Simulations of Decaying Kinetic Alfvén Wave Turbulence: Coherent and Intermittent Structures

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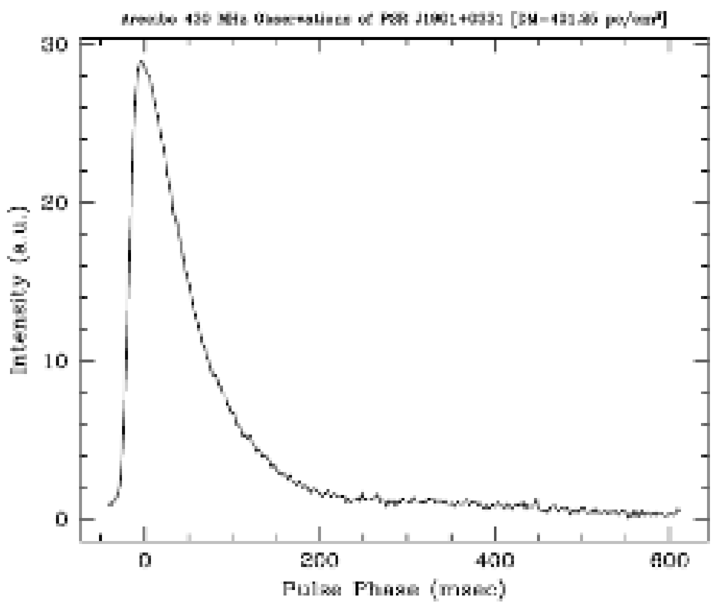
9 July 2008

Motivation



- The time width τ of pulsar signals through the interstellar medium (ISM) probe the distribution of electron density n_e fluctuations.
- If n_e is Gaussian distributed $\Rightarrow \tau \sim D^2$, where D is the distance to source.
- Observation yields $\tau \sim D^4 \Rightarrow$ Lévy distributed n_e.

- Expect n_e pdf to have long tails and deviate from Gaussian statistics.
- The ISM turbulent spectrum range spans many decades, even scales smaller than ρ_i.
- Scintillation is dominated by small-scale n_e fluctuations.
- Can electron microturbulence in a Kinetic Alfvén Wave model develop interesting structures?
- Intermittent structures?
- Moreover, can KAW turbulence yield Lévy distributed n_e?

Kinetic Alfvén Wave Model

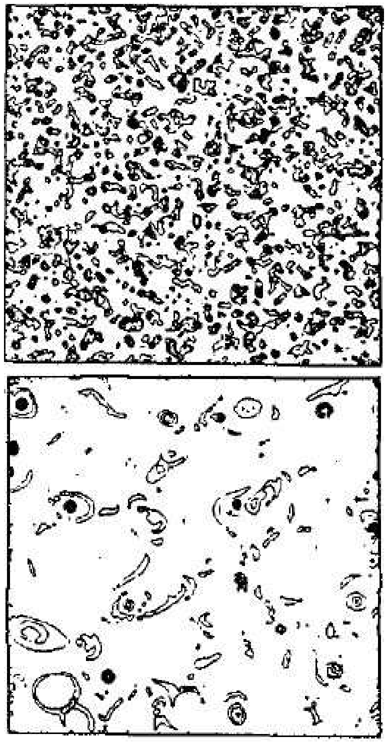
$$\partial_t \psi = (\rho_s/L)^2 \nabla_{\|} n + \eta J \qquad \text{Ohm's Law}$$

$$\partial_t n = -\nabla_{\|} J + \mu \nabla^2 n \qquad \text{Electron Continuity}$$

$$\nabla_{\|} = \partial_z + \nabla\psi \times \hat{\mathbf{z}} \cdot \nabla \qquad J = \nabla^2 \psi \, \hat{\mathbf{z}} \qquad \mathbf{B} = \mathbf{B}_0 + \nabla\psi \times \hat{\mathbf{z}}$$

- Model of small-scale n_e and B fluctuations with $L \leq 10\rho_s$. For the ISM, $L \sim 10^{10}\mathrm{cm}$.
- Ions form a neutralizing background.
- Ion flow is not dynamically active in this small-scale regime; ϕ equation drops from the usual 3-field equations, reducing to the system above.
- Electric field is inductive and is balanced by parallel electron density gradients.
- Diffusion in density is ad-hoc, allows control of η/μ ratio.

