Reconnection and Line Tying

Ellen Zweibel

zweibel@astro.wisc.edu

Departments of Astronomy & Physics

University of Wisconsin, Madison

and

Center for Magnetic Self-Organization

in

Laboratory and Astrophysical Plasmas

Why is this Important?

- Fieldlines in stellar & accretion disk coronae rooted in dense medium; what is effect on resistive stability?
  - energy storage
  - energy release
- Laboratory experiments have line tying: RWX, RSX
- Line tying has a dual role
  - Suppresses reconnection through magnetic tension
  - Forces reconnection through current sheet formation

The Plan of This Talk

- Effect of line tied boundary conditions
- Reconnection in a line tied slab **with Yi-Min Huang**
- Example of current sheet formation

Boundary Conditions

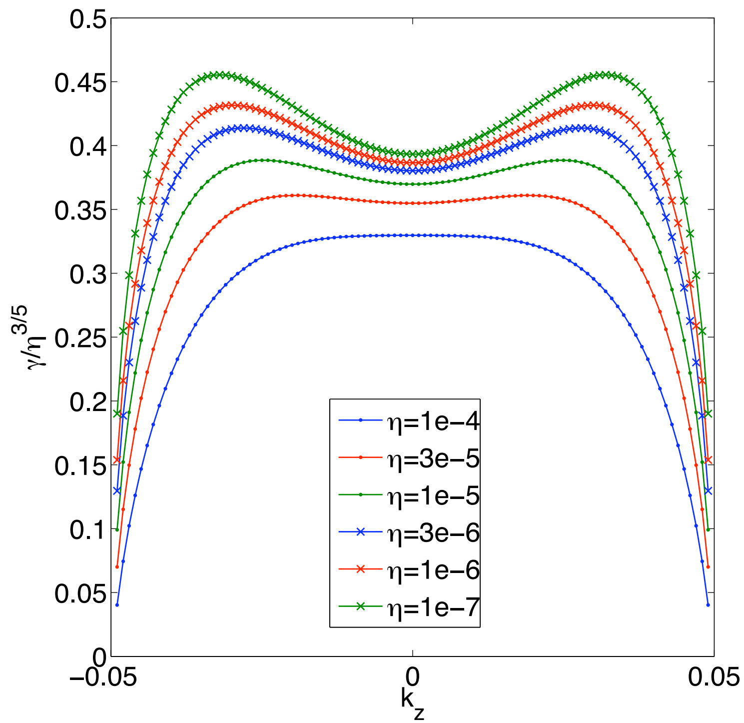
- Sheared field $\mathbf{B} = \hat{y}B_y(x) + \hat{z}B_z(x)$
- Perturbations $\delta Q = e^{\gamma t + i(k_y y + k_z z)} q(x)$
- Tearing modes localized to rational surfaces x_r where $\mathbf{k} \cdot \mathbf{B} \equiv 0$
- Single Fourier component cannot satisfy line tied boundary conditions $\delta \mathbf{V} = \delta B_z = 0$ at $z = (0, L)$.

Recall Ideal Case

- Line tied kink stable if $L < L_c \sim V_A/\gamma_p$
- If same here, $L_c \propto \eta^{-3/5}$.

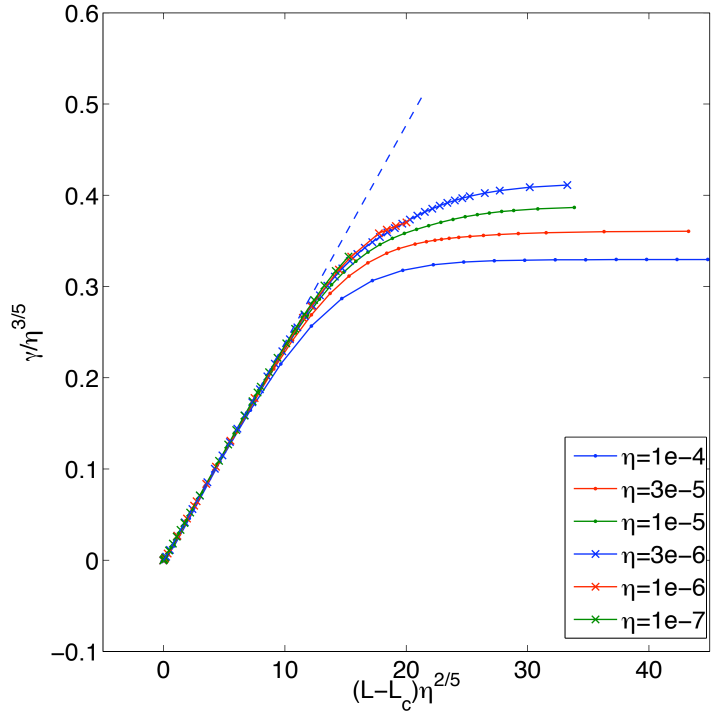
We will show that $L_c \propto \eta^{-2/5}$.

