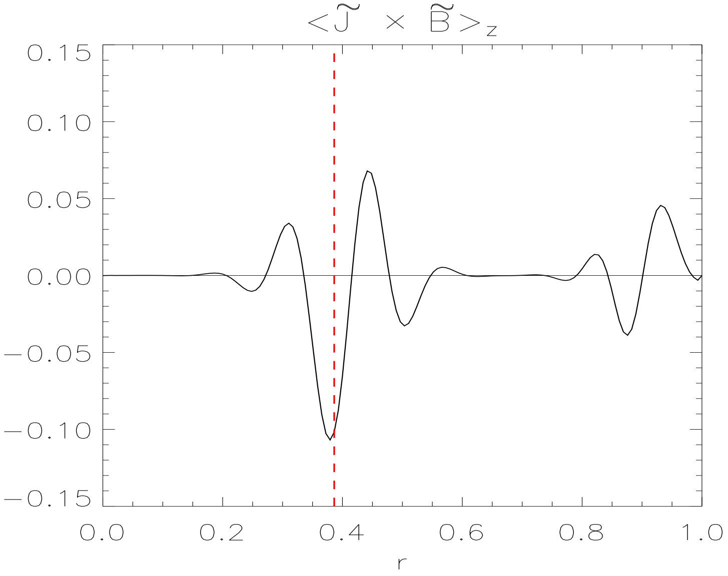
$$\begin{aligned}
\frac{\partial \overline{A}}{\partial t} &= S\overline{V} \times \overline{B} - \eta \overline{J} \\
\rho \frac{\partial \overline{V}}{\partial t} &= -S\rho \overline{V}.\nabla \overline{V} + S\overline{J} \times \overline{B} + \nu \nabla^2 \overline{V} + \mathbf{F}(r) \\
\overline{B} &= \nabla \times \overline{A} \\
\overline{J} &= \nabla \times \overline{B}
\end{aligned}$$

  where $S = \frac{\tau_R}{\tau_A}$ (Lundquist number) , $\nu = \frac{\tau_R}{\tau_{vis}}$ (magnetic Prandtl number = viscosity / resistivity).

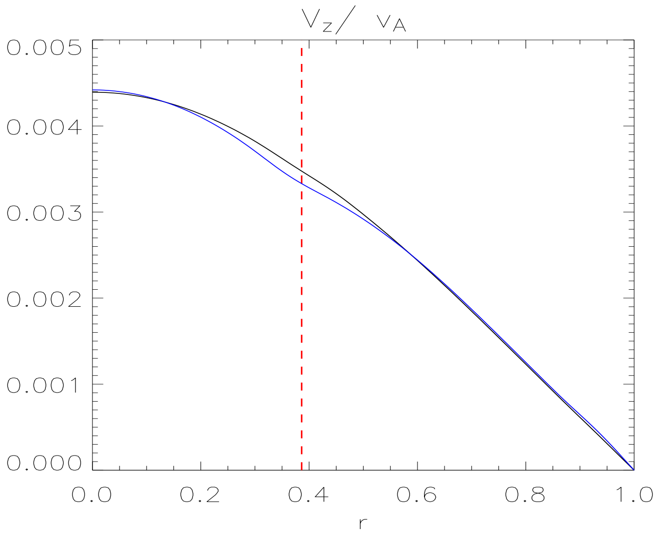
- With an axial ad-hoc momentum source term $\mathbf{F}$(r) added to the momentum equation.

Single tearing mode does transport momentum

during the linear growth



- Single helicity $<\tilde{J} \times \tilde{B}>$ from computation agrees with the analytical calculations.