Intermittency in KAW Turbulence



- Previous decaying 2D fluid turbulence simulation (McWilliams) and 2D decaying KAW system (Craddock): Gaussian initial conditions $\Rightarrow$ spontaneous intermittent fields.
- Craddock saw highly intermittent current filaments, large kurtosis, but damped n_e heavily for numerical stability.
- We investigate n_e behavior in the same decaying system, with particular interest in:

- What preserves intermittent structures when interacting with turbulence?
- Can structures form in n_e?
- What are they like? (Sheets, filaments, etc.)
- Can n_e become non-Gaussian from Gaussian initial conditions?
- Can it become Lévy distributed?

Simulation

- Two fields (n_e and ψ) are evolved in a domain of size $[2\pi] \times [2\pi]$.
- Resolution of 512^2.
- System integrated in Fourier space, with fully dealiased pseudospectral explicit fourth-order Runge-Kutta time stepping for the nonlinearities.
- Linear damping is handled via an integrating factor $\Rightarrow$ integrated exactly, with no timestepping constraints.
- All timestep constraints stem from nonlinear terms.
- Initial Conditions:
  - n_e and $k\psi$ are initialized with either random or correlated phases.
  - Internal energy E_I and magnetic energy E_M are equipartitioned, where

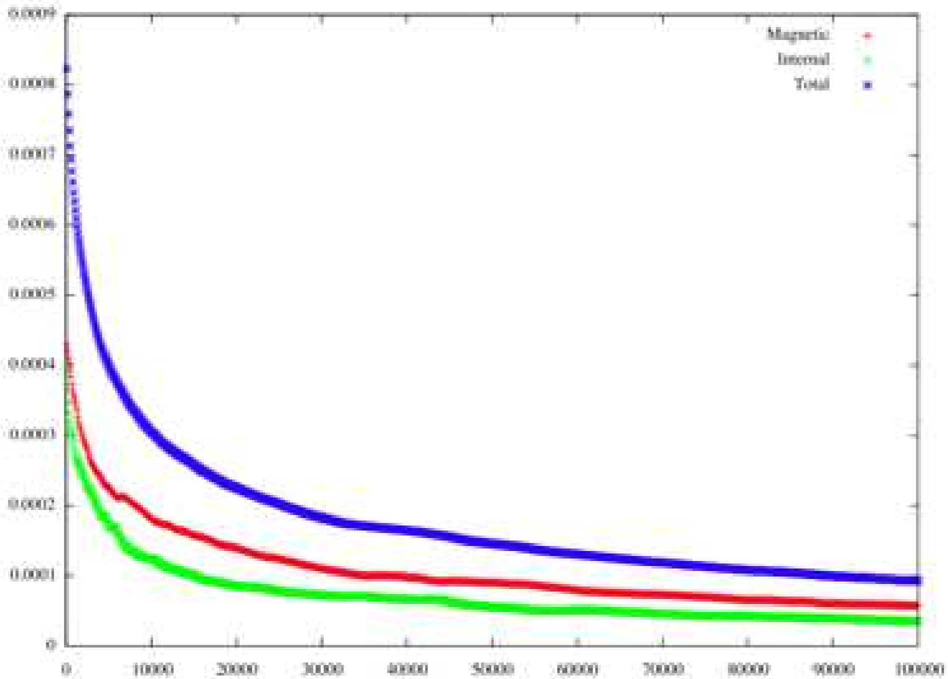
$$E_I = \sum_k n_k^2 \qquad \text{and} \qquad E_M = \sum_k k^2 \psi_k^2.$$

  - Spectra $\sim k^{-3}$ with peak in spectrum $k_0 \sim 6$.

