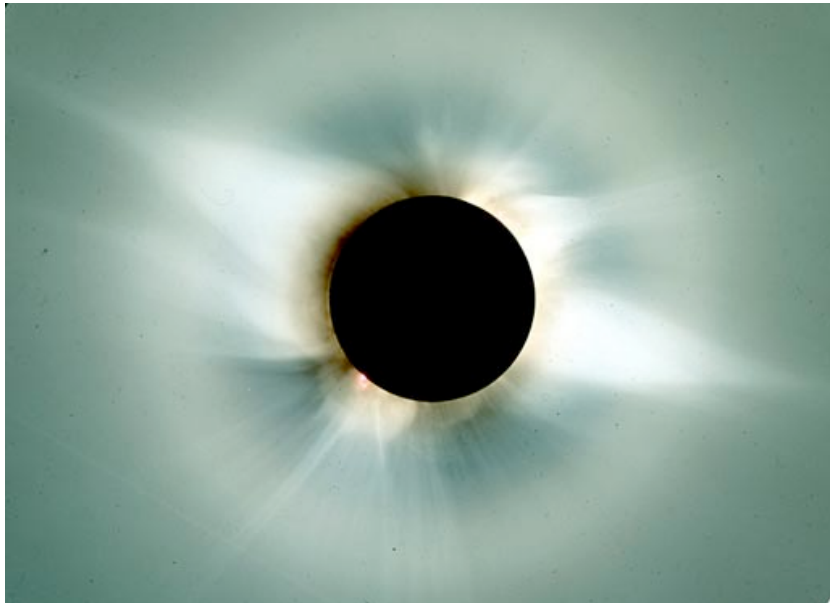


# Reconnection Onset and Self-Organization in Solar and Stellar Coronae



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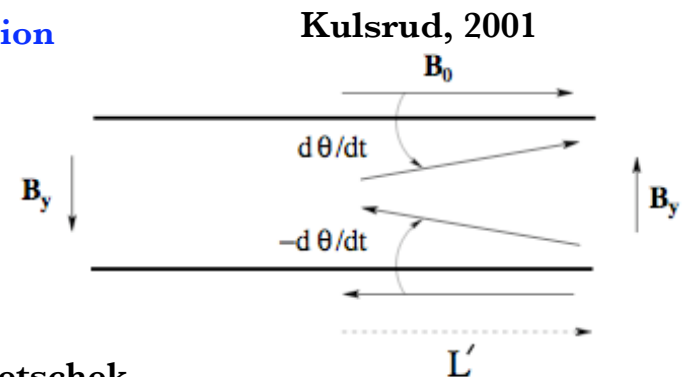
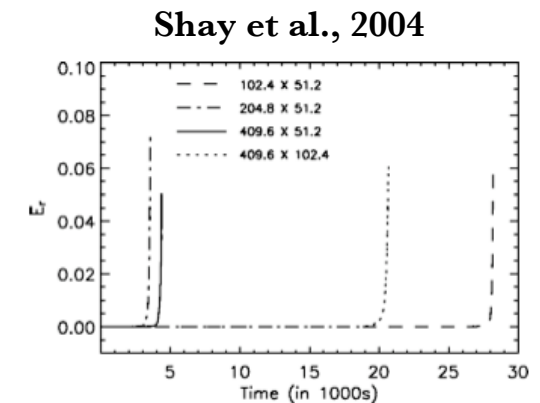
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**CMSO General Meeting  
July 10, 2008**

# Reconnection: From Slow to Fast

- **Fast reconnection onset**
  - Wang and Bhattacharjee, 1993 - rapid development of **Hall reconnection** could explain sawtooth crash onset. (Bhattacharjee, 1999 - flares and substorms too!)
  - In simulations, reconnection often is slow (**Sweet-Parker** like) for a long time before **fast (Hall) reconnection** begins (Bhattacharjee et al., 1999; Shay et al., 2004)
- **Goal -**
  - to understand the dynamics of the transition from **Sweet-Parker** to **Hall** reconnection
- **We present -**
  - The nonlinear dynamics of reconnection
  - A model for the **catastrophic** onset of **Hall reconnection**
  - A prediction of the model
  - Observational evidence from solar and stellar flares
  - Implications for flares, coronal heating and **self-organization** in the corona
- **Kulsrud, *Earth, Planets and Space*, 53, 417 (2001)**
  - Performed scaling analysis which showed that the Petschek model does not work for uniform resistivity because there is nothing to bend the upstream field in the normal direction



# When Sweet-Parker? When Hall?

- Consider the generalized Ohm's Law (electron eqn. of motion)

$$\vec{E} + \frac{\vec{v}_i \times \vec{B}}{c} = \eta \vec{J} + \frac{1}{nec} \vec{J} \times \vec{B} - \frac{1}{ne} \vec{\nabla} \cdot \vec{p}_e - \frac{m_e}{e} \frac{d\vec{v}_e}{dt}$$

Convection     Resistivity     Hall Effect     Electron Pressure     Electron Inertia

Sweet-Parker is valid when  
Convection ~ Resistivity » Hall

$$\delta_{SP} > \rho_i = \frac{c}{\omega_{pi}} \quad \text{or} \quad \frac{c_s}{\Omega_{ci}}$$

$$\hat{\eta} > \frac{\rho_i}{L_{SP}}$$

Hall reconnection is valid when  
Convection ~ Hall » Resistivity

