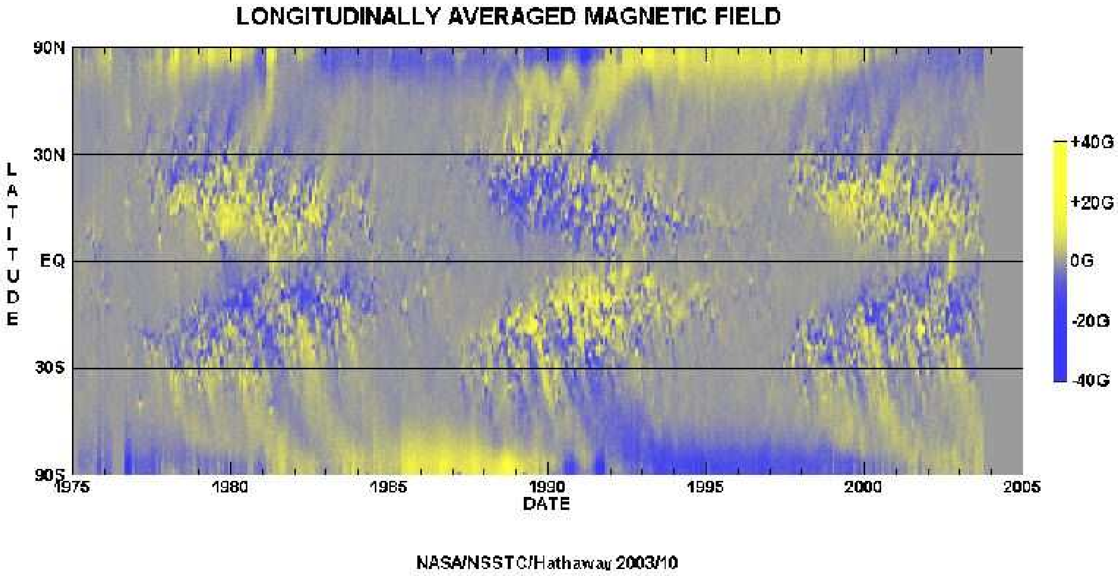
# Aspects of Helical Dynamo Theory

(Eric Blackman, U. Rochester)



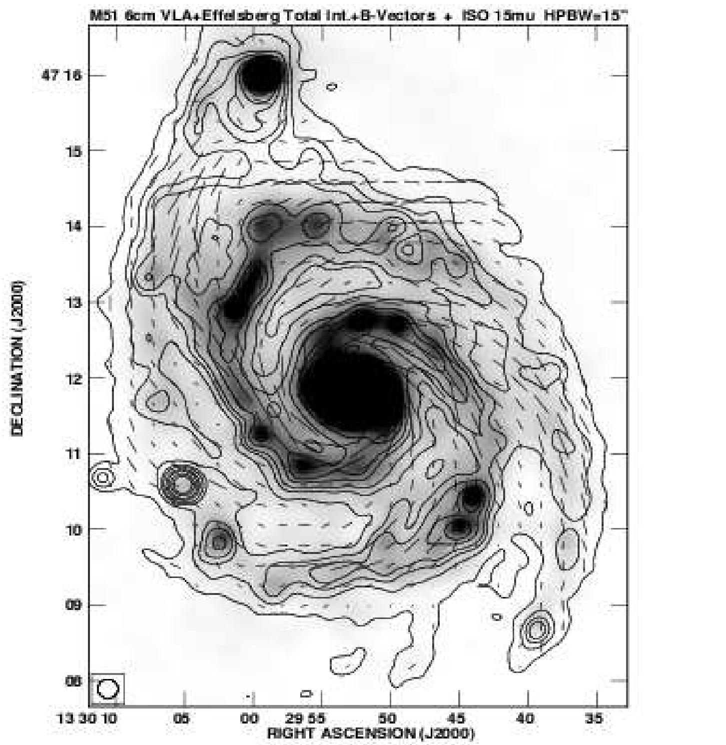




*Figure 1.* Total radio continuum intensity (contours) and **B**–vectors of polarized intensity of M 51 at 15″ resolution, combined from VLA and Effelsberg observations at λ6 cm. The grey-scale ISOCAM image shows the λ15 μm emission (kindly provided by M. Sauvage), smoothed to 15″ resolution. (Beck, unpublished)

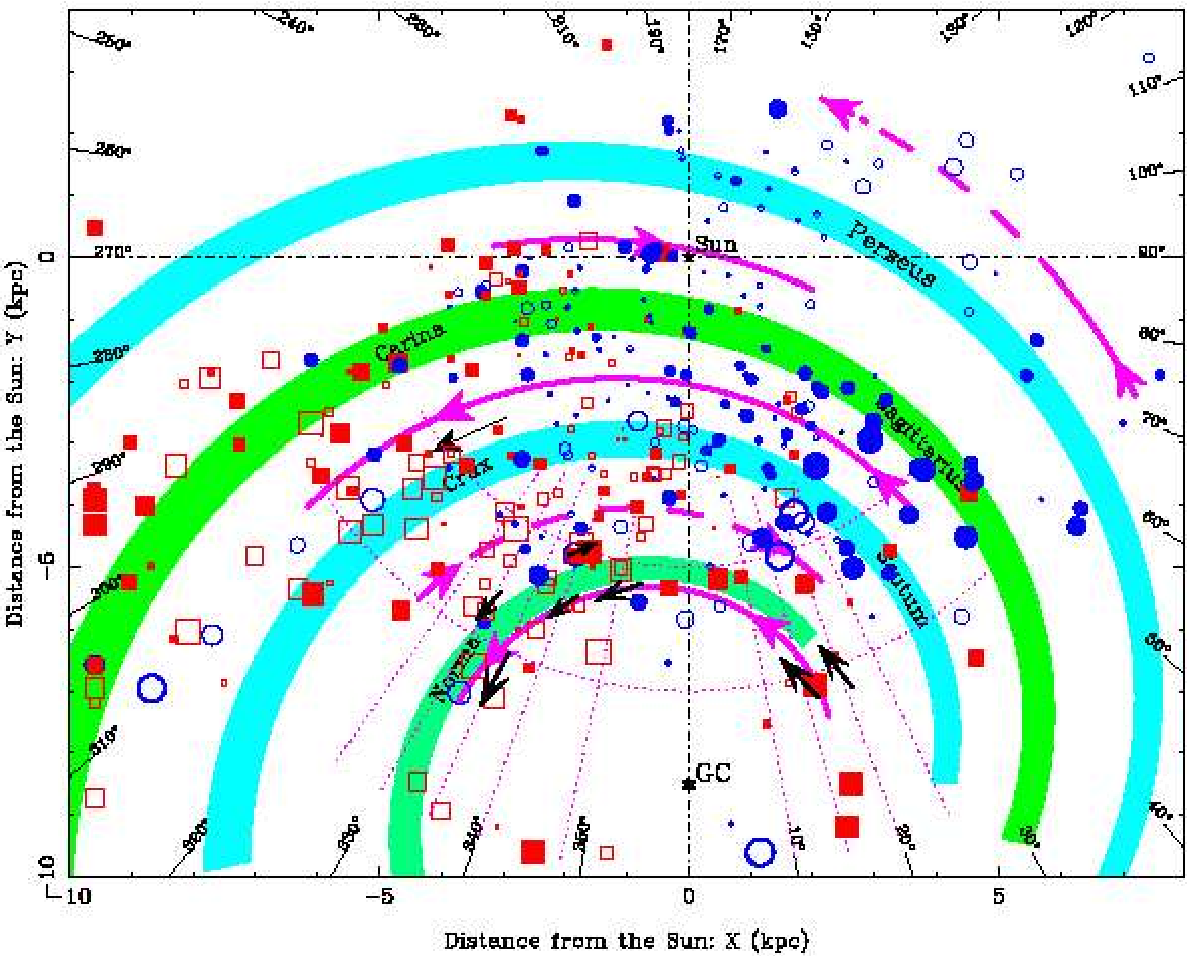



Figure 2. The distribution of pulsar RMs projected onto the Galactic plane reveals the field structure in the Galactic disk, which has direction reversals from arm to arm (after Han et al. 2002). The square symbols are newly determined RMs from Parkes observations, and filled

**Nonhelical Small Scale Velocity Driven Dynamo:**

- field amplification on scales **at or smaller** than largest turbulent scale
- $\overline{\boldsymbol{\mathcal{E}}}_{||} = 0$, magnetic helicity irrelevant
- Neither kinetic nor magnetic helicity required
- What is the shape of saturated magnetic energy spectrum?

**Large Scale or Helical Velocity Driven Dynamo**:

- field amplification on **scales larger** than the largest "turbulent" scale.
- finite $\overline{\boldsymbol{\mathcal{E}}}_{||}$ (mean pseudosalar or pseudovector required)
- magnetic helicity transfer and evolution important
- accompanied by a velocity driven small scale dynamo
- What is large scale field growth rate and saturation value?

**Lg. Scale Mag. Driven Dynamo=Dynamical Mag. Relaxation**:

- Converts strong **B**-field from unrelaxed state to relaxed state, and sustains the state against decay.
- finite $\overline{\boldsymbol{\mathcal{E}}}_{||}$ (mean pseudosalar or pseudovector required)
- magnetic helicity transfer and evolution important

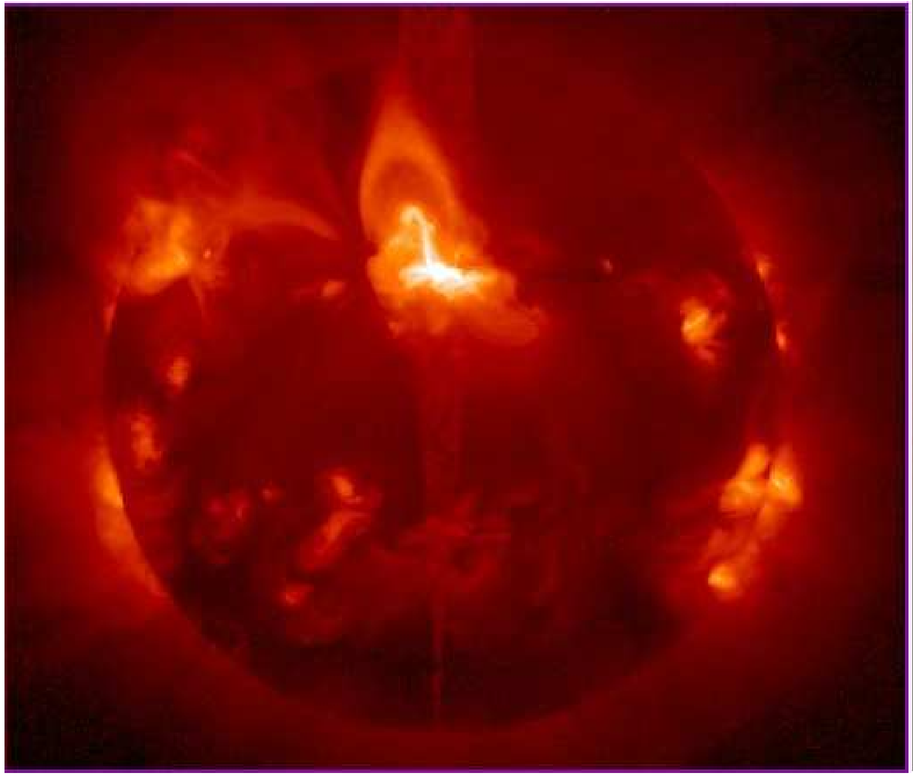
**TWO STAGE PARADIGM FOR STARS AND DISKS**:

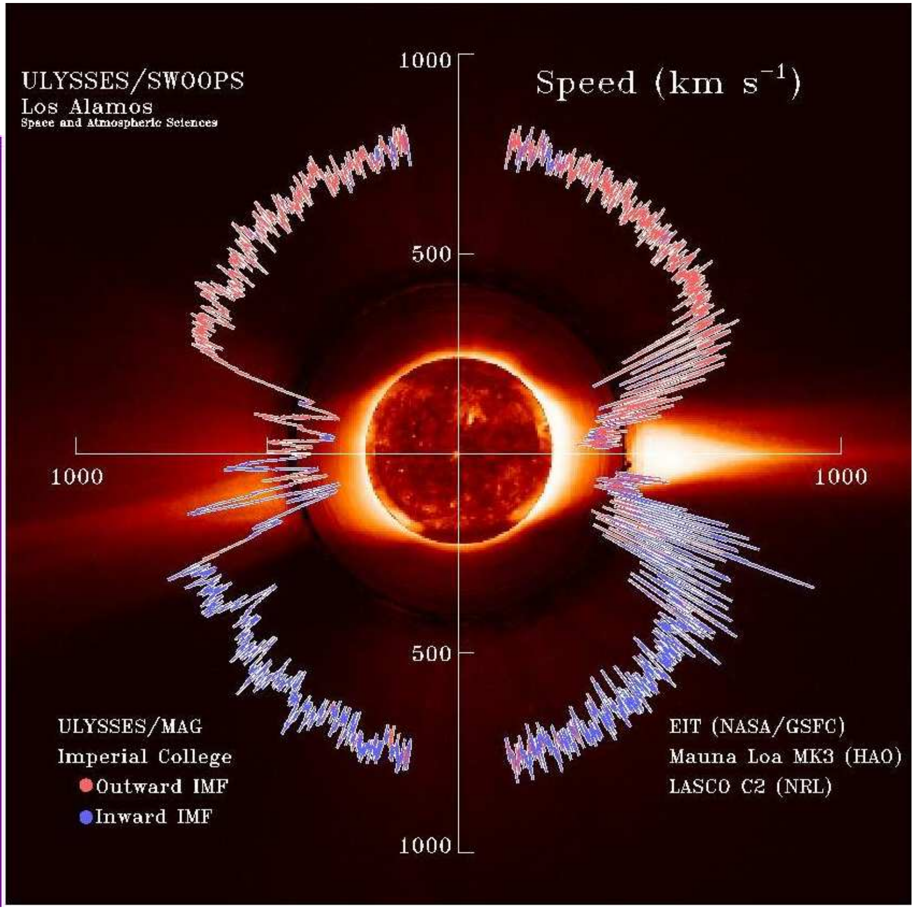
• (1) Velocity driven helical dynamo and footpoint motions inject helical fields to the corona

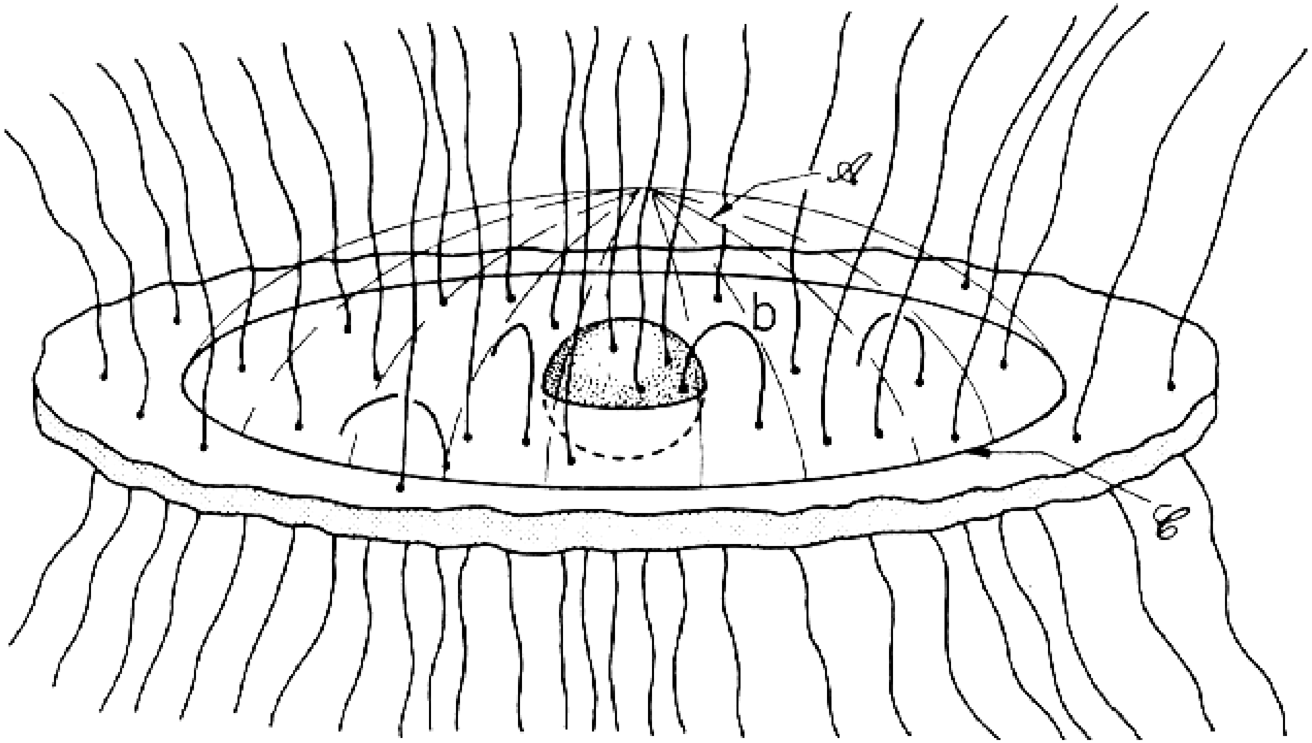
• (2) Dynamical relaxation (= magnetically dominated dynamo) in corona to open up fields to very large scales, e.g. for jets and coronal hole.

• **NOTE**: For stars and inner accretion disks, we probe at best, the coronal field.

## SUN PROVIDES INSIGHT








(Thorne et al. 1986)

## DISKS, CORONAL HOLES & LARGE SCALE $B$ GROWTH

• **In situ field production is helpful, if not essential**

*Disks*:

(1) Poloidal flux freezing in thin disks not guaranteed (but note Vishniac talk)
(2) **B**-field to corona requires $t_{dif} > t_{buo} \rightarrow$ favors larger scales

*Stars*:

(1) Solar cycle shows flux is not frozen in sun
(2) Helical dynamo overcomes turbulent diffusion

• **Regardless of need:** understand $\overline{\mathbf{B}}$ growth/evolution from first principles

• **Different flavors of field growth studies**

(1) study of nonlinear dynamo for idealized systems to gain some understanding of backreaction at expense of realism

(2) phenomenological linear models to make contact with observations, at expense of self-consistency

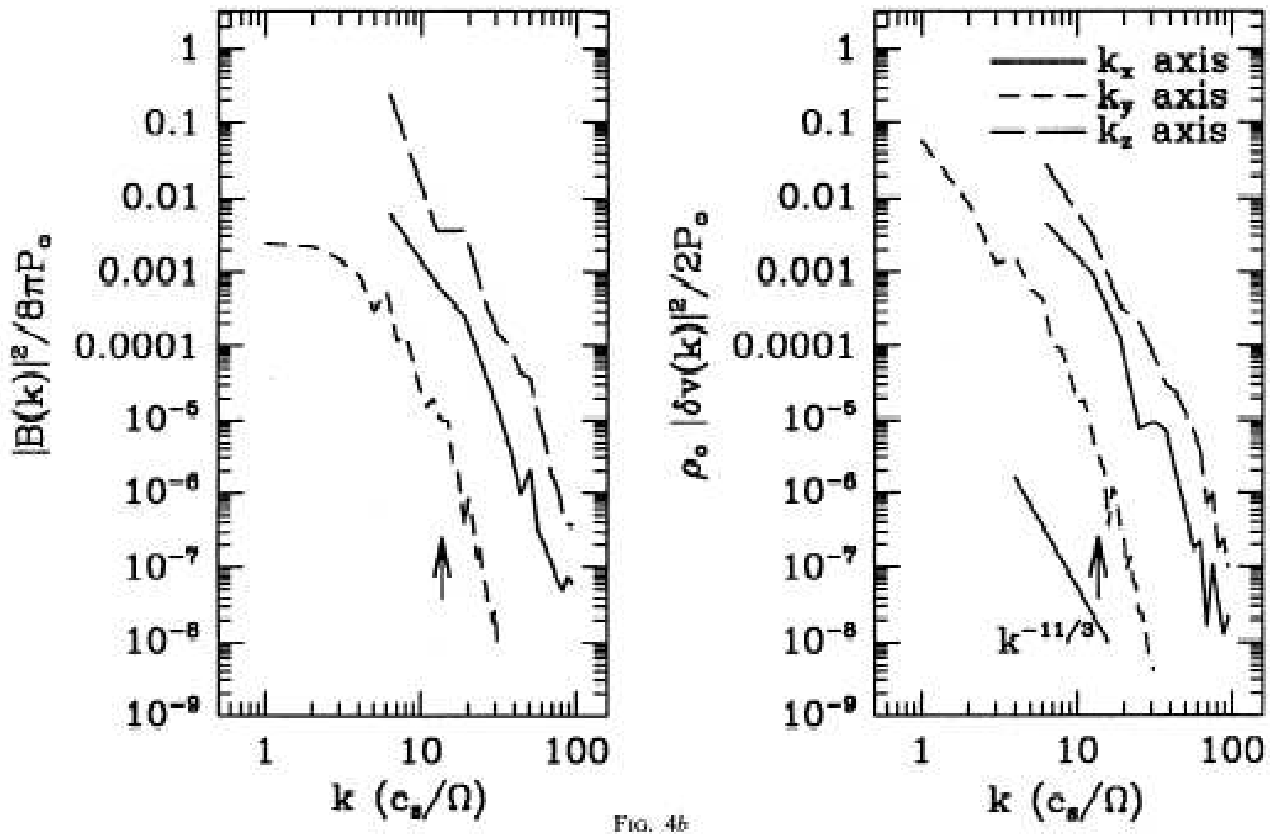
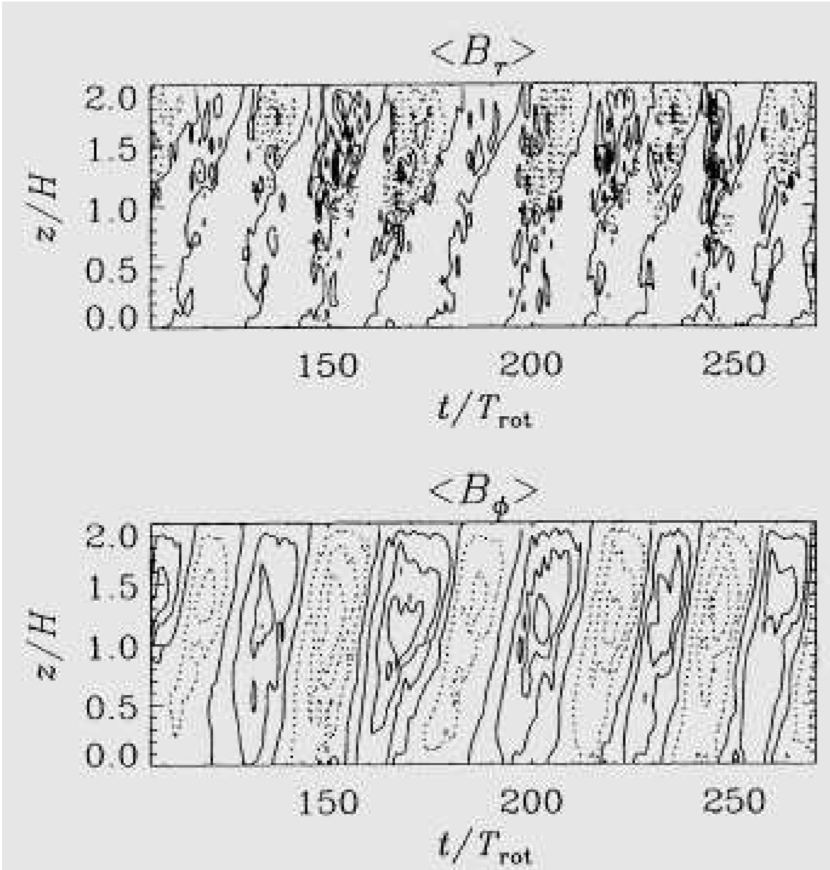



FIG. 4b

Non-stratified, nonhelical MRI spectra from Hawley et al. (1995).