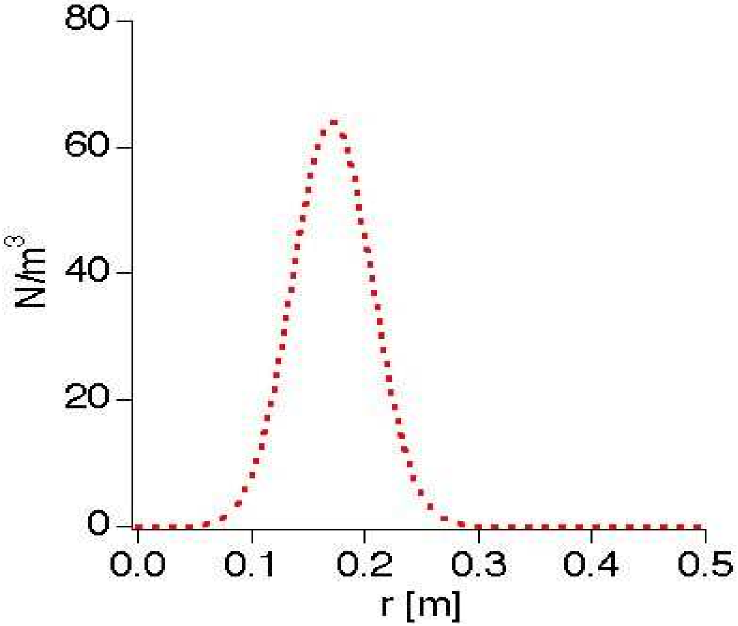
- Quasi-linear Lorentz force from the single mode perturbations affect the flow profile. <==== V_Z/V_A

Multiple helicity computations

Three wave coupling generates localized electromagnetic torques .

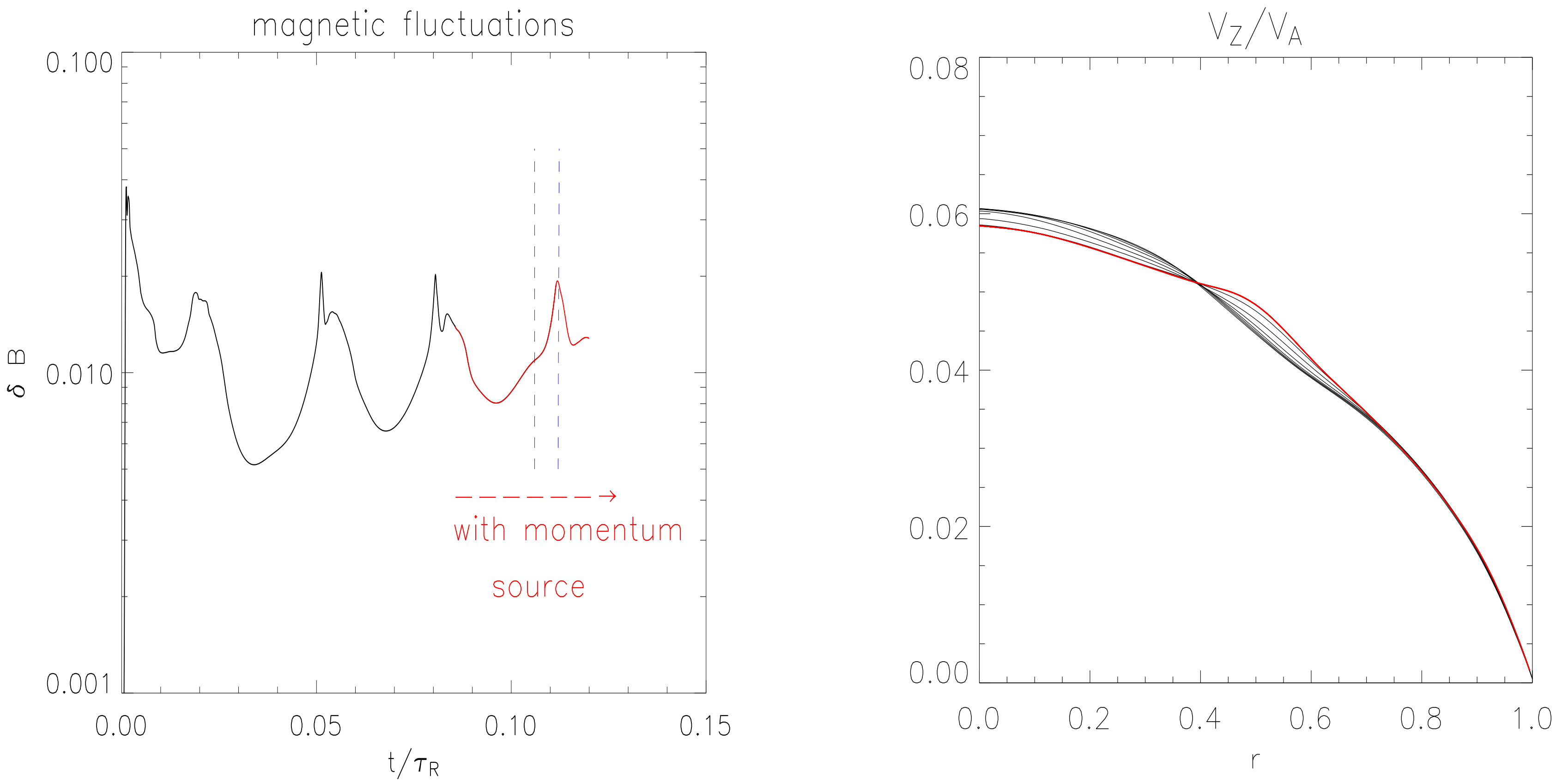
- $T_1 \approx < \widetilde{J}_{k1} \times \widetilde{B}_{k1} >$ torque at the rational surface k1.
- If $J_{k1} \approx B_{k2} B_{k3}$ is nonlinearly generated
- $T_1 \approx < B_{k2} B_{k3} B_{k1} >$ three wave coupling [Hegna and Fitzpatrick]
- $\boxed{T_1 + T_2 + T3 = 0}$
- The torque generated from nonlinear mode coupling is correlated with momentum transport.