Time Evolution

J, n_e, B movies

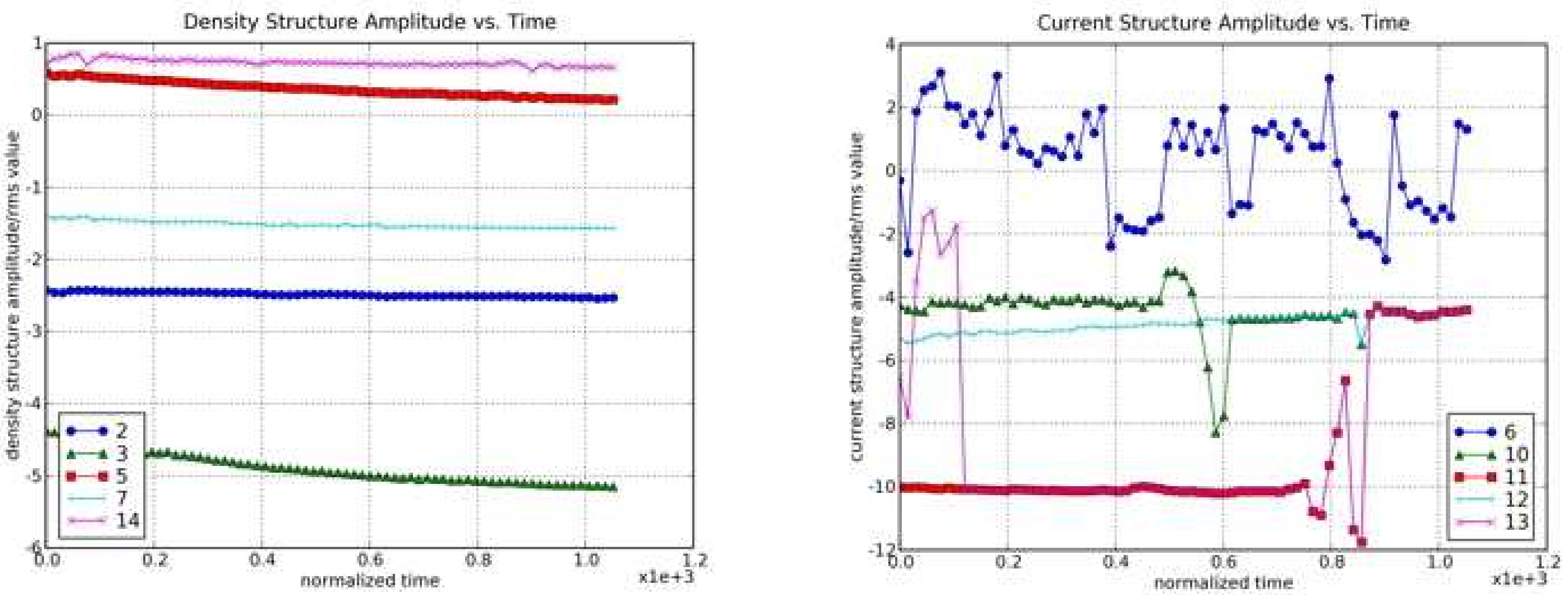
Energy vs. Time



- Although decaying, system is in quasi steady-state after a few Alfvén times.
- Quasi steady-state has many structure-structure and structure-eddy interactions.
- Initial decay is exponential. Turbulent energy cascade is disrupted as non-decaying structures stabilize; total energy levels off.

- Quasi steady-state phase ends when background turbulence dies away, many structures merge, and dynamics of system dominated by structure-structure interactions.
- Eventually system will evolve to a state with two oppositely-signed current structures.
- A driven system would replenish the turbulence, increasing its amplitude so that structure-eddy interactions persist.