• All $k < k_c$ are unstable and turbulent. Maximum growth wavenumber is $k_{max} \sim \Omega/v_A$: decreases as $v_A$ increases




Stratified (Helical) MRI driven helical dynamo in stratified non-periodic box. (from Brandenburg & Donner (1997))

## SIMPLE FLUX FREEZING: NOT FOR TURBULENT FLOWS

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \nu_M \nabla^2 \mathbf{B}$$

$$\frac{1}{2}\partial_t \langle \mathbf{B}^2 \rangle = \langle \mathbf{B} \cdot \nabla \times (\mathbf{V} \times \mathbf{B}) \rangle + \nu_M \langle \mathbf{B} \nabla^2 \mathbf{B} \rangle$$

- flux freezing argument requires advection term >> dissipation term
- turbulence can violate this assumption even for small $\nu_M$:

**Fluctuating Field**

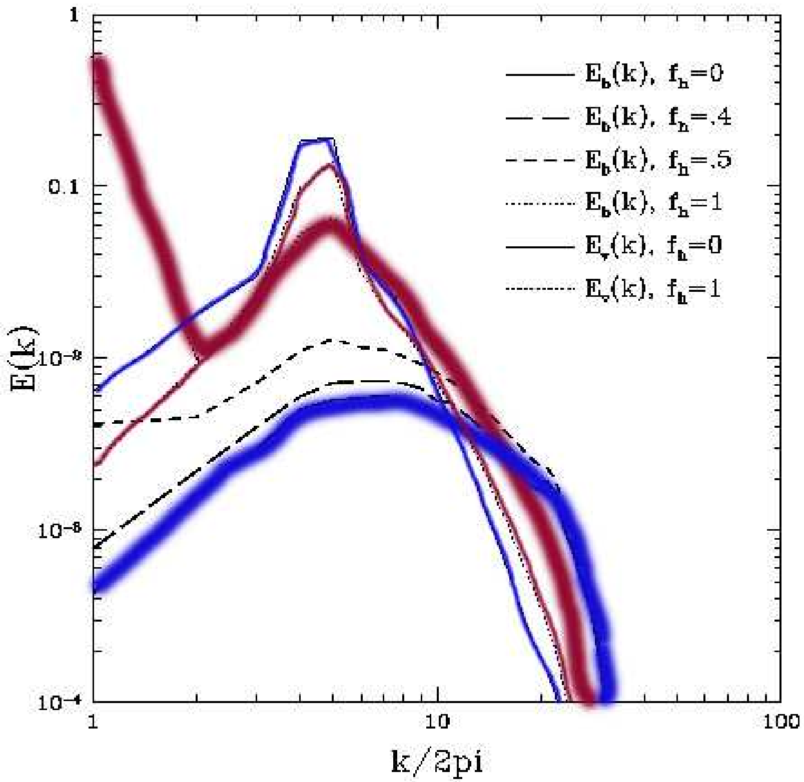
$$\frac{1}{2}\partial_t \langle b^2 \rangle = \langle \mathbf{b} \cdot \nabla \times (\mathbf{v} \times \overline{\mathbf{B}}) \rangle + \langle \mathbf{b} \cdot \nabla \times (\mathbf{v} \times \mathbf{b}) \rangle + \nu_M \langle \mathbf{b} \cdot \nabla^2 \mathbf{b} \rangle$$

**Large Scale field**

$$\partial_t \overline{\mathbf{B}} = \nabla \times (\overline{\mathbf{V}} \times \overline{\mathbf{B}}) + \nabla \times \langle \mathbf{v} \times \mathbf{b} \rangle + \nu_M \nabla^2 \overline{\mathbf{B}}$$

$$= \nabla \times (\overline{\mathbf{V}} \times \overline{\mathbf{B}}) + \nabla \times \alpha \overline{\mathbf{B}} + (\beta + \nu_M) \nabla^2 \overline{\mathbf{B}} + \nabla \times \mathbf{Q}_A + ...$$

## VELOCITY DRIVEN HELICAL DYNAMO: SIMULATIONS




(simulation from Maron & Blackman )

- NOTE: "**Doubly Relaxed State**" for $f_h = 1$

## Classical Kinematic MFD Theory: Useful but Incomplete

$$\partial_t\overline{\mathbf{B}} = -\nabla\times\overline{\mathbf{E}} = \nabla\times\overline{\boldsymbol{\mathcal{E}}} + \nabla\times(\overline{\mathbf{V}}\times\overline{\mathbf{B}}) - \eta\overline{\mathbf{J}}$$

$$\overline{\boldsymbol{\mathcal{E}}} = \int\langle\mathbf{v}(t)\times\partial_t\mathbf{b}(t')\rangle dt'$$

First order smoothing approximation: ignore $\nabla\times(\mathbf{v}\times\mathbf{b})$, then

$$\overline{\boldsymbol{\mathcal{E}}} = \int\frac{1}{3}\langle\mathbf{v}(t)\cdot\nabla\times\mathbf{v}(t')\rangle dt'\ \overline{\mathbf{B}} + \int\frac{1}{3}\langle\mathbf{v}(t)\cdot\mathbf{v}(t')\rangle dt'\ \nabla\times\overline{\mathbf{B}}$$

$$\simeq \alpha\overline{\mathbf{B}} - \beta\nabla\times\overline{\mathbf{B}}$$

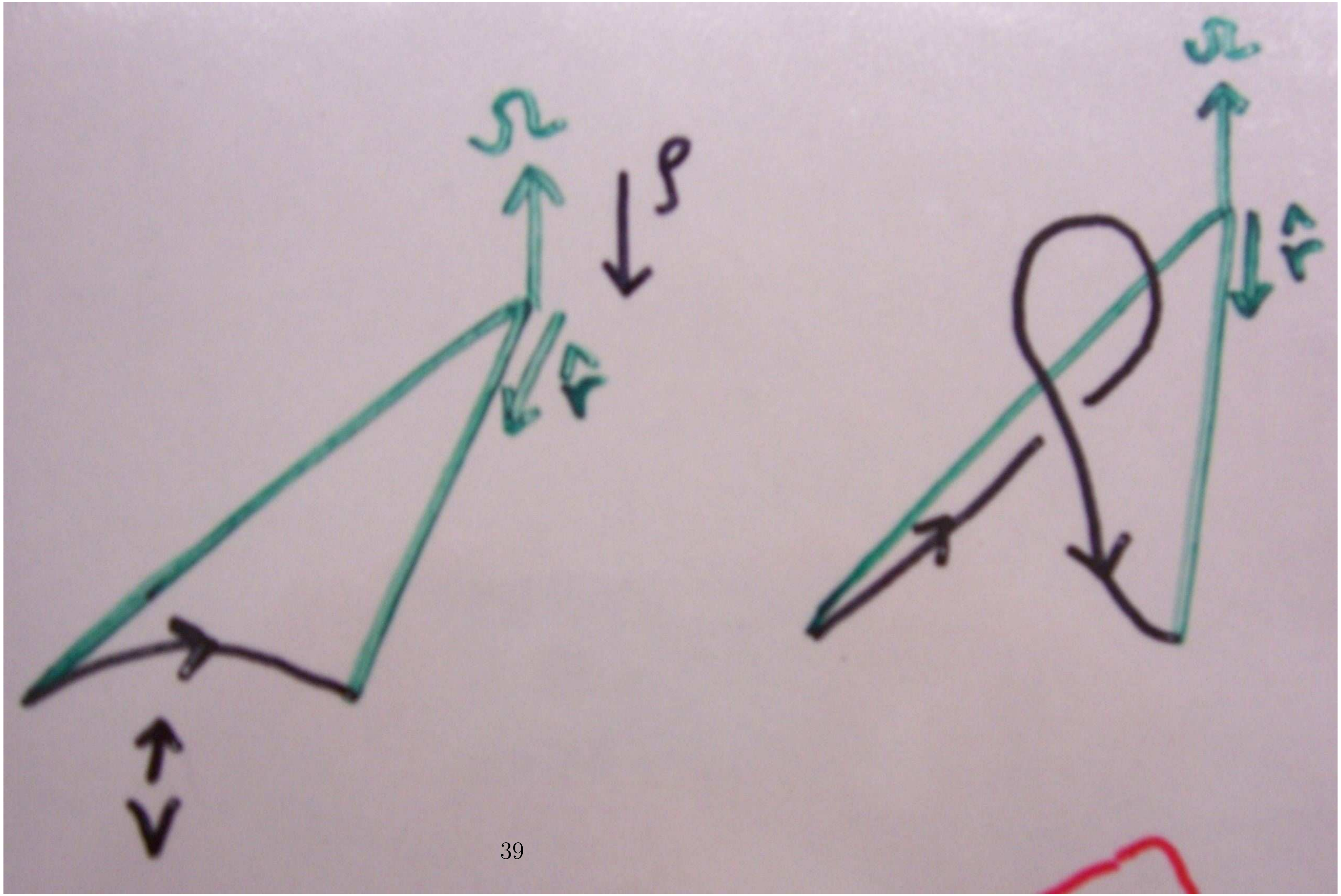
**But this is incomplete with respect to real MHD:**

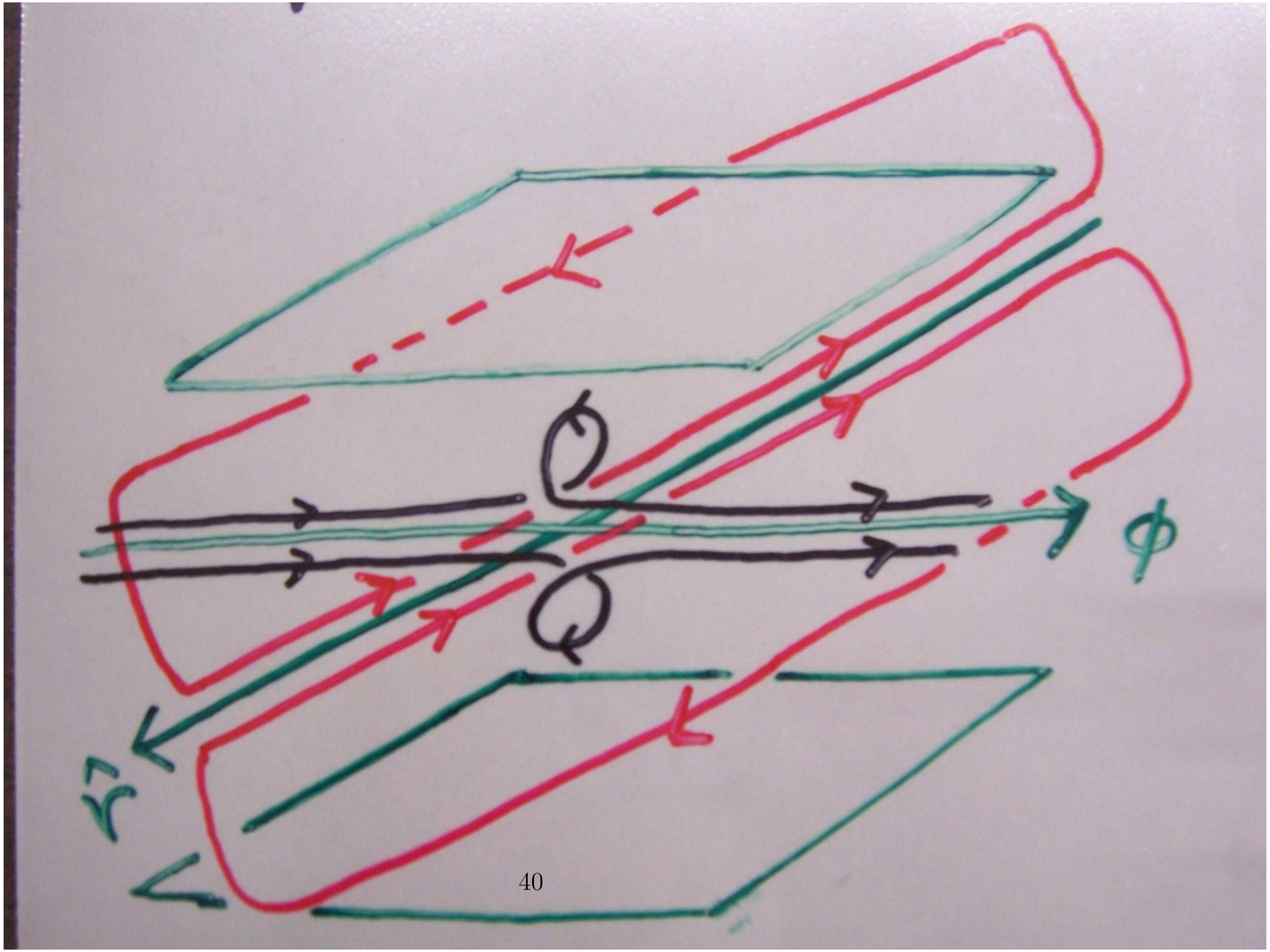
(1) What about backraction of field on velocity (Navier-Stokes)?

(2) What about growth of small scale field?

(3) What about turbulence? $\tau_c/\tau_{ed}$ not $<< 1$.

(4) Finite $\langle\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle$ grows $\langle\mathbf{A}\cdot\mathbf{B}\rangle = \langle\overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\rangle$: conservation of mag. helicity violated in "textbook" approaches.