core mode $< \widetilde{J} \times \widetilde{B} >$ torque from experiment



Flow profile is affected by the tearing fluctuations during sawtooth oscillations.

$S = 5 \times 10^4$

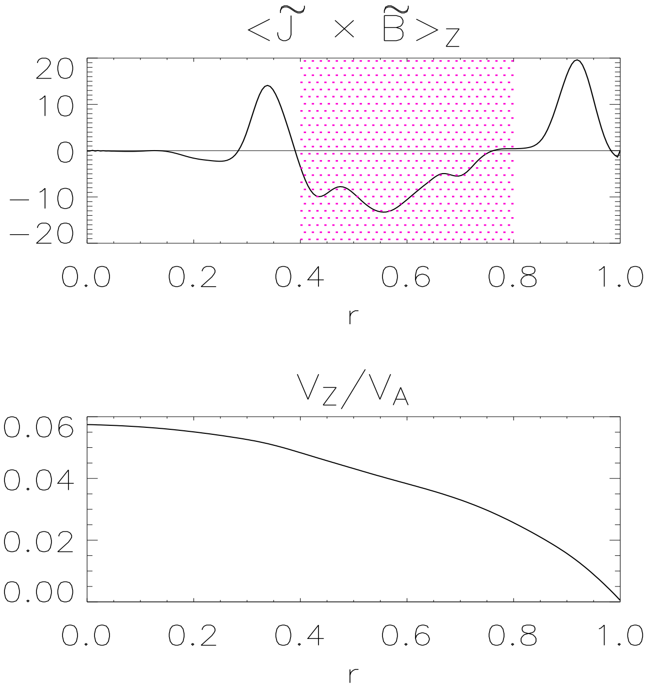


- Higher Lundquist number computation show more pronounced changes of the flow profile during a sawtooth.

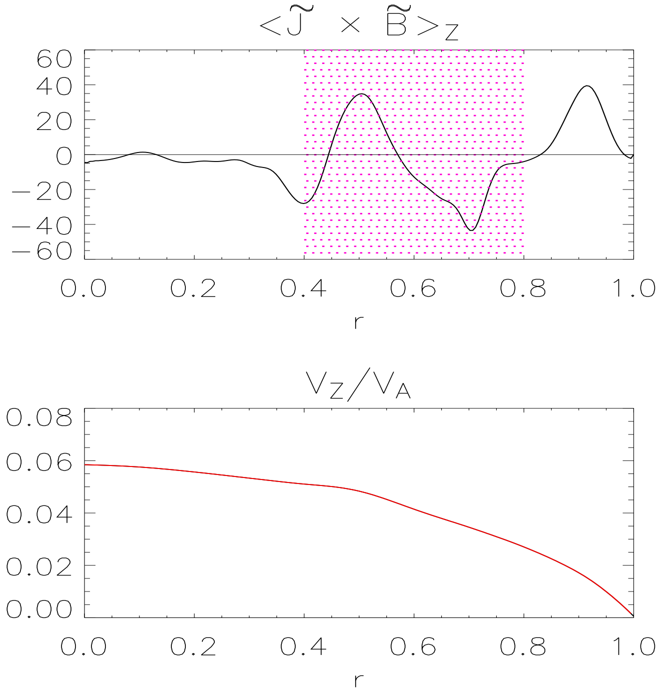
There is correlation between the momentum transport and the fluctuation-induced forces.