Periodic System



Tearing mode growth rates γ_p for the Harris sheet equilibrium ($B_y \propto \tanh x \ll B_z$) with periodic boundary conditions, for different η.

Line Tied System



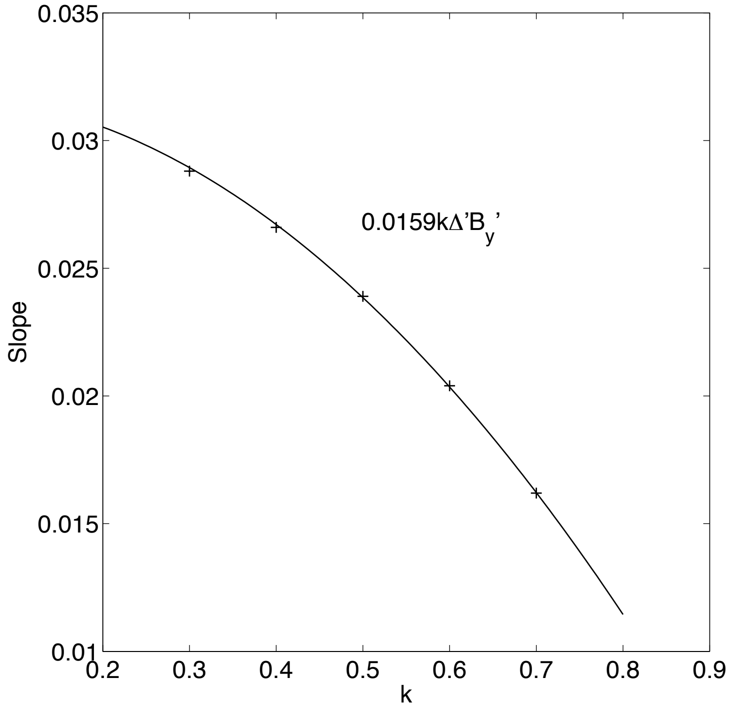
Scaled growth rates vs scaled length for Harris sheet with line tied b.c.. As L increases beyond critical length L_c, $\gamma \to \gamma_p$. At first $\gamma \propto \eta$, but as L increases there is a transition to $\gamma \propto \eta^{3/5}$. See also Delzanno & Finn.

The Transition

- At long L, a pair of modes $\pm k_z = \pm \frac{n\pi}{L}$ can combine to satisfy line tied B.C.
- Corresponding rational surfaces are $x_n \sim \frac{n\pi}{k_y L} \frac{B_z}{B_y'}$.
- Tearing mode layer width $\delta \propto (\Delta^{'})^{1/5} \eta^{2/5}$.
- $\gamma_p \propto (\Delta^{'})^{4/5} \eta^{3/5}$

Transition is at $L_t \propto \eta^{-2/5}$, at which $\delta \sim x_n$. For $L_c \ll L < L_t$, $\gamma \propto \eta L$.

