Helical Dynamo

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Kazantsev (1968); Kraichnan (1968); Vainshtein & Kichatinov (1986); Kulsrud & Anderson (1992); Berger & Rosner (1995)

Origin of Large-Scale Magnetic Fields

ISM turbulence:

Correlation length: $l_V \sim 100\,pc$

Velocity: $V \sim 10^6\,cm/s$

Magnetic Field in the Galaxy:

Scales: up to $\boxed{10kpc}$

Strength: $1-10\mu G$

?

Fluctuations ~ mean field

[Zweibel & Heiles, Nature 385 (1997) 131]

Magnetic Dynamo

Kinetic Helicity: $h = \langle V \cdot [\nabla \times V] \rangle$

$h = 0 \Rightarrow$ magnetic fluctuations scale: $l_B < l_V$

$h > 0 \Rightarrow$ $\partial_t \bar{B} = \nabla \times (\alpha \bar{B}) + \beta \Delta \bar{B}$

$\bar{B}$ is the mean field, $\alpha \sim h l_V / V < V$ $\beta \sim V l_V$

$l_B \sim \beta / \alpha > l_V$ Large-scale field is possible

[Steenbeck, Krause & Radler, Z. Naturforsch 21a (1966) 369]

α–dynamo Mechanism

$$B(x,t) = \overline{B}(x,t) + b(x,t)$$

mean field fluctuations

assumption: $l_b << l_{\overline{B}}$

$$\partial_t \overline{B} = \nabla \times \left(\alpha \overline{B}\right) + \beta \Delta \overline{B} \quad (1)$$

Magnetic energy is related to the magnetic correlator:

$$\langle B(x,t) B(x',t) \rangle = \overline{B}\,\overline{B} + \langle bb \rangle \quad (2)$$

small by assumption

Numerics: Large-scale field is not described by (1)
[Fausto Cattaneo talk]

Can we neglect fluctuations $\langle bb \rangle$ in the correlator (2)?

Kazantsev-Kraichnan Dynamo Model

Velocity field is given:

$$\langle v^i(\mathbf{x},t)v^j(\mathbf{x}',t')\rangle = \kappa^{ij}(|\mathbf{x}-\mathbf{x}'|)\delta(t-t') \qquad \langle \mathbf{v} \rangle = 0$$

$$\kappa^{ij}(x) = \kappa_N\left(\delta^{ij} - \frac{x^i x^j}{x^2}\right) + \kappa_L \frac{x^i x^j}{x^2} + g\epsilon^{ijk}x^k$$

Need to find magnetic correlator:

$$H^{ij}(x,t) = \langle B^i(\mathbf{x},t)B^j(0,t)\rangle$$

$$H^{ij} = M_N\left(\delta^{ij} - \frac{x^i x^j}{x^2}\right) + M_L \frac{x^i x^j}{x^2} + K\epsilon^{ijk}x^k \qquad M_N = M_L + xM_L'/2$$

Need to find:

M_L - magnetic energy

K - magnetic helicity

Vainshtein-Kichatinov Equations

$$\partial_t M = \frac{1}{x^4}\frac{\partial}{\partial x}\left(x^4 \kappa \frac{\partial M}{\partial x}\right) + GM - 4hK$$

$$\partial_t K = \frac{1}{x^4}\frac{\partial}{\partial x}\left(x^4 \frac{\partial}{\partial x}\left[\kappa K + hM\right]\right)$$

$h = g(0) - g(x)$ kinetic helicity

$\kappa = 2\eta + \kappa_L(0) - \kappa_L(x)$ kinetic energy

$G = \kappa'' + 4\kappa'/x$

[Vainshtein & Kichatinov, J. Fluid Mech. 168 (1986) 73]

Self-Adjoint Dynamo Equations

$$\partial_t M = \frac{1}{x^4}\frac{\partial}{\partial x}\left(x^4 \kappa \frac{\partial M}{\partial x}\right) + GM - 4hK$$

$$\partial_t K = \frac{1}{x^4}\frac{\partial}{\partial x}\left(x^4 \frac{\partial}{\partial x}\left[\kappa K + hM\right]\right)$$

Change of variables:

$$M = \frac{\sqrt{2}}{x^2} W_2 \qquad K = -\frac{1}{\sqrt{2}x^4}\frac{\partial}{\partial x}\left(x^2 W_3\right)$$

transforms the system to the self-adjoint form!
Has not been known before.

