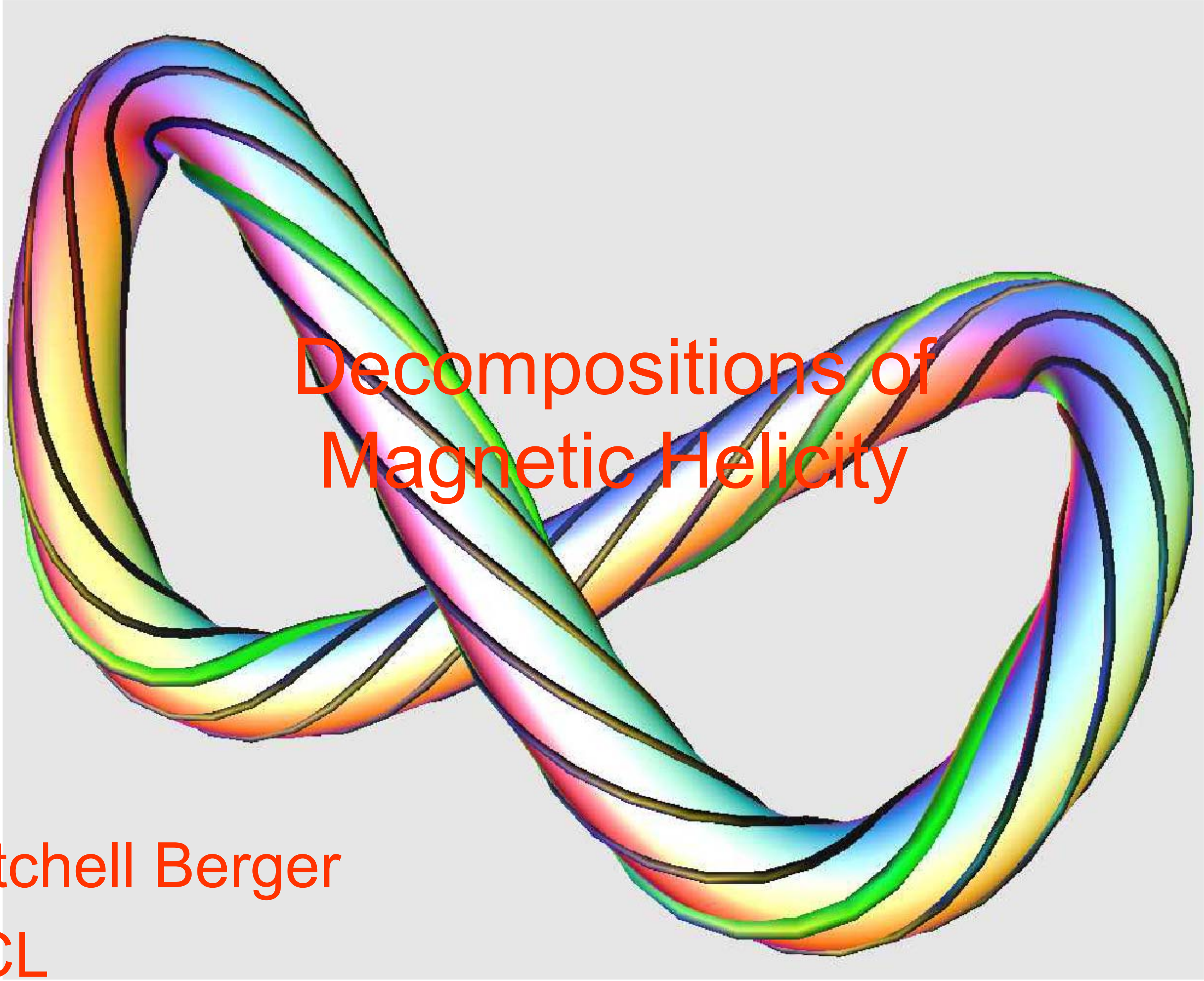

Decompositions of Magnetic Helicity

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Decompositions of Magnetic Helicity

1. Fourier Spectra
2. Poloidal-Toroidal
3. Wavelet Transform
4. Regions of Space
5. Self and Mutual Helicity
6. Twist and Writhe

Fourier Spectra

Physical Meaning: Represent field as sum of circularly polarized modes. Each mode has self linking, but there is no net linking between modes.