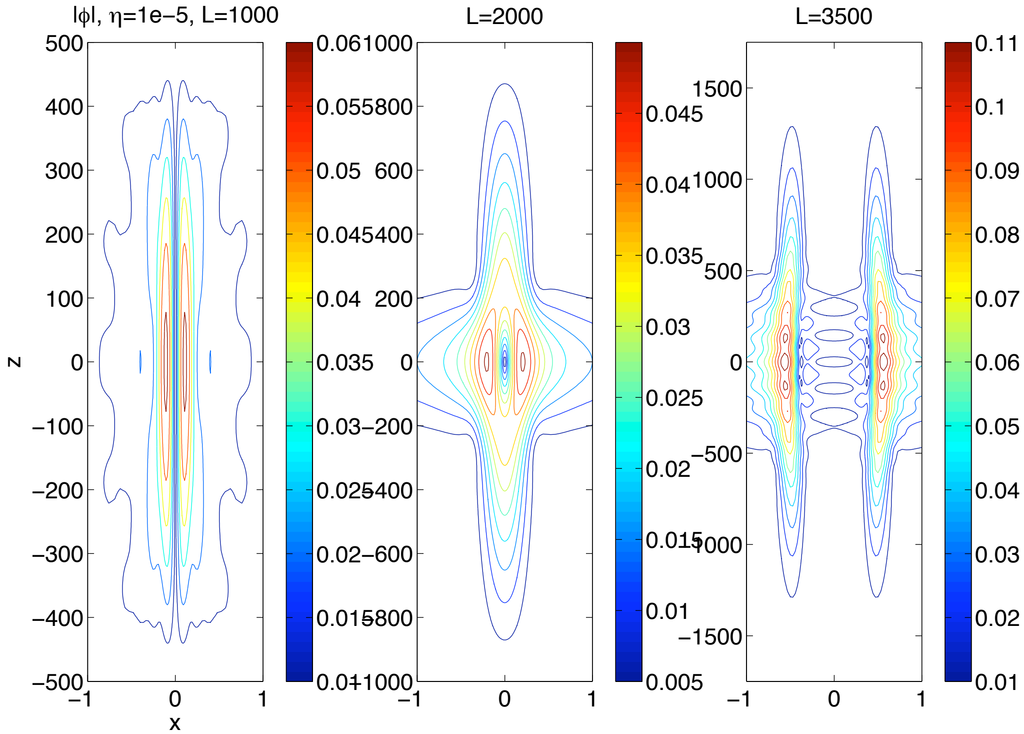
The Slope



Find the slope of the $\gamma - L$ relation such that $\gamma = \gamma_p$ at $L = L_t$.

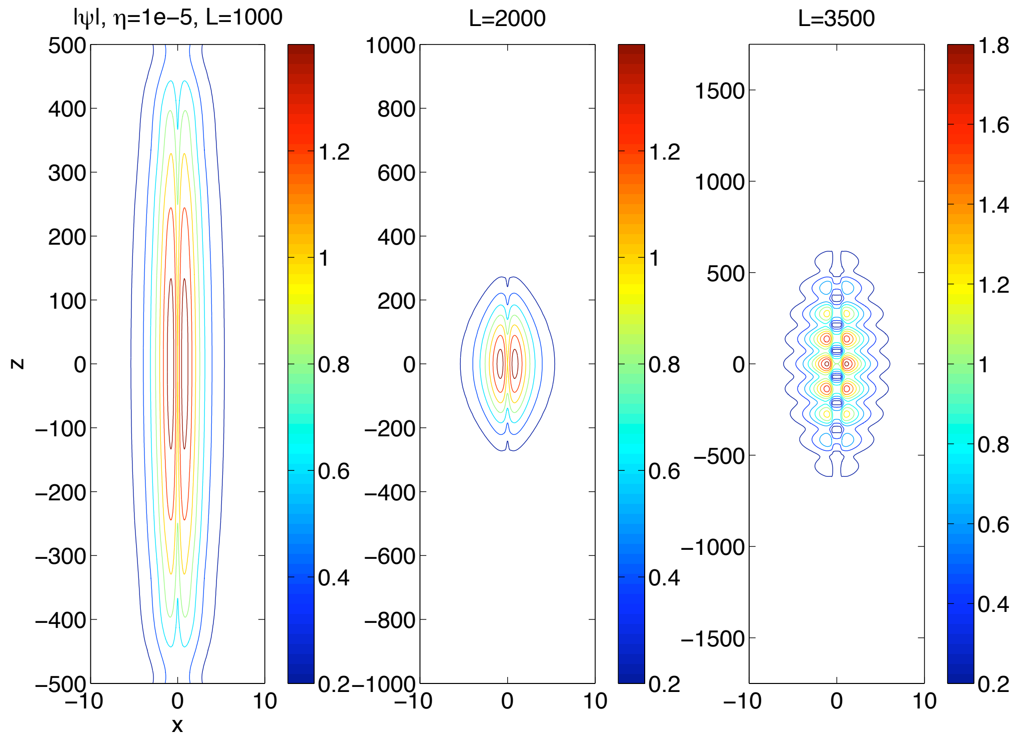
The fit is very good.