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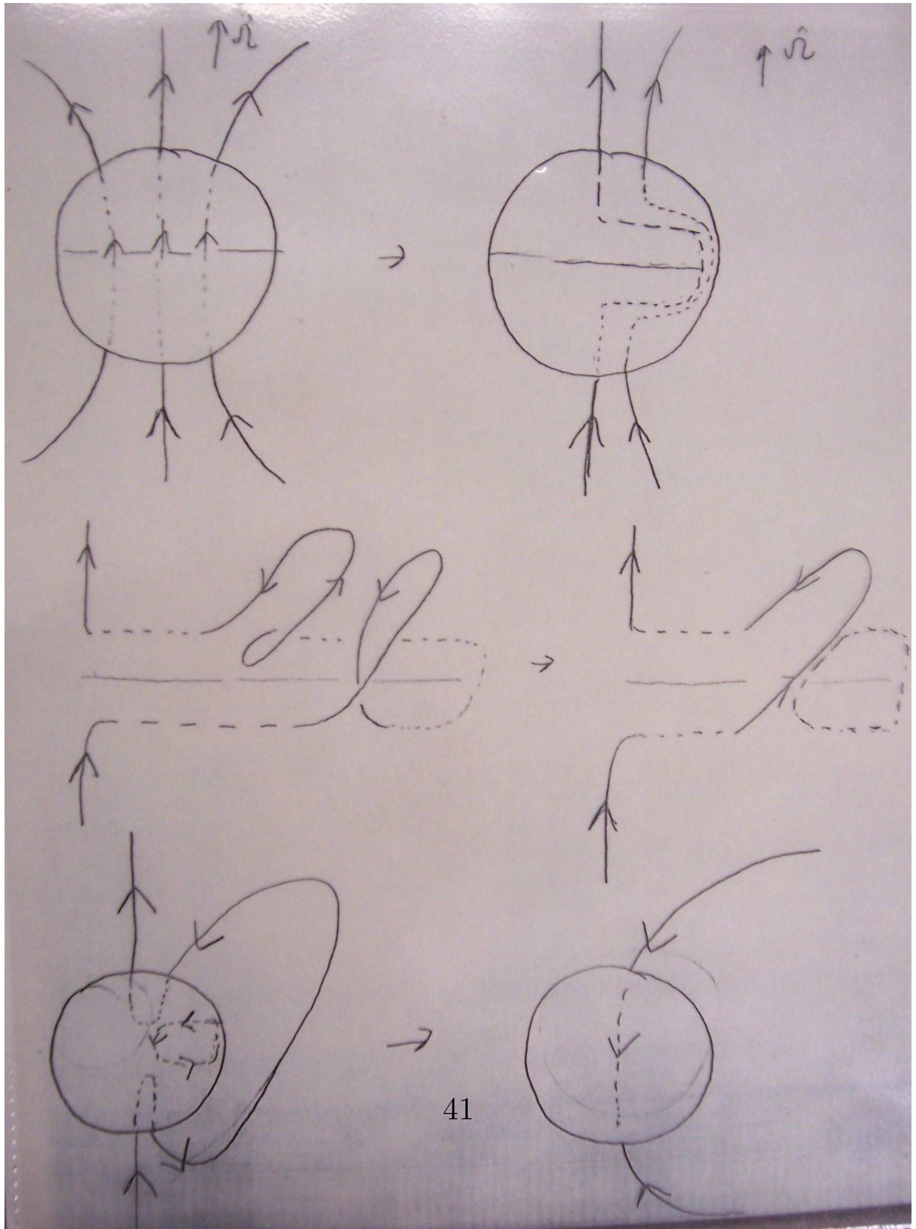


**MAGNETIC HELICITY:** $H \equiv \int \mathbf{A}\cdot\mathbf{B}dV$

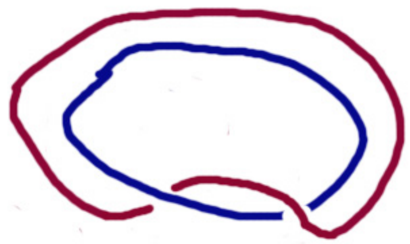
• **Conserved in ideal MHD for closed system**

$$\partial_t\langle\mathbf{A}\cdot\mathbf{B}\rangle = -2\langle\mathbf{E}\cdot\mathbf{B}\rangle - \nabla\cdot\langle\mathbf{A}\times\partial_t\mathbf{A} + 2\phi\mathbf{B}\rangle$$

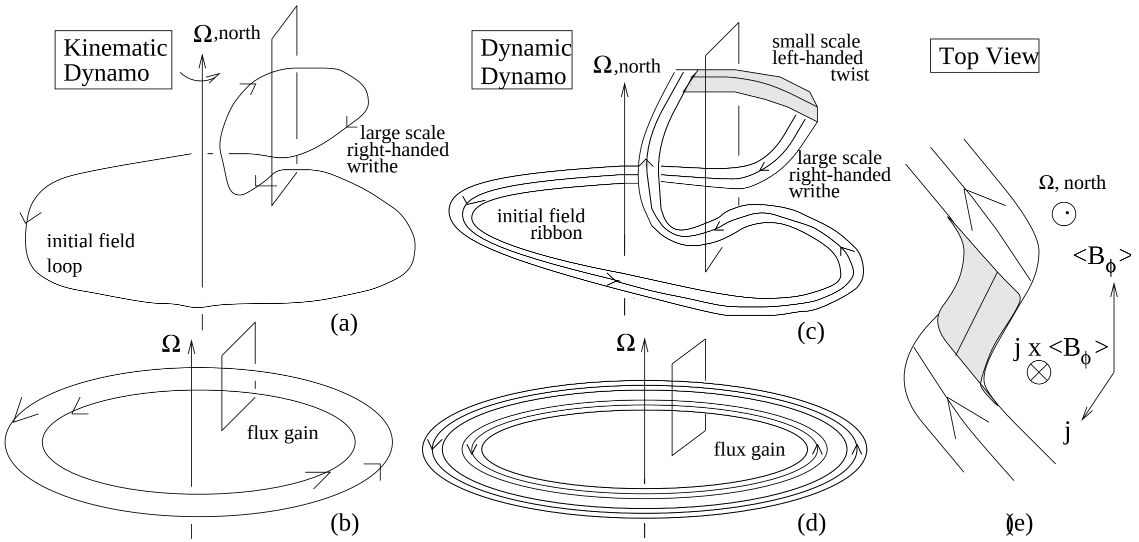
• **Measures linkage**: For two flux tubes $A$ and $B$: $H = H_A + H_B$.

$$H_B = H_A \equiv \int \mathbf{A}\cdot\mathbf{B}dV = \int\mathbf{B}\cdot d\mathbf{S}_A \int \mathbf{A}\cdot d\mathbf{l}_A = \Phi_A \int \mathbf{B}\cdot d\mathbf{S}_B = \Phi_A\Phi_B$$

• **Actually, it measures twist and linkage**: Two linked tubes can be viewed as a twisted ribbon.



# HELICAL DYNAMO: REVISING THE "TEXTBOOK" PICTURE




• **Traditional:** dynamics between field and flow approximated, but not with dynamical theory

• **Modern:** principle to understand the backreaction included (magnetic helicity conservation)

**Toward including v evolution:**

$$\partial_t \overline{\mathbf{B}} = \nabla \times \langle \mathbf{v} \times \mathbf{b} \rangle - \eta \overline{\mathbf{J}}$$

Pouquet et al. (76), spectral MHD turbulence theory, interpreted to imply

$$\langle \mathbf{v} \times \mathbf{b} \rangle = \frac{\tau_c}{3}(\langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle - \langle \mathbf{b} \cdot \mathbf{j} \rangle)\overline{\mathbf{B}} + ...$$

But when $\tau_c$ is a correlation time, then this is a misinterpretation.

**Cognitive Dissonance 1:**

$$\langle \mathbf{v} \times \mathbf{b} \rangle = \int \langle \mathbf{v}(t) \times \partial_t \mathbf{b}(t') \rangle dt' = \int \langle \partial_t \mathbf{v}(t) \times \mathbf{b}(t') \rangle dt' \neq \int \langle \mathbf{v}(t) \times \partial_t \mathbf{b}(t') \rangle dt' + \int \langle \partial_t \mathbf{v}(t) \times \mathbf{b}(t') \rangle dt'$$

**Cognitive Dissonance 2:** FOSA haunts intergral approaches.

**Cognitive Dissonance 3:** Ambiguity in "ordering" approaches:

$$\langle \mathbf{v}(t) \times \mathbf{b}(t) \rangle^{(1)} = \int \langle \mathbf{v}^{(0)}(t) \times \partial_t \mathbf{b}^{(1)}(t') \rangle dt' + \int \langle \partial_t \mathbf{v}^{(1)}(t) \times \mathbf{b}^{(0)}(t') \rangle dt'$$

or

$$\langle \mathbf{v}(t) \times \mathbf{b}(t) \rangle^{(1)} = f(\overline{\mathbf{B}}) \tfrac{\tau_c}{3} (\langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle^{(0)} - \langle \mathbf{j} \cdot \mathbf{b} \rangle^{(0)}) \overline{\mathbf{B}} + ...$$

BUT helicity conservation equations involve $\langle \mathbf{j} \cdot \mathbf{b} \rangle$ unambiguously to all orders.

## SPHERICAL COWING THE ASTROPHYSICS

- **Ingredients of Astrophysical Problem**

(1) $\beta >> 1$ helical dynamo in sheared rotator
(2) buoyant injection of $\mathbf{B}$ and mag. helicity flow into $\beta << 1$ corona
(3) coronal relaxation subject to $\mathbf{B}$-injection from below

- **Toy Problems to Capture Some Key Physical Principles**

(1) closed volume, helical velocity driven dynamo
(2) closed volume, dynamical magnetic relaxation

- **What the Toy Problems Capture**

(1) self-consistent backreaction and role of magnetic helicity dynamics
(2) theory of field saturation

- absence of shear and boundaries: first focus minimalistically on scale transfer of magnetic helicity rather than spatial flow.

**Dynamical $\alpha^2$ Nonlinear Mean-Field Theory** (e.g.BF02,B03)

**Basic relations**:

$$\partial_t\overline{\mathbf{A}} = -\overline{\mathbf{E}} - \nabla\overline{\phi} \qquad \partial_t\mathbf{a} = -\mathbf{e} - \nabla\phi$$

**Induction equations and Navier Stokes**:

$$\partial_t\overline{\mathbf{B}} = -\nabla\times\overline{\mathbf{E}} = \nabla\times\overline{\boldsymbol{\mathcal{E}}} - \eta\overline{\mathbf{J}} + \overline{\mathbf{Q}}(\overline{\mathbf{V}}), \quad \text{where } \overline{\boldsymbol{\mathcal{E}}} \equiv \overline{\mathbf{v}\times\mathbf{b}}$$

$$\partial_t\mathbf{b} = -\nabla\times\mathbf{e} = \nabla\times(\mathbf{v}\times\overline{\mathbf{B}}) + \nabla\times(\mathbf{v}\times\mathbf{b}) - \nabla\times\overline{\mathbf{v}\times\mathbf{b}} + \lambda\nabla^2\mathbf{b} + g(\overline{\mathbf{V}})$$

$$\partial_t\mathbf{v} = \mathbf{f} - \mathbf{v}\cdot\nabla\mathbf{v} + \overline{\mathbf{v}\cdot\nabla\mathbf{v}} - \nabla p + \mathbf{j}\times\overline{\mathbf{B}} + \overline{\mathbf{J}}\times\mathbf{b} + \mathbf{j}\times\mathbf{b} - \overline{\mathbf{j}\times\mathbf{b}} + \nu\nabla^2\mathbf{v} + f(\overline{\mathbf{V}})$$

**Differential Equations to be solved:**

$$\partial_t\langle\overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\rangle = 2\langle\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle - 2\lambda\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle + \nabla\cdot\langle\rangle_1$$

$$\partial_t\langle\mathbf{a}\cdot\mathbf{b}\rangle = -2\langle\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle - 2\lambda\langle\mathbf{j}\cdot\mathbf{b}\rangle + \nabla\cdot\langle\rangle_2$$

$$\partial_t\overline{\boldsymbol{\mathcal{E}}} = \partial_t\overline{\mathbf{v}\times\mathbf{b}} = \overline{\mathbf{v}\times\partial_t\mathbf{b}} + \overline{\partial_t\mathbf{v}\times\mathbf{b}}$$

$$= -\tfrac{1}{3}(\overline{\mathbf{v}\cdot\nabla\times\mathbf{v}} - \overline{\mathbf{b}\cdot\nabla\times\mathbf{b}})\overline{\mathbf{B}} - \tfrac{1}{3}\overline{\mathbf{v}^2}f(\overline{\mathbf{B}})\nabla\times\overline{\mathbf{B}} - \overline{\mathbf{v}\times\mathbf{b}}/\tau$$

- <u>minimal $\tau$</u> closure for triple corr. in $\partial_t\overline{\boldsymbol{\mathcal{E}}}$ (BF02)
  $\tau$ is damping time, not correl. time.
- <u>No first order smoothing needed</u>