$$\nabla \cdot \mathbf{B} = 0 \qquad \Rightarrow \qquad \mathbf{k} \cdot \mathbf{B}(\mathbf{k}) = 0.$$

$$\mathbf{A}(\mathbf{k}) = \frac{-i\mathbf{k} \times \mathbf{B}}{k^2}.$$

Define

$$h(\mathbf{k}) = \mathbf{A}^*(\mathbf{k}) \cdot \mathbf{B}(\mathbf{k}).$$

Gauge invariance:

$$\begin{aligned} \text{let} \qquad \delta\mathbf{A}(\mathbf{x}) &= \nabla\psi(\mathbf{x}). \\ \Rightarrow \delta\mathbf{A}(\mathbf{k}) &= -i\mathbf{k}\psi(\mathbf{k}) \\ \text{and} \qquad \delta h(\mathbf{k}) &= i\psi(\mathbf{k})\mathbf{k} \cdot \mathbf{B}(\mathbf{k}) \\ &= 0. \end{aligned}$$

But …

Spectral technique does not work when a mean field is present

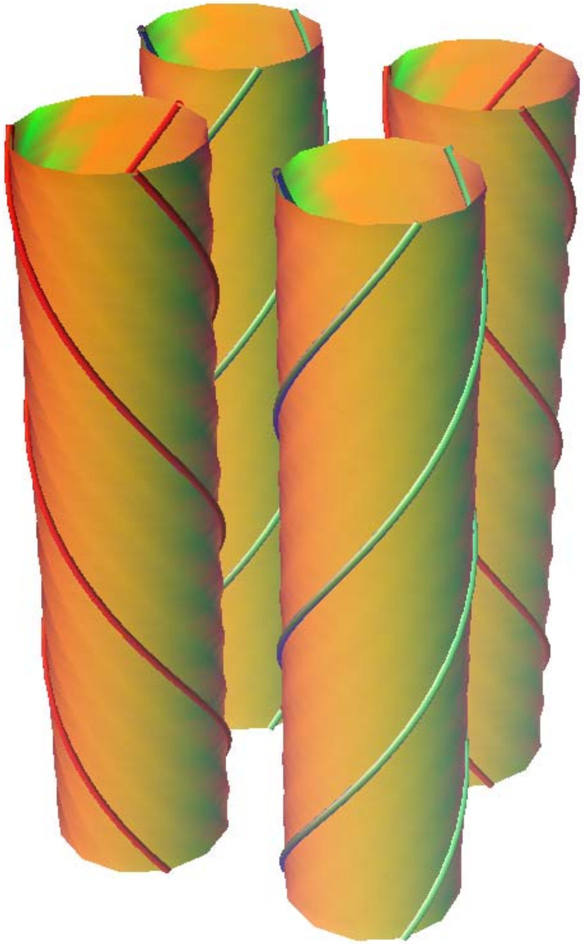
Berger 1997

Sun: negative helicity in North, positive in South – spectrum may miss most of the helical structure!

Fourier helicity spectra does not always detect helical structure …

$$\mathbf{B}_{poloidal} = (0, 0, \sin x \sin y)$$
$$\mathbf{B}_{toroidal} = (\sin 2x \cos 2y, -\cos 2x \sin 2y, 0)$$

Helicity spectrum is identically zero!



Poloidal - Toroidal

Physical Meaning: Poloidal and Toroidal Fields link each other, but not themselves.

Let

$$\begin{aligned}\mathcal{L} &\equiv \hat{z} \times \nabla && \text{planar} \\ &\equiv \hat{r} \times \nabla && \text{spherical}\end{aligned}$$

and

$$\mathbf{B} = \mathcal{L}T + \nabla \times \mathcal{L}P$$

The helicity between planes $z = z_1$ and $z = z_2$ can be written

$$\boxed{H = 2 \int \mathcal{L}T \cdot \mathcal{L}P \, \mathrm{d}^3x}.$$

We can write this as

$$\boxed{H = \int_{z_1}^{z_2} F(z) \, \mathrm{d}z},$$

$$F(z) = 2 \int \mathcal{L}T \cdot \mathcal{L}P \, \mathrm{d}x \, \mathrm{d}y.$$

Wavelets

Physical Meaning: detect helicity fluctuations of arbitrary scales near arbitrary positions

Let $f(x, r)$ be the helicity of a spherical shell centred at x of radius r. Take integrals weighted by favourite weighting function (mother wavelet) to get $\psi(x, r)$.

Regions of Space

Physical Meaning: Only measure helicity arising from internal structure (source currents) in each region. Measure global helicity by placing vacuum fields in each region.

global and regional helicities

Let space be divided into N regions. Let H_i be the helicity in region i (relative to vacuum field). Let H_{global} give the helicity of all space, replacing the field in each region by a vacuum field with the same fluxes.

$$H(\text{all space}) = H_1 + H_2 + \cdots + H_N + H_{global}$$

The H_{global} term is necessary! It contains all the data concerning fluxes in multiply connected regions and all fluxes through boundaries. Without it we could let $N \to \infty$ and let H_i become a helicity density. But helicity density does not exist!