The Stream Function



Stream function near L_c, near L_t, and $L \gg L_t$.

The Flux Function



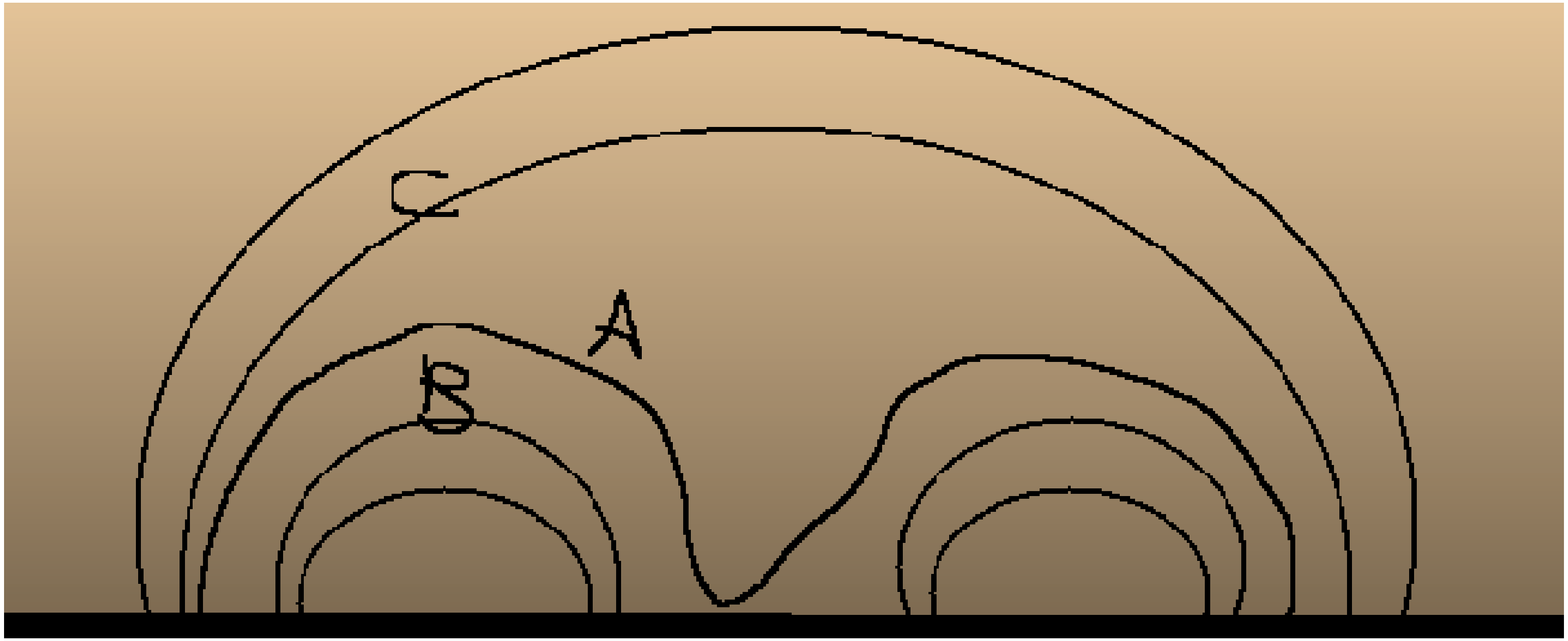
Flux function for the same 3 values of L. The mode approaches a constant length and high harmonics play an increasing role in localizing it.

Is $\gamma \propto \eta$ Tearing?