Structure Persistence



- Structure cores are undisturbed for many τ_A, despite turbulent background.
- Merger of structures evident as time history coalesces into one structure.
- As compared with local turbulent amplitude, structure core amplitude is several σ and larger, and only decays due to linear diffusion.

What is the mechanism of structure persistence?

Structure Persistence Mechanism

- Presume a large current filament, parabolic profile, radial scale a.
- Inside filament, B_θ increases linearly with r.
- $\mathrm{B}_\theta \sim 1/r$ outside $\Rightarrow$ radial shear in B_θ.
- Turbulent fluctuations interacting with B_θ shear leads to refraction of turbulence.
- Enhances turbulent decorrelation, turbulence cannot persist to disrupt core of structure.
- Key points:
  - Structures only form when sufficiently large B_θ shear preserves it from turbulent mixing.
  - Localized current structures yield non-localized B.

Structure Mantles

Outside an isolated current filament, B field is fixed by Biot-Savart Law

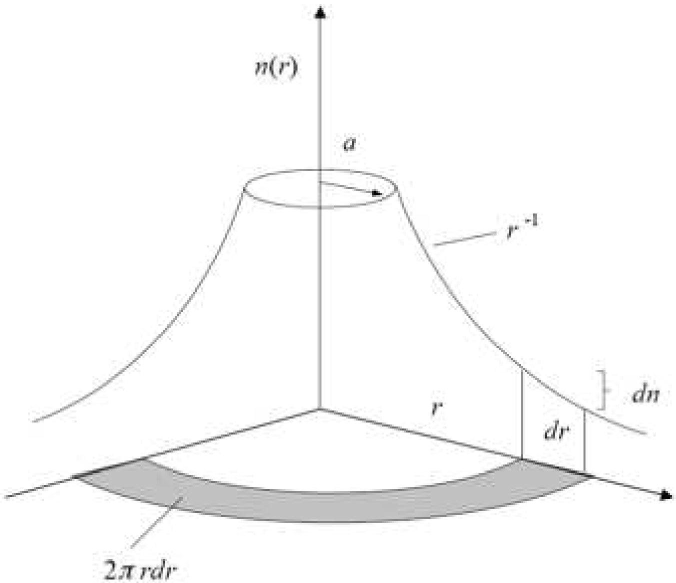
$$\mathrm{B}_\theta \sim 1/r.$$

With many interacting structures and background turbulence, the actual B field is very complex.
How does n_e behave outside structure cores?
KAW is an interchange between fluctuating internal and magnetic energies.
Equipartition implies

$$E_I \sim E_M$$

Supposing $n_e \sim 1/r$ outside structure core $\Rightarrow$ Lévy distributed n_e.

Lévy Distributed n_e

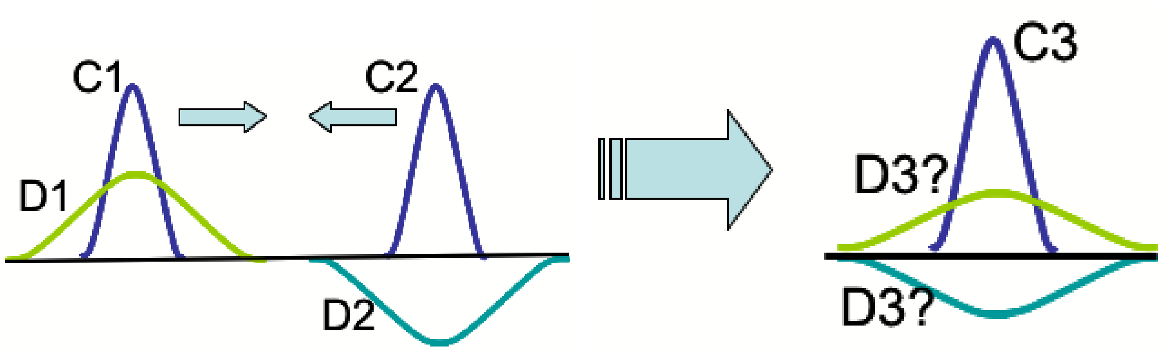


$$\left.\begin{array}{c} P(n)\,dn \approx 2\pi r\,dr \\ n(r) \sim r^{-1} \\ dn \sim r^{-2}\,dr \end{array}\right\} \begin{array}{c} P(n)\,dn \approx n^{-3}\,dn \\ P(n')\,dn' \approx (n')^{-2}\,dn \end{array}$$

Both $P(n)$ and $P(n')$ are Lévy distributed $\Rightarrow$ can we find $1/r\ n_e$ mantles in simulations?