The fluctuation-induced forces at two times shown in previous slide.

low magnetic fluctuations



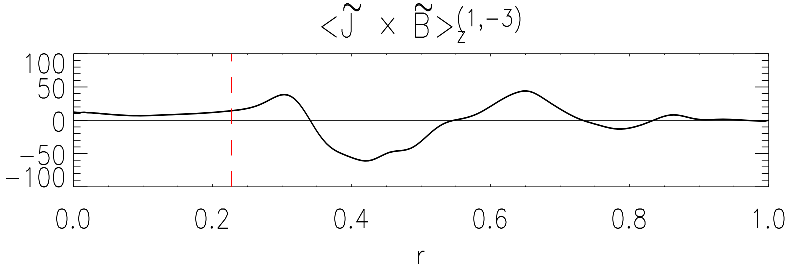
high magnetic fluctuations



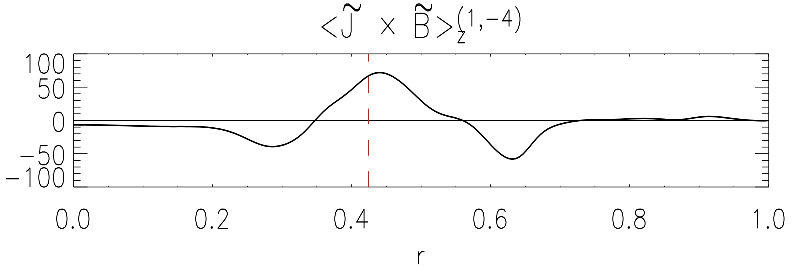
- The fluctuation-induced Lorentz force during the peak of the magnetic fluctuations results in flattening of the flow profile around r=0.4–0.6.

Total fluctuation-induced forces are the superposition of the single mode Lorentz forces.

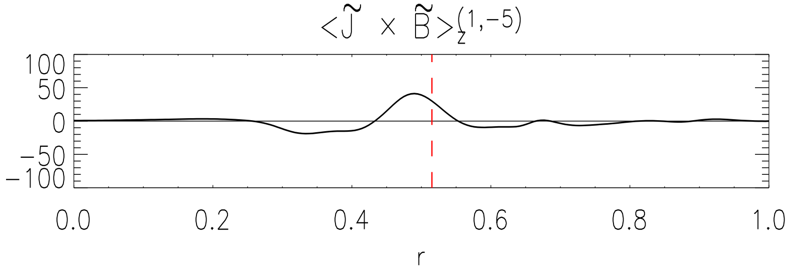
Axial $<\tilde{J} \times \tilde{B}>$ forces for core modes, (m,n)



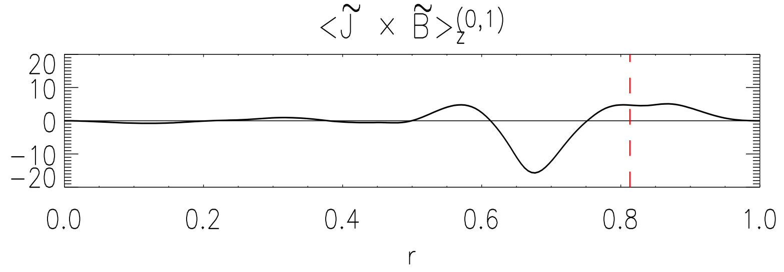
- (1, -3)



- (1, -4)