## BUOYANT TUBE IN ROTATOR: BIHELICITY

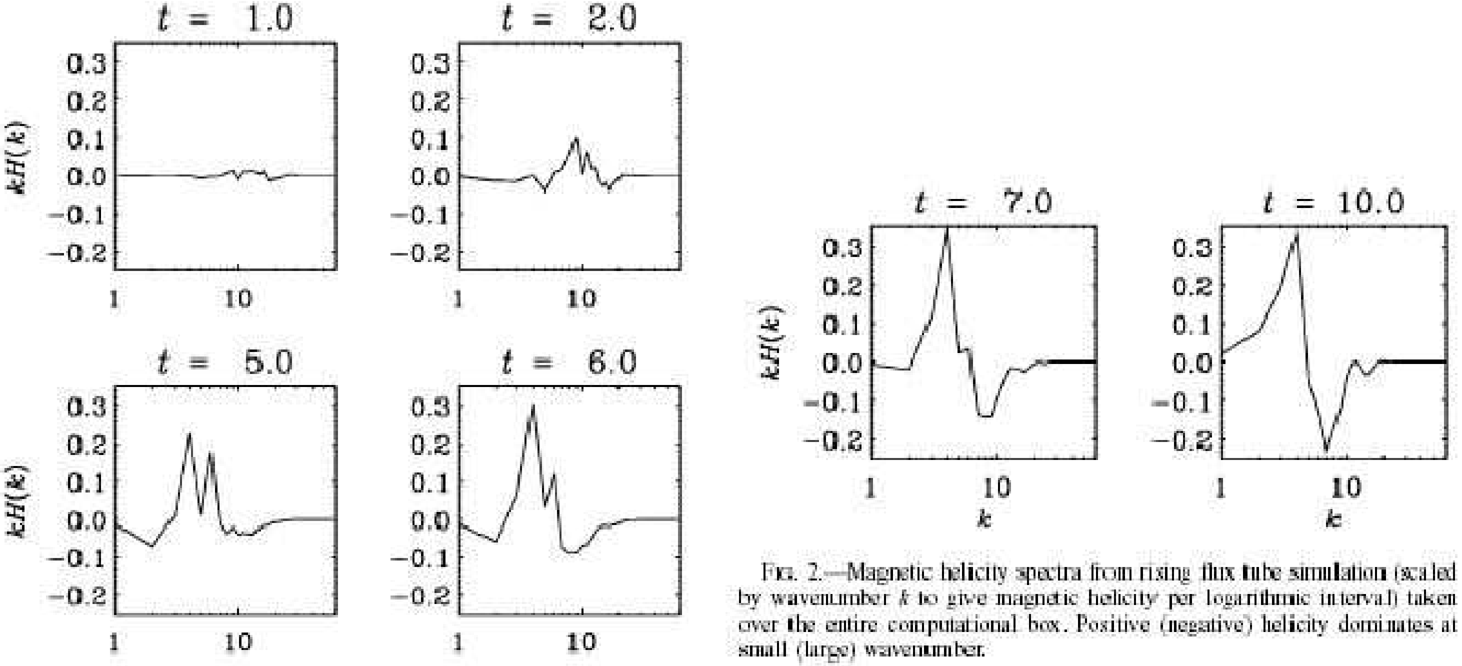



FIG. 2.—Magnetic helicity spectra from rising flux tube simulation (scaled by wavenumber $k$ to give magnetic helicity per logarithmic interval) taken over the entire computational box. Positive (negative) helicity dominates at small (large) wavenumber.

(Blackman & Brandenburg 03)

- NOTE: Berger's discussion
- NOTE: Kusano's discussion of small net helicity in AR
- NOTE: Demoullin 2002 scale of shear and sign of helicity injection

## HELICAL DYNAMO: THEORY

$$\partial_t H_1^M = \frac{2k_1\tau}{3}\left(k_2^2 H_2^M - H_2^V\right) H_1^M - 2\beta k_1^2 H_1^M - 2\nu_M k_1^2 H_1^M$$

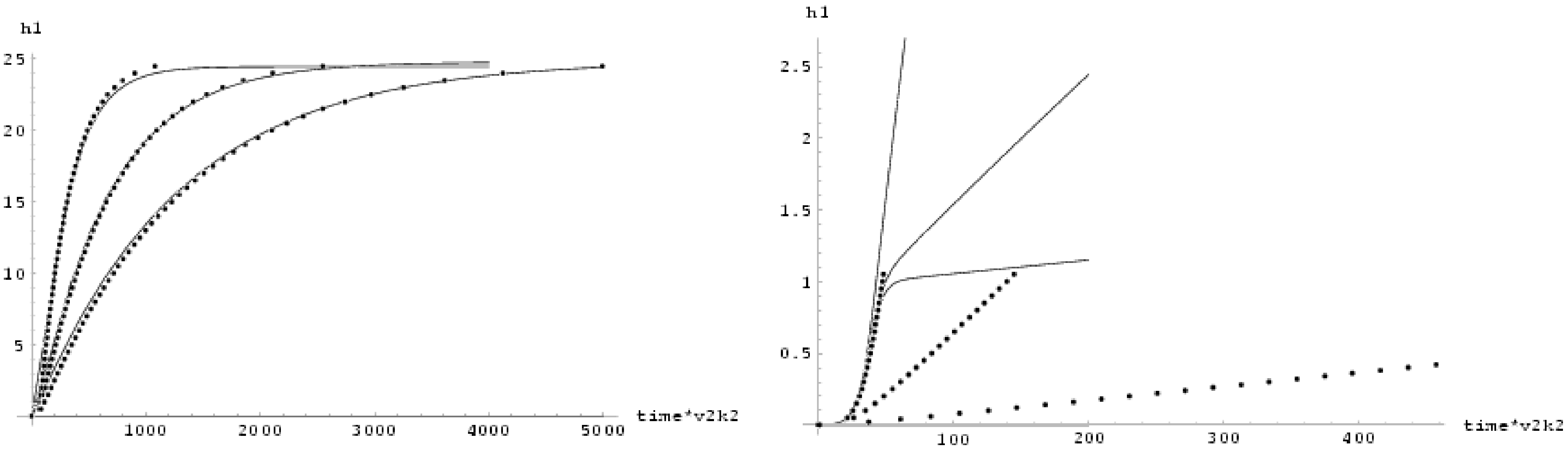
$$\partial_t H_2^M = -\frac{2k_1\tau}{3}\left(k_2^2 H_2^M - H_2^V\right) H_1^M + 2\beta k_1^2 H_1^M - 2\nu_M k_2^2 H_2^M$$

$$\partial_t H_2^V = 0$$

• Helical velocity driven dynamo "pumps" magnetic helicity of one sign to large scales and the other sign to small scales

## HELICAL DYNAMO: THEORY VS. SIMULUATION

**Large Scale Helical Field Growth:** 2-scale (Blackman & Field 02) vs. empirical fit formula of Brandenburg (01) Curves left to right: $R_M = 100, 250, 500$:




- At end of kinematic regime: $\overline{\mathbf{B}}^2 \sim f_h v_2^2 \left(\frac{k_1}{k_2}\right)(1+\Omega^2 t_{dif}^2)$
- At end of resistive regime: $\overline{\mathbf{B}}^2 \sim f_h v_2^2 \left(\frac{k_2}{k_1}\right)(1+\Omega^2 t_{dif}^2)$

## MAGNETIC HELICITY INJECTION INTO A CORONA

- Relative helicity injection

$$\frac{dH_c}{dt} = -2\int_c \mathbf{E}\cdot\mathbf{B}dV_c + 2\int_c (\mathbf{E}\times\mathbf{A}_p)\cdot d\mathbf{S} \quad (1)$$

$$= -\frac{1}{2\pi}\int\int_{\mathbf{B}_n\cdot\mathbf{B}'_n>0}\frac{d\theta}{dt}B_nB'_ndSdS' + \frac{1}{2\pi}\int\int_{\mathbf{B}_n\cdot\mathbf{B}'_n<0}\frac{d\theta}{dt}|B_nB'_n|dSdS' \quad (2)$$

$$= \frac{dH_r}{dt}(\mathbf{twist}) \qquad + \qquad \frac{dH_r}{dt}(\mathbf{writhe}) \quad (3)$$

(e.g. Berger & Ruzmaikin 2000; Demoulin et al. 2002, 2003)

# IMPLICATIONS OF FOOTPOINT SHEAR

(Demoulin et al. 2002)

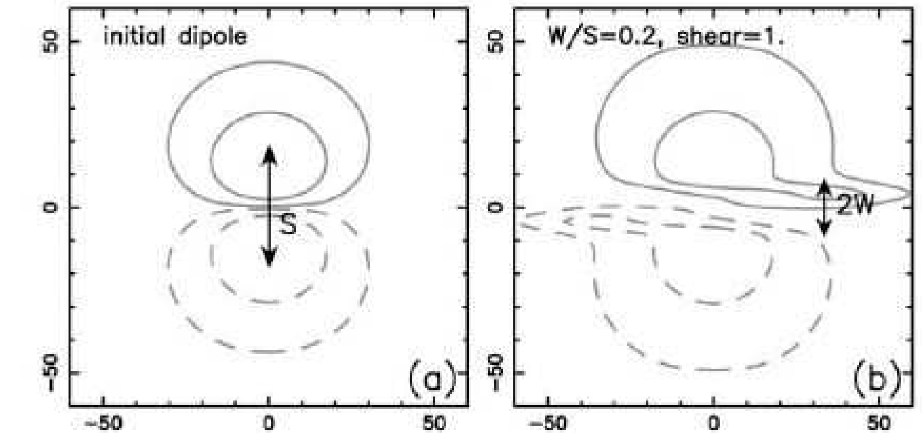



*Figure 1.* Magnetic field distribution for: (a) the initial bipolar configuration, and (b) after applying a shear concentrated in the vicinity of the PIL (see Equation (13). Isocontours of positive/negative $B_n$ are drawn with *continuous/dashed lines*.