[Boldyrev, Cattaneo & Rosner, astro-ph/0504588]

Analogy with Shrodinger equation….

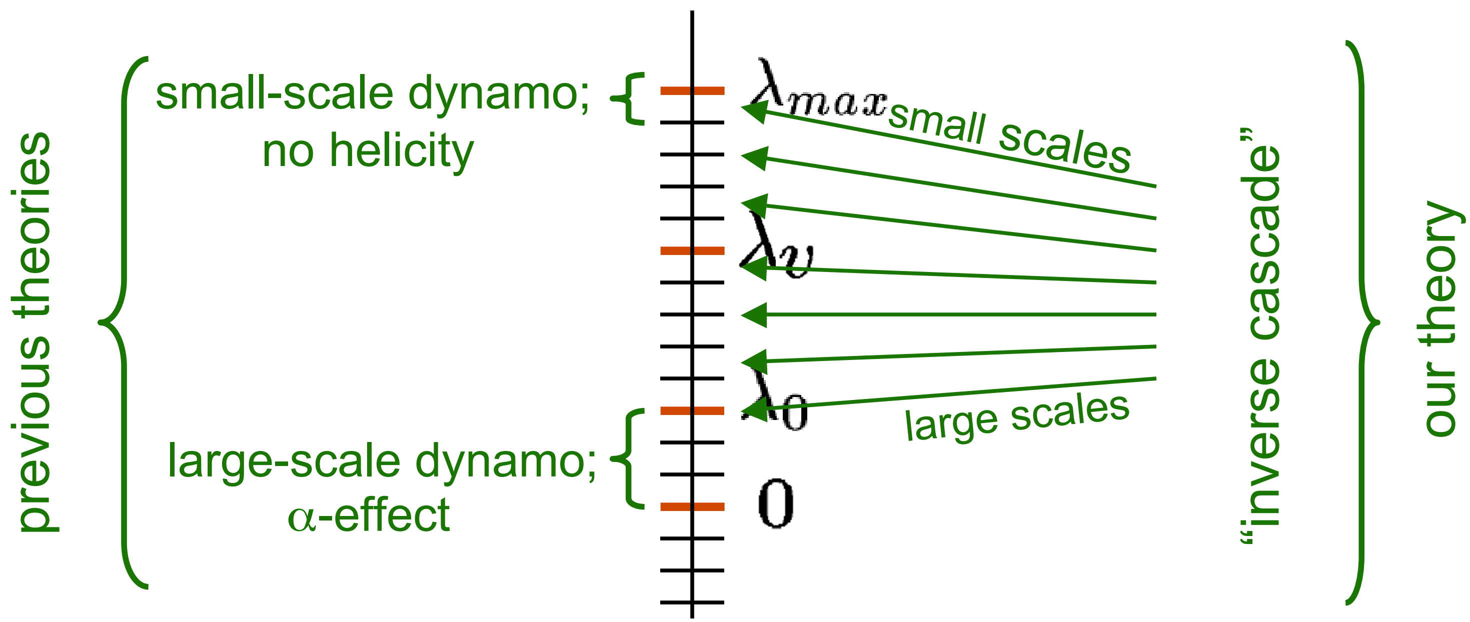
Eigenmodes….

Variational principle….

Large-Scale Dynamo Action

$$M_\lambda, K_\lambda \propto \frac{1}{x^2} \exp\left(\lambda t - k_\lambda x\right) \times [\text{Oscillatory terms}]$$

$$k_\lambda = [(\lambda - \lambda_0)/\kappa_0]^{1/2}$$



Conclusions

1. Dynamo action is self-adjoint.
   Magnetic energy and helicity grow as eigenmodes.
   Only fastest and slowest modes were known before.

2. Scale separation does not generally hold.
   Large and small scales are interdependent.

3. Helps understand nonlinear results:

   [Fausto talk, this session]…,
   [Vainshtein & Cattaneo (1992)]…
   [Maron & Blackman (2002)]…
   …