Has proven to be elusive.

Density structure intensity is strongly determined by history of mergers.



n_e Mantle

- Two processes strongly influence n_e structure:
  1. Structure persistence mechanism preserves structure against disruption by turbulence.
  2. Mergers with other structures can reduce or increase its amplitude and change its radial profile.

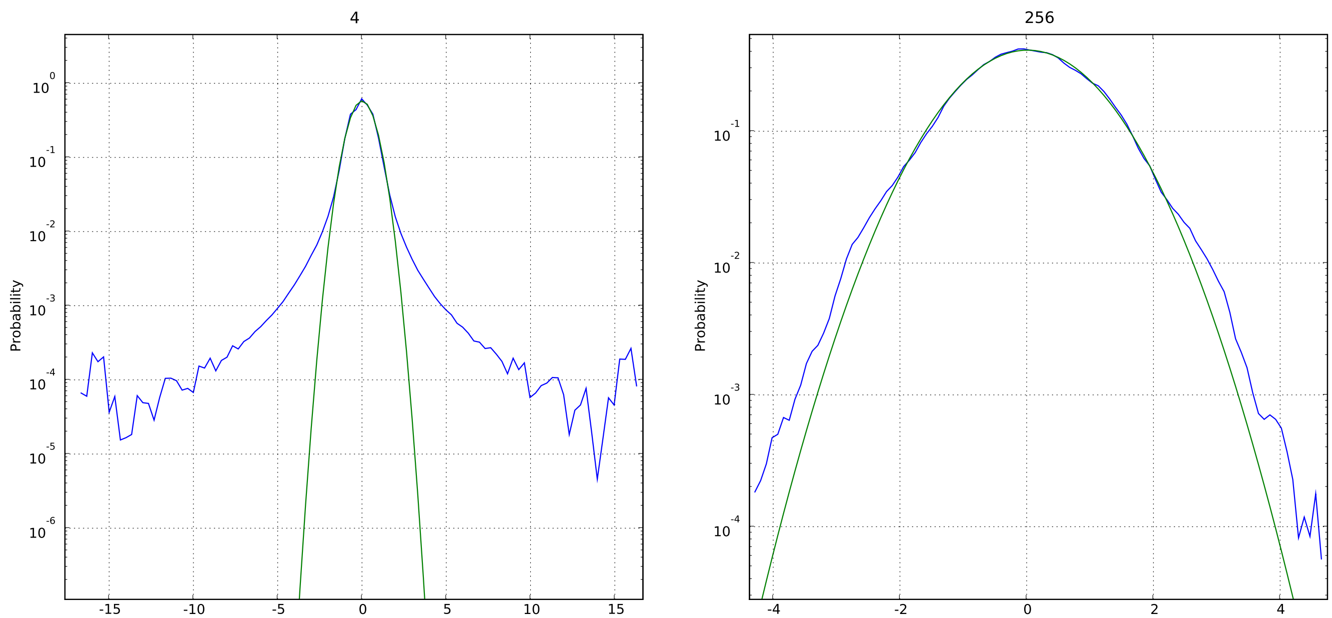
- As number of mergers increase, expect n_e structures to decrease in amplitude.
- In decaying system, time between structure mergers increases $\Rightarrow$ structure mantles are preserved for a long time.

n_e Mantle

It has proven difficult to directly measure the n_e mantle profile:

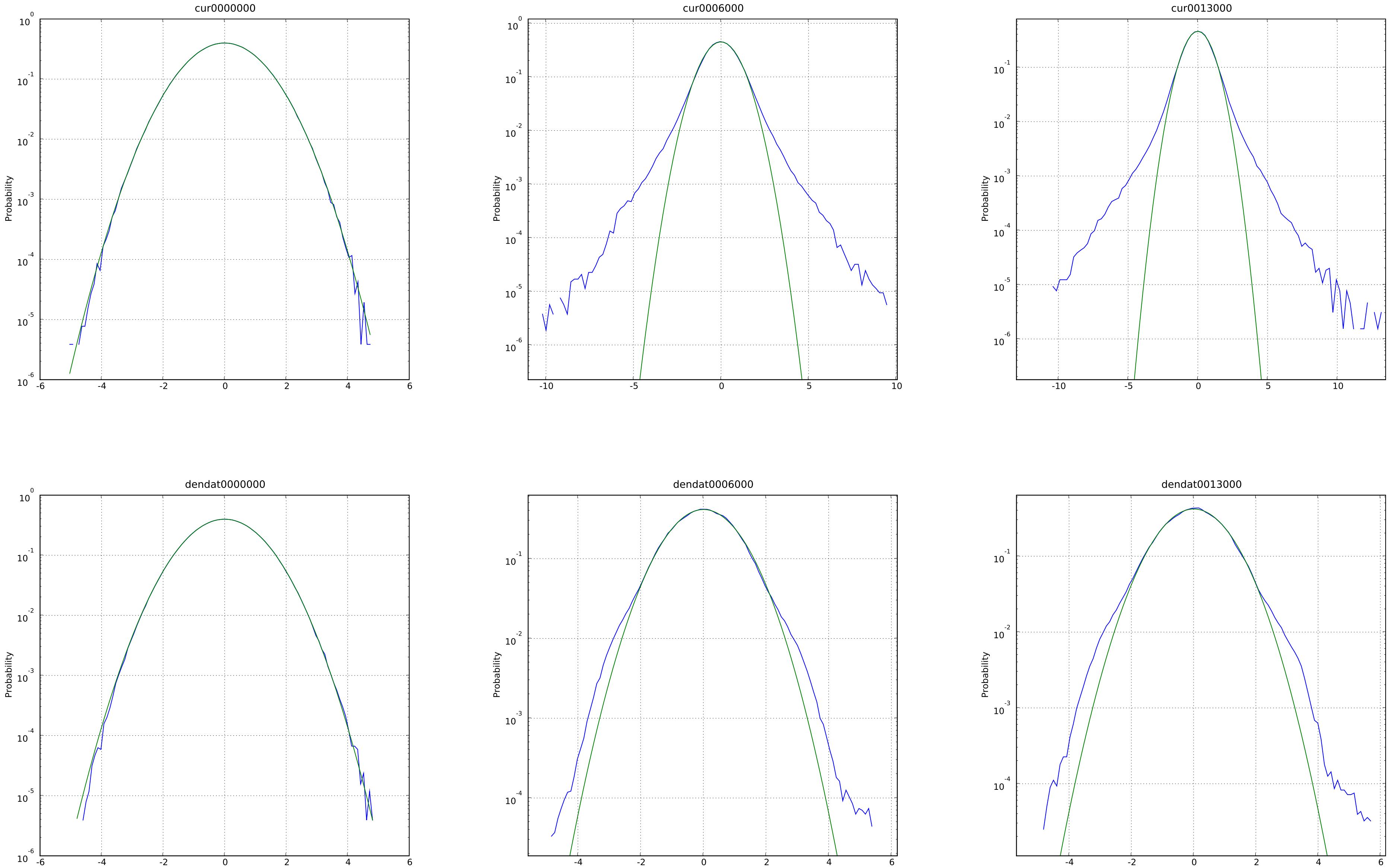
1. Structure core radius a and amplitude can vary greatly;
2. Sheet structures?
3. These aren't as large in amplitude, but make it hard to look at just the circular structures.
4. Have tried to isolate just the circular structures with wavelet filtering – work in progress.