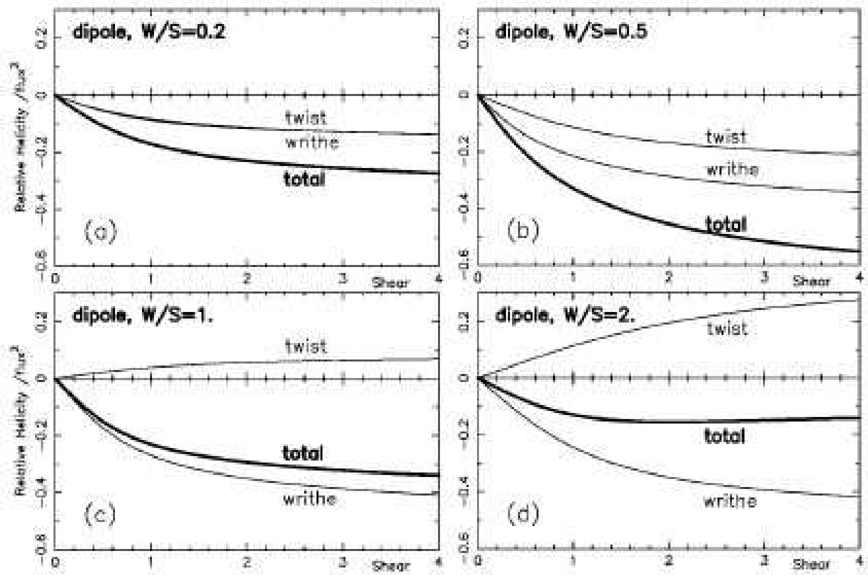
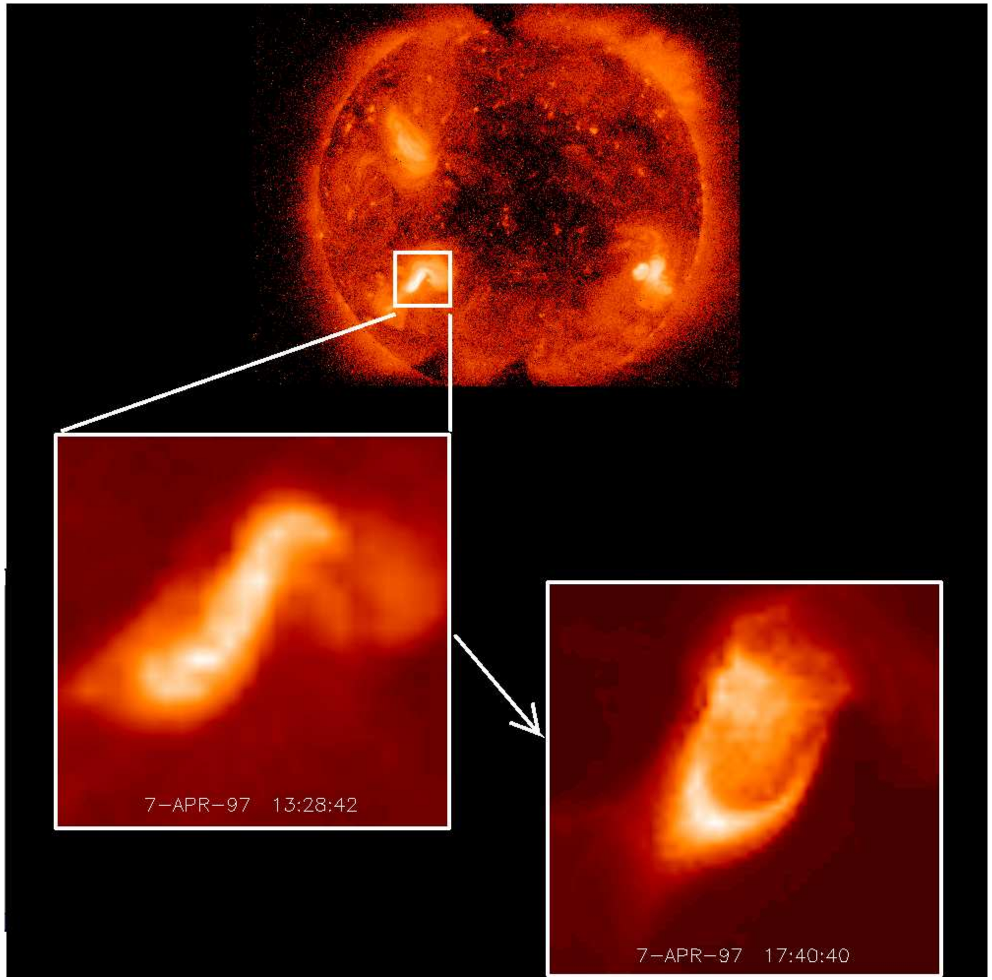
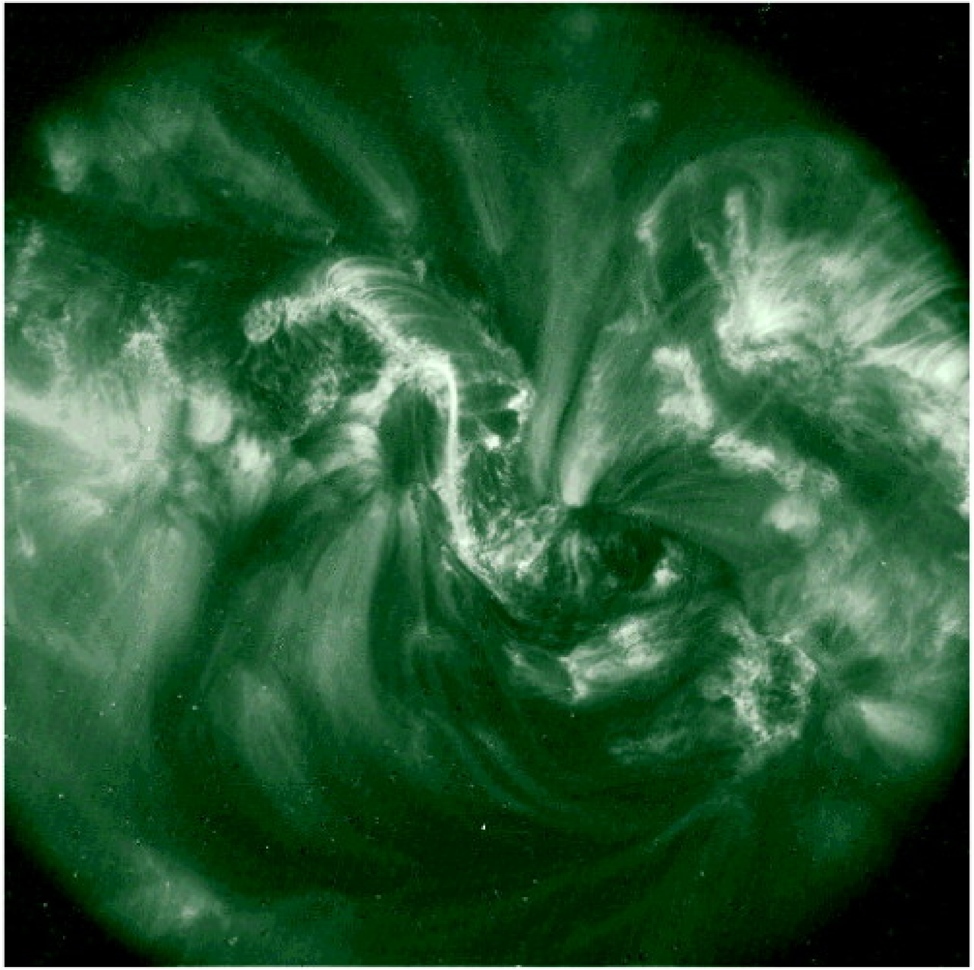



*Figure 2.* Injection of magnetic helicity by shearing motions in the dipolar configuration of Figure 1 for different extension of the shearing motions around the PIL. The abscissa is the shear distance in units of the bipole size $S$, and the helicity is written in units of $\Phi^2$ (where $\Phi$ is the magnetic flux of one polarity). Three quantities are plotted: twist, writhe and total magnetic helicity (see Equation (11)). The total injected helicity is drawn with *thicker lines*.




(YOHKOH X-ray image: Yohkoh Project, SXT Group, NASA, ISAS)

TRACE Image of a Sigmoid (Gibson et al. 2003) (195 Å)



## MORE PROPERTIES OF MAGNETIC HELICITY

• **Large Scale Helical Fields Minimize Energy**: For fixed $\langle \mathbf{A} \cdot \mathbf{B} \rangle$, min. energy state has $\nabla \times \mathbf{B} = \lambda \mathbf{B}$, with $\lambda$ = constant, ( $P, v^2 << B^2$).

• **Spectral flow:** $\lambda = \langle \mathbf{J} \cdot \mathbf{B} \rangle / \langle \mathbf{B}^2 \rangle = \langle \mathbf{B}^2 \rangle / \langle \mathbf{A} \cdot \mathbf{B} \rangle$ minimized

• **Spatial flow:** Minimizing energy implies $\nabla \lambda \rightarrow 0$;

e.g. Merge force-free tubes conserving $\langle \mathbf{A} \cdot \mathbf{B} \rangle = \langle \mathbf{A} \cdot \mathbf{B} \rangle_1 + \langle \mathbf{A} \cdot \mathbf{B} \rangle_2$:

$$\Delta \langle \mathbf{B}^2 \rangle = \Delta \langle \mathbf{B}^2 \rangle_2 + \Delta \langle \mathbf{B}^2 \rangle_1 = 0 = (\lambda_2 - \lambda_1) \Delta \langle \mathbf{A} \cdot \mathbf{B} \rangle_1$$

**MULTI-SCALE THEORY**

$$\partial_\tau h_1 = \frac{2}{3}\left(f_h + h_2 + \left(\frac{k_3}{k_2}\right)^{\frac{4}{3}} h_3\right)\frac{k_1}{k_2}h_1 - 2\left(\frac{q(h_1)}{3} + \frac{q(h_1)}{3}\left(\frac{k_2}{k_3}\right)^{\frac{4}{3}} + \frac{1}{R_M}\right)\left(\frac{k_1}{k_2}\right)^2 h_1 \tag{4}$$

$$\begin{aligned}\partial_\tau h_2 = \frac{-2}{3}\left(\frac{k_3}{k_2}\right)^{\frac{4}{3}} f_u h_3 h_2 - \frac{2}{3}\left(f_h + h_2\right)\frac{k_1}{k_2}h_1 + \frac{2q(h_1)}{3}\left(\frac{k_1}{k_2}\right)^2 h_1 \\ -2\left(\frac{q(h_1)}{3}g_u\left(\frac{k_2}{k_3}\right)^{\frac{4}{3}} + \frac{1}{R_M}\right)h_2\end{aligned} \tag{5}$$

and

$$\begin{aligned}\partial_\tau h_3 = \frac{2}{3}\left(\frac{k_3}{k_2}\right)^{\frac{4}{3}} f_u h_3 h_2 - \frac{2}{3}\left(\frac{k_3}{k_2}\right)^{\frac{4}{3}}\left(\frac{k_1}{k_2}\right) h_3 h_1 + \frac{2q(h_1)}{3}\left(\frac{k_2}{k_3}\right)^{\frac{4}{3}}\left(\frac{k_1}{k_2}\right)^2 h_1 \\ +\frac{2q(h_1)}{3}g_u\left(\frac{k_2}{k_3}\right)^{\frac{4}{3}} h_2 - \frac{2}{R_M}\left(\frac{k_3}{k_2}\right)^2 h_3\end{aligned} \tag{6}$$

**At end of fast growth regime:**

$$h_1 \simeq 1 - (k_2/k_3)^{4/3} + (k_2/k_3)^{8/3} \tag{7}$$

$$h_2 \simeq -1 + (k_2/k_3)^{4/3} \tag{8}$$

and

$$h_3 \simeq -(k_2/k_3)^{8/3} \tag{9}$$

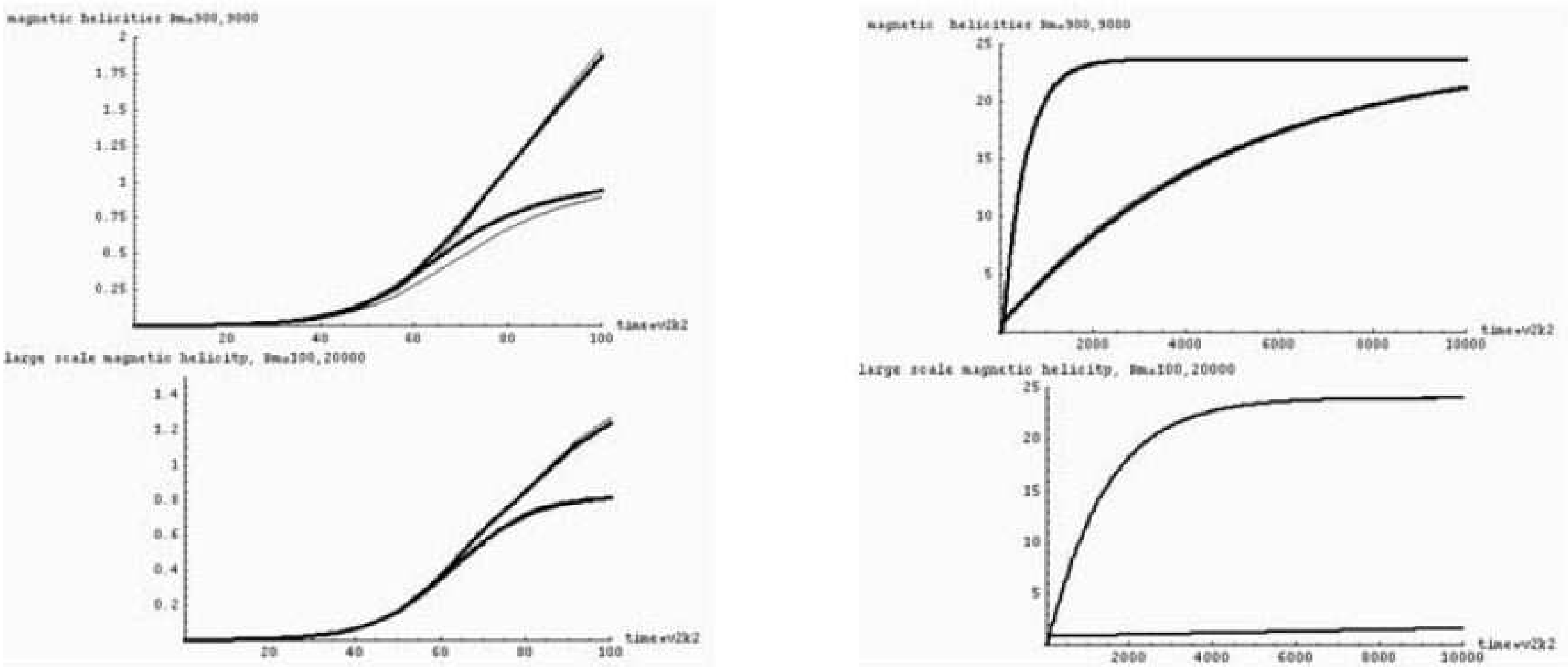



**Figure 1.** The magnetic helicity, $h_1$, for the four-scale approach (thin lines) and the two-scale approach (thick lines). The left-hand column of plots is for early times and the right-hand column is for a broader time range. For the top row, $k_1 = 1$, $k_2 = 5$, $k_3 = 30$, and $R_M = 900$ (top pair of curves), $R_M = 9000$ (bottom pair of curves). For the bottom row, $k_1 = 1$, $k_2 = 5$, $k_3 = 160$ and $R_M = 100$ (top pair of curves) $R_M = 2 \times 10^4$ (bottom pair of curves). Note that the two- and four-scale approaches are largely indistinguishable in all but the upper left-hand plot (see the text). [The high-$R_M$ curve in the lower right-hand side will eventually saturate at the same value $h_1 = (k_2/k_1)^2$ as the low-$R_M$ case but at much later times and thus undershoots the top curve for the plotted time range.]