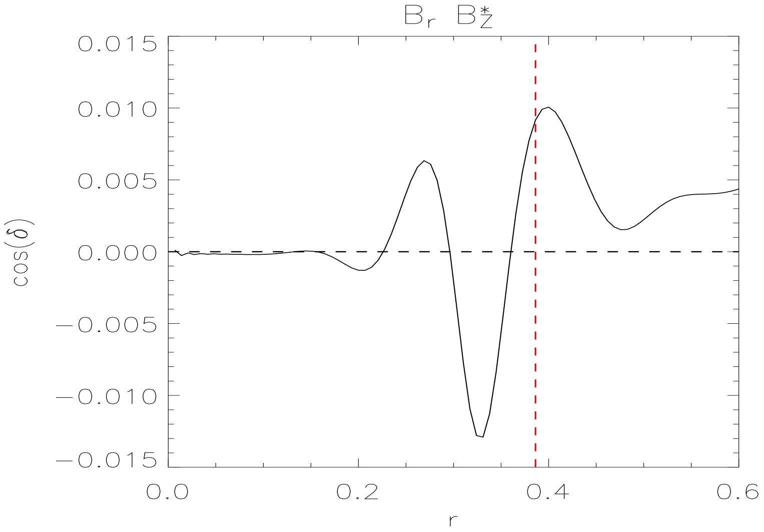
- (1,-5)



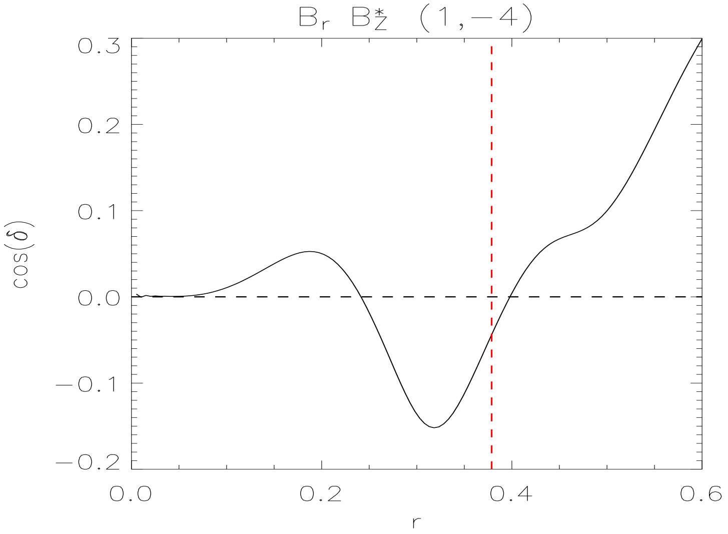
- (0, 1)

Nonlinear mode coupling alters the radial phase dependency.

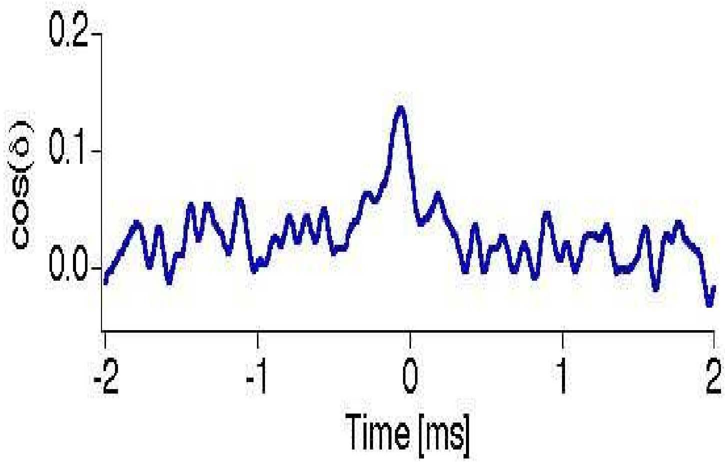
single helicity phase



multiple helicity phase



phase in experiment



- momentum transport is enhanced due to nonlinear mode coupling.

$$< \widetilde{J} \times \widetilde{B} >= \frac{1}{r}\left[r|\widetilde{B}_z||\widetilde{B}_r|\cos(\delta)\right]'$$

Conclusions

- Quasilinear analytical calculations and single helicity computations show:

  momentum transport can occur due to single tearing mode (spontaneous reconnection)

- Nonlinear computations:

  momentum transport can arise from multiple mode coupling (coupled spontaneous and driven reconnection)

- Whether current-driven reconnection can transport momentum in astrophysical plasmas?