- Growth rate depends on $\Delta^{'}$ and only k_y values for which the periodic system is unstable are unstable here.
- But equilibrium also evolves on resistive timescale.
- May be too slow to be interesting,
- Except for nonlinear evolution (high k structure).

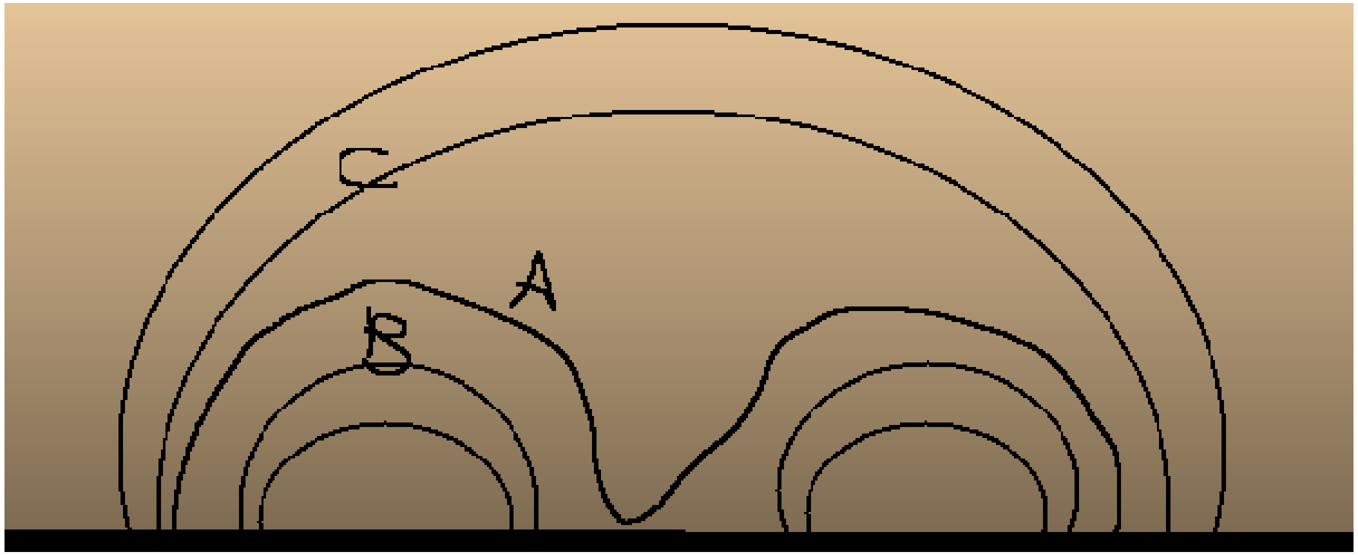
This strengthens the case that energy release in line tied systems begins with an ideal MHD process. Parker proposed current sheet formation is ubiquitous.

Example



Line tied configuration with a separatrix (A). Line length doubles going from B type to C type across A.

Current Sheet Formation



Shear the field by moving footpoints perpendicular to plane of screen & let field adjust to a new equilibrium. The shear per unit length will be $\sim$ constant. Unless footpoint displacement doubles going from B to C, the shear will be discontinuous across A (see Low, also Zweibel & Proctor).

Current Sheet Properties

- Viscously or resistively resolved: $P_m \equiv \frac{\nu}{\eta} \sim \left(\beta_i \frac{m_i}{m_e}\right)^{1/2}$
- Equilibrium shear determines Poynting flux $B_n \mathbf{v}_\perp \cdot \mathbf{B}_\perp$.
  - What is the heating rate?
  - Is equilibrium shear ideal stable?

N.B. This current sheet has a simple topology but theorems on current sheet formation in a field extending between 2 plates do not apply. Separatrix is crucial & not unrealistic.

Summary

- Linetying is an important astrophysical & lab boundary condition but precludes rational surfaces
- Resistive instability grows as S^{-1} for systems above a moderate length
- Rational surface behavior emerges in systems where tearing layers overlap for modes of periodic system
  - Transition length $\propto S^{2/5}$.
- Effective suppression of tearing strengthens case for MHD initiated energy release
- Current sheet formation across separatrices may be one way to achieve this.