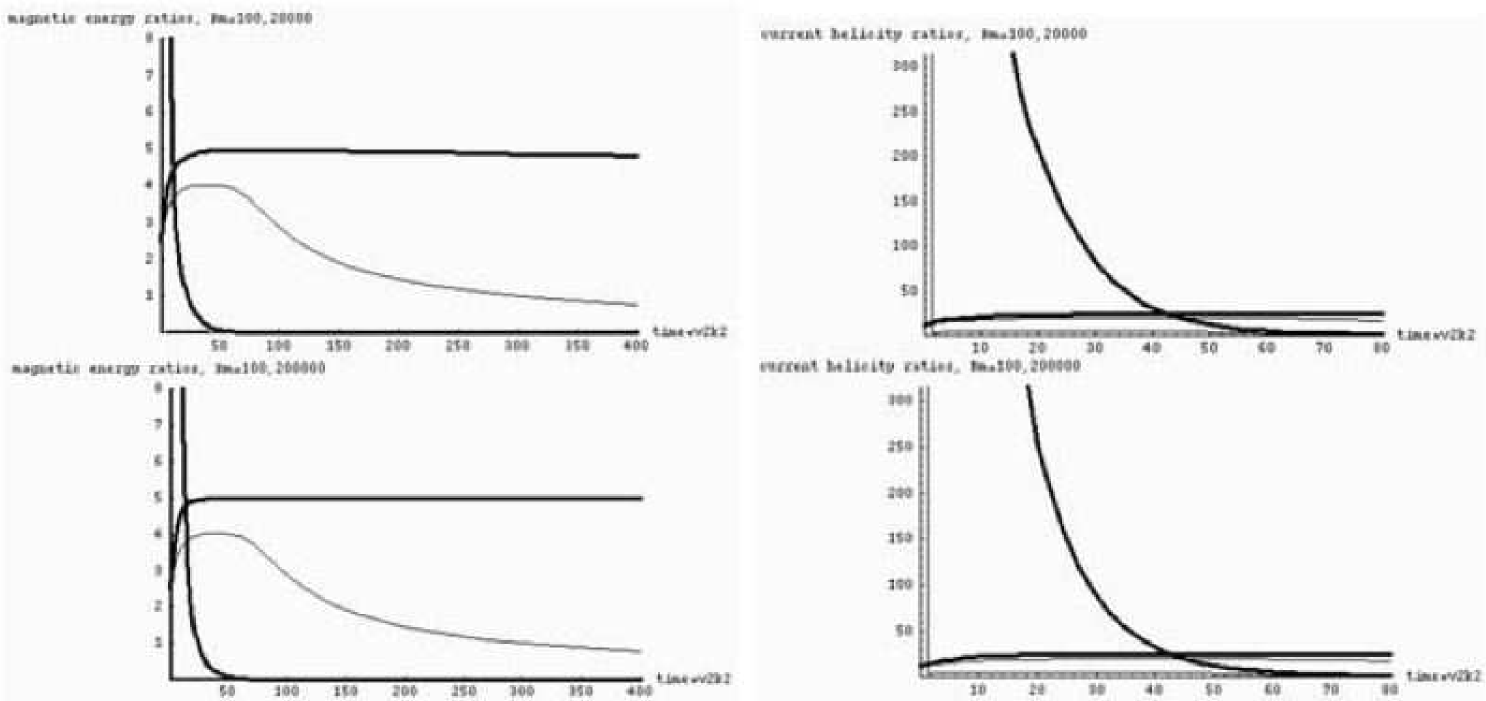
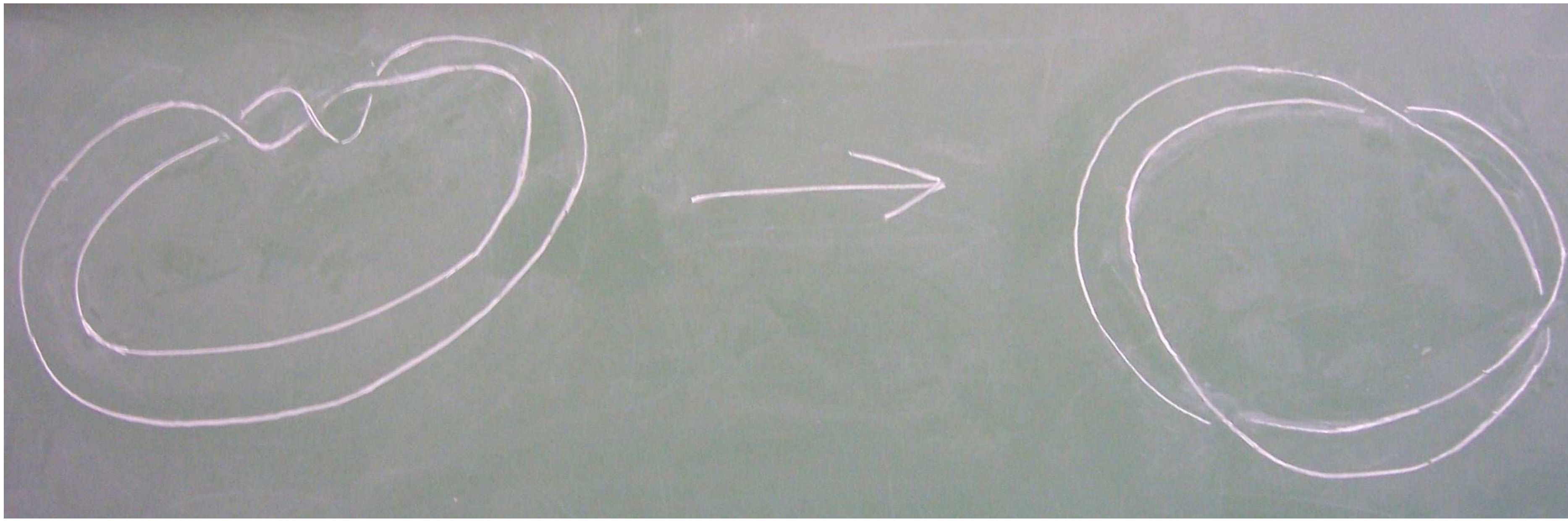


**Figure 3.** The absolute magnitude of the current helicity ratios $|k_2^2h_2/k_1^2h_1|$ and $|k_3^2h_2/k_1^2h_1|$, and helical magnetic energy ratios $|k_2h_2/k_1h_1|$ and $|k_2h_2/k_1h_1|$ assuming an initial seed of $h_2(0) = h_3(0) = -h_1(0)/2 = 0.0005$. for $k_1 = 1$, $k_2 = 5$, $k_3 = 160$. The top row shows plots for $R_{\rm M} = 100$ (thin lined curves) and $R_{\rm M} = 2 \times 10^4$ (thick lined curves) and the bottom row shows plots for $R_{\rm M} = 100$ (thin lined curves) and $R_{\rm M} = 2 \times 10^5$ (thick lined curves). In each plot, the quantities at $k_3$ dominate at early times, but then become subdominant to the values at $k_2$ at later times. The magnetic energy plot is shown for a broader time range. The location of the crossover for the large-$R_{\rm M}$ cases of $2 \times 10^4$ and $2 \times 10^5$ is independent of $R_{\rm M}$: the crossovers occur for the thick pairs of lines at the same time in the top and bottom rows. The crossover occurs much earlier for $R_{\rm M} = 100$ because the resistive wavenumber is closer to $k_3$ and thus the resistivity is more effective at early times in draining the current helicity and magnetic energy at $k_3$ than in the larger-$R_{\rm M}$ cases. For the current helicity, the crossover for $R_{\rm M} = 100$ occurs so early that it is not visible on the graph. Before the end of the kinematic regime ($t \lesssim 100$), the crossovers are complete and $k_2$ emerges as the dominant small scale. Since $k_3 = 160$ here, $R_{\rm M} = 100$ corresponds to $Pr_{\rm M} \simeq 1$.

## DYNAMICAL MAGNETIC RELAXATION

• idealized but relevant for "stage two"; field evolution in coronae



**Equations for Driven Two-Scale Helical Dynamo**

$$\partial_t H_1^M = \frac{2k_1\tau}{3}\left(k_2^2 H_2^M - H_2^V\right) H_1^M - 2\beta k_1^2 H_1^M - 2\nu_M k_1^2 H_1^M$$

$$\partial_t H_2^M = -\frac{2k_1\tau}{3}\left(k_2^2 H_2^M - H_2^V\right) H_1^M + 2\beta k_1^2 H_1^M - 2\nu_M k_2^2 H_2^M$$

$$\partial_t H_2^V = 0$$

**Equations for Driven Unihelical Magnetic Relaxation**

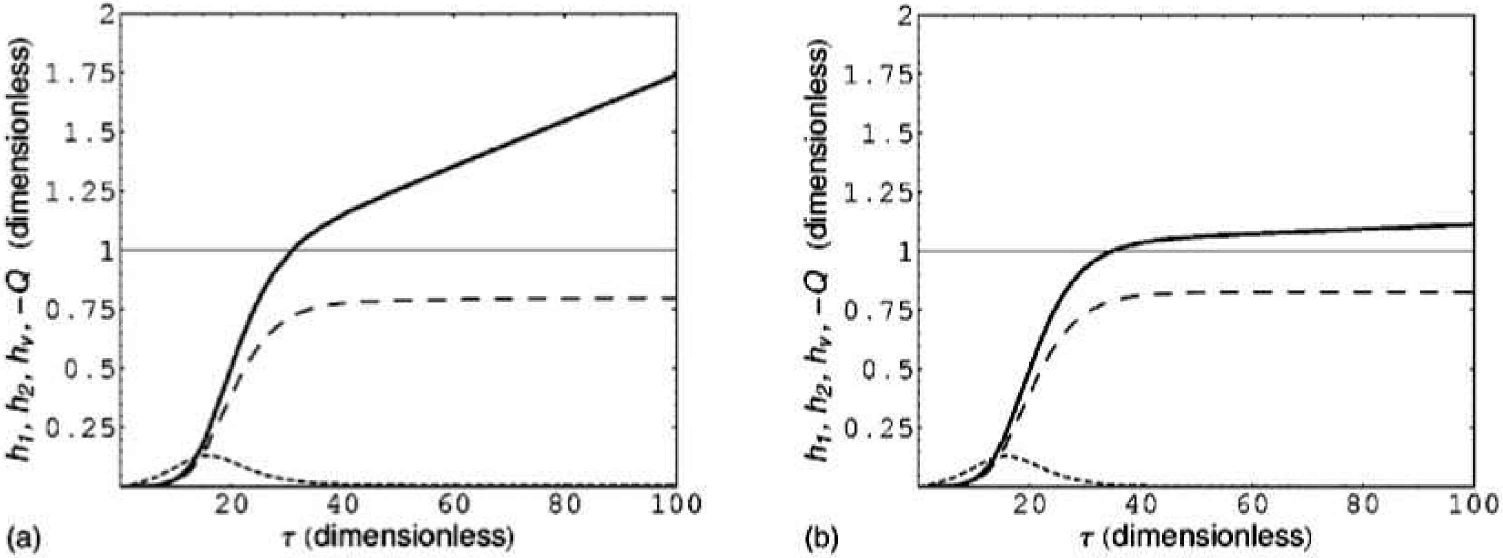
$$\partial_t H_1^M = \frac{2k_1\tau}{3}\left(k_2^2 H_2^M - H_2^V\right) H_1^M - 2\beta k_1^2 H_1^M - 2\nu_M k_1^2 H_1^M$$

$$\partial_t H_2^M = 0$$

$$\partial_t H_2^V \simeq \frac{2k_1\tau}{3} k_2^2 \left(k_2^2 H_2^M - H_2^V\right) H_1^M - 2\beta k_2^2 k_1^2 H_1^M - 2\nu k_2^2 H_2^V$$

- Unihelical relaxation drives injected magnetic helicity to large scales

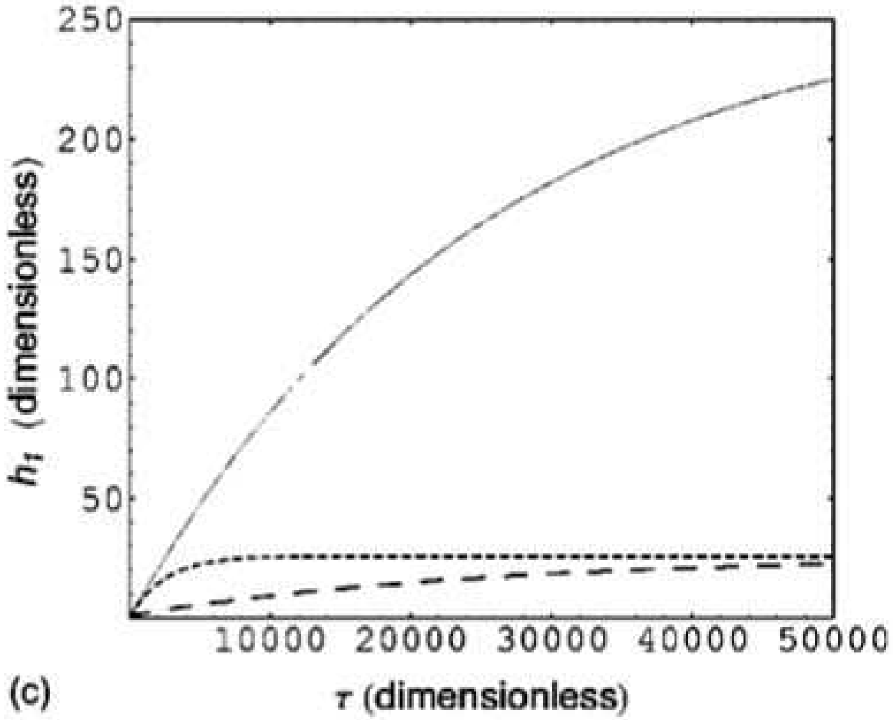
## Driven Unihelical Relaxation, Closed System