As a baseline for comparison, we consider fields populated only with $1/r$ structures.



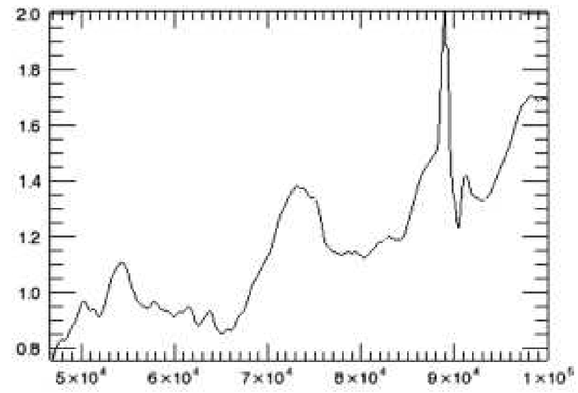
Simulation Statistics: PDFs

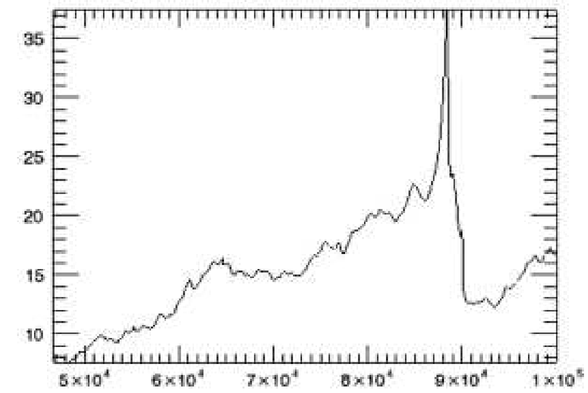
Ensemble of simulations $\Rightarrow J$ and n_e pdfs.



Simulation Statistics: Kurtosis

$$\kappa(X) = \frac{\int dX\ X^4 P(X)}{\sigma^4}$$





- Large kurtosis $\kappa \Rightarrow$ PDF has non-Gaussian tail.
- How large is "large"?
- If a (discrete) field is Gaussian distributed, its kurtosis is 3 with $\sigma = \sqrt{96/N}$.
- For $N = 512^2$, $\sigma \approx 0.019$.
- $\kappa(J) \geq 10$ for later times $\Rightarrow$ non-Gaussian tail in current.
- Current has spikes that rise above background.
- $\kappa(n) \sim 4.5 \leq \kappa(\mathrm{B})$
- n_e many σ deviation from Gaussian κ.
- Suggestive of non-Gaussian structures in n_e.

Lévy Statistics

- Lévy distributed PDF has tails $\sim |x|^{-(1+\alpha)}$, with $\alpha \in [0, 2]$.
- This distribution has $\kappa \to \infty$.
- $\kappa(n_e)$ from simulation is bounded by (still finite) $\kappa(J)$.
- Remains to be seen if n_e field can nonetheless yield τ pulsar signal scaling.

Conclusions

- Decaying KAW simulations give rise to long-lived, coherent structures.
- After structure formation, turbulence unable to mix structures.
- Structure persistence due to radial shear in B_θ.
- J field intermittent with isolated filaments and $\kappa(J) \sim 10 - 20$.
- n_e and B fields have similar kurtosis. While less than $\kappa(J)$, they are still non-Gaussian.
- n_e PDFs indicate deviation from Gaussian statistics over time.

Future Work

- Steady state?
  - Will replenish turbulence and affect n_e distribution, but
  - Sensitive to energy input rate ϵ.
  - Small ϵ will allow structures to form and persist.
  - Large ϵ will disrupt structures and keep fields Gaussian.
- Can we isolate circular structures via wavelet analysis? (Farge et al.)
- With n_e non-Gaussian, can we recover τ scaling?

Questions?