**Left:** Solutions for $k_1 = 1$, $f = 1$, $k_2 = 5$, $R_M = R_V = 200$, $h_2 = 1$ $h_2$ =solid thin; $h_1$ =solid thick, $h_V$ =long dashed

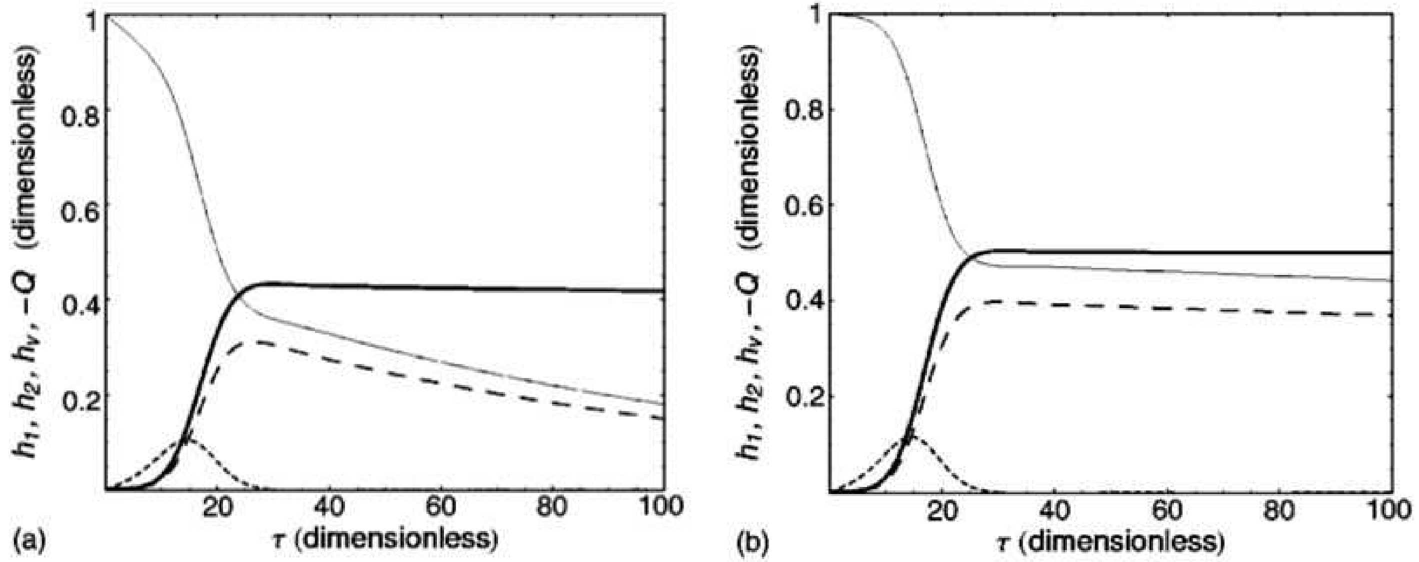
**Right:** Solutions for $k_1 = 1$, $f = 1$, $k_2 = 5$, $R_M = R_V = 2000$, $h_2 = 1$ $h_2$ =solid thin; $h_1$ =solid thick, $h_V$ =long dashed

**Driven Unihelical Relaxation, Closed System**




Growth of $h_1$ for late times, $h_2 = 1$. Top curve $R_M = 10R_V = 2000$, middle curve $R_M = R_V = 200$, bottom curve, $R_M = R_V = 2000$.

## Free Unihelical Magnetic Relaxation, Closed System




**Left:** Solutions for $k_1 = 1$, $f = 1$, $k_2 = 5$, $R_M = R_V = 200$.
$h_2$ =solid thin; $h_1$ =solid thick, $h_V$ =long dashed

**Right:** Solutions for $k_1 = 1$, $f = 1$, $k_2 = 5$, $R_M = R_V = 2000$. $h_2$ =solid thin; $h_1$ =solid thick, $h_V$ =long dashed

## SPATIAL MAGNETIC HELICITY FLOW AND SHEAR

• Local averaging implies boundary terms. $\overline{\boldsymbol{\mathcal{E}}}$ can be supplied by flux (e.g. Bhattacharjee - Hameri for fusion dyn 86)

• In reality: have time evolution, flux, and shear terms: need explicit forms

• Different approaches/different approximations: e.g Vishniac & Cho 01: mag. helicity current for a kinematic dynamo; but utitlizes anisotropic turbulence.

$$-2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}} = -\nabla\cdot\langle 2\overline{\Phi}\overline{\mathbf{B}} - \overline{\mathbf{A}}\times\partial_t\overline{\mathbf{A}}\rangle - 2\eta\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle - \partial_t\langle\overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\rangle \tag{10}$$

$$2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}} = -\nabla\cdot\langle 2\phi\mathbf{b} + \mathbf{a}\times\partial_t\mathbf{a}\rangle - 2\eta\langle\mathbf{j}\cdot\mathbf{b}\rangle - \partial_t\langle\mathbf{a}\cdot\mathbf{b}\rangle \tag{11}$$

• Then assume $\langle\mathbf{a}\cdot\mathbf{b}\rangle$ steady, drop resistive term, inverse F.T. $\nabla\phi$, use $\mathbf{a} = \tau(\mathbf{e} - \nabla\phi)$, use kinematic $\mathbf{v}$, and use FOSA:

$$\simeq \nabla\cdot\langle(\nabla\phi + \mathbf{e})\times\mathbf{a}\rangle \propto \nabla\cdot\langle(\overline{\mathbf{B}}\cdot\omega)(\overline{\mathbf{B}}\cdot\nabla)\mathbf{v}\rangle. \tag{12}$$

$$\partial_t\overline{\mathbf{B}} = \nabla\times\overline{\boldsymbol{\mathcal{E}}} + ... \tag{13}$$

## CURRENT HELICITY FLOW

• Subramanian & Brandenurg 2004 consider "current helicity evolution"

$$\partial\langle \mathbf{j}\cdot\mathbf{b}\rangle = -2\langle \mathbf{e}\cdot\nabla\times j\rangle - \nabla\cdot\langle 2\mathbf{e}\times\mathbf{j} + (\nabla\times\mathbf{e})\times\mathbf{b}\rangle$$

• For case of anisotropic turbulence from, one of the terms in the current helicity flow is the Vishniac-Cho flux term.

## STILL MORE CONTRIBUTIONS FROM SHEAR

• Including anisotropic turbulence using the "minimal tau closure (BF03)" for the full MHD equations leads also to extra contributions to $\overline{\boldsymbol{\mathcal{E}}}$: (e.g. Subramanian (05); Rudiger 68; Rogachevskii & Kleeorin 03; Rädler et al. 03)

• In 2 scale approximation:

$$\overline{\boldsymbol{\mathcal{E}}}_{sc} \propto c_1(b^2, v^2)\nabla(\mathbf{\Omega}\cdot\mathbf{B}) + c_2(b^2)\mathbf{\Omega}\cdot\nabla\overline{\mathbf{B}} \tag{14}$$

(For constant $c_1$ first term on right grows no field. For constant $c_2$ and closed volume $\langle\overline{\boldsymbol{\mathcal{E}}}_{sc}\cdot\overline{\mathbf{B}}\rangle$=0.)

• Simulations trying to test such terms (e.g. Brandenburg...)

**SIMPLEST KINEMATIC "SHEAR-CURRENT" DYNAMO**
(e.g. Rogachevskii- Kleeroin 03)

$$\overline{\boldsymbol{\mathcal{E}}} \propto c_3(v^2)(\nabla \times \overline{\mathbf{V}}) \times \overline{\mathbf{J}}. \tag{15}$$

• need only field gradient and shear for exponential growth(!)
• simple example: $\partial_z B_r(z) < 0, \quad \partial_r V_\phi(r) < 0$
• my interpretation:
1) Differential rotation plays role of vorticity on an eddy $\rightarrow$ Clockwise twist.
2) Gradient in mean field favors more eddy displacement toward $+z$.
3) Statistically provides same mean sense of twist for eddies: poloidal fields superimpose and add $\rightarrow$ dynamo.
4) key pseudoscalar: $\langle(\nabla \times \overline{\mathbf{V}}) \cdot \nabla \overline{B}^2\rangle$.
• message: pseudo scalar is always essential for large scale field growth!

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• Aside: **cross-helicity** equations are needed to be fully complete picture. Not yet been done..

## Steady-State Solution and subtleties for $\alpha$ quenching

In steady-state

$$\alpha_d = \tau\tilde{\alpha} = \frac{\alpha_{d0} + R_{M,2}(\beta\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle + \nabla\cdot\langle\ \rangle_2)/B_{eq}^2}{1 + R_{M,2}\langle\overline{B}^2\rangle/B_{eq}^2}$$

• Kleeorin Ruzmaikin 1982 (=Gruzinov Diamond 1995) results emerge. Cattaneo & Hughes (1996) results for uniform $\overline{\mathbf{B}}$ also emerge.
• For nonuniform $\overline{B}$, growth depends on: $\langle\boldsymbol{\overline{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle$

$$= \frac{\alpha_{d0} + R_{M,2}\beta\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle/B_{eq}^2}{1 + R_{M,2}\langle\overline{B}^2\rangle/B_{eq}^2}\langle\overline{B}^2\rangle - \beta\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle$$

$$= \frac{\alpha_{d0}}{1 + R_{M,2}\langle\overline{B}^2\rangle/B_{eq}^2}\langle\overline{B}^2\rangle - \frac{\beta}{1 + R_{M,2}\langle\overline{B}^2\rangle/B_{eq}^2}\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle$$

• degeneracy (BB02): constant $\beta$ emerges as same as an artificially imposed symmetric, resisitive quenching of $\alpha_d$ and $\beta$.
• Current helicities are equal and opposite in steady state so in fact $\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle \propto \alpha_d - \alpha_{d0}$. This implies,

$$\alpha_d = \frac{1 + R_{M,2}\beta/\beta_0}{1 + R_{M,2}(\beta/\beta_0 + \overline{B}^2/B_{eq}^2)}$$

.

- can break the degeneracy by solving for the saturated value of $\overline{\mathbf{B}}^2$.

## SUMMARY OF CONCEPTS

• Kinetic helicity affects both large and small scale spectra

• Ask not whether mean field theory is correct but whether we have the correct mean field theory

• Simple $\alpha^2$ helical dynamo growth and saturation explained with multiscale "spectral transfer" theory incorporting mag. hel. conservation dynamically.

• no first order smoothing approximation

• Closed helical velocity drivern dynamo pumps opposite sings of mag. hel. to large & small scales (spectral transfer)

• Fast amplfication and sufficiently strong fields to $\overline{B}^2 \sim (k_1/k_f)\mathbf{b}^2$ before resistive evolution even without boundaries or shear.

• No contradiction with formulae of the form $\alpha = \alpha_0/(1 + R_M\overline{B}^2/\mathbf{b}^2)$: this emerges as the late time steady state limit.

• build up of small scale magnetic energy is not a threat: (1) small scale mag. helicity rather than mag. energy is what matters (2) Moreover, for a system driven with velocity, small scale field falls short of equipartition by factor of order unity in steady state.

**BUT**

- Higher resolution simulations needed even for $\alpha^2$ dynamo
- Role of shear and boundary terms are important: anisotropic turb, heli. currents
- In reality: non-helical growth, helical growth, and large and small scale helicity currents.
- Boundaries imply spatial helicty transfer as well as spectral helicity transfer
- Boundary flow of helicity in sun keeps dynamo "fast" and unsaturated $\rightarrow$ FAST solar cycle
- Important to understand how specific cases relate to the same fundamental equations: distinguish and understand self-consistent theories that arise from different approximations (BJ05). (e.g. BF compared to Vishniac-Cho, compared to Fusion dynamo)
- Separate debate is which generalized or simplified version applies to nature.
- Dominant fields need not be helical
- Note: no contradiction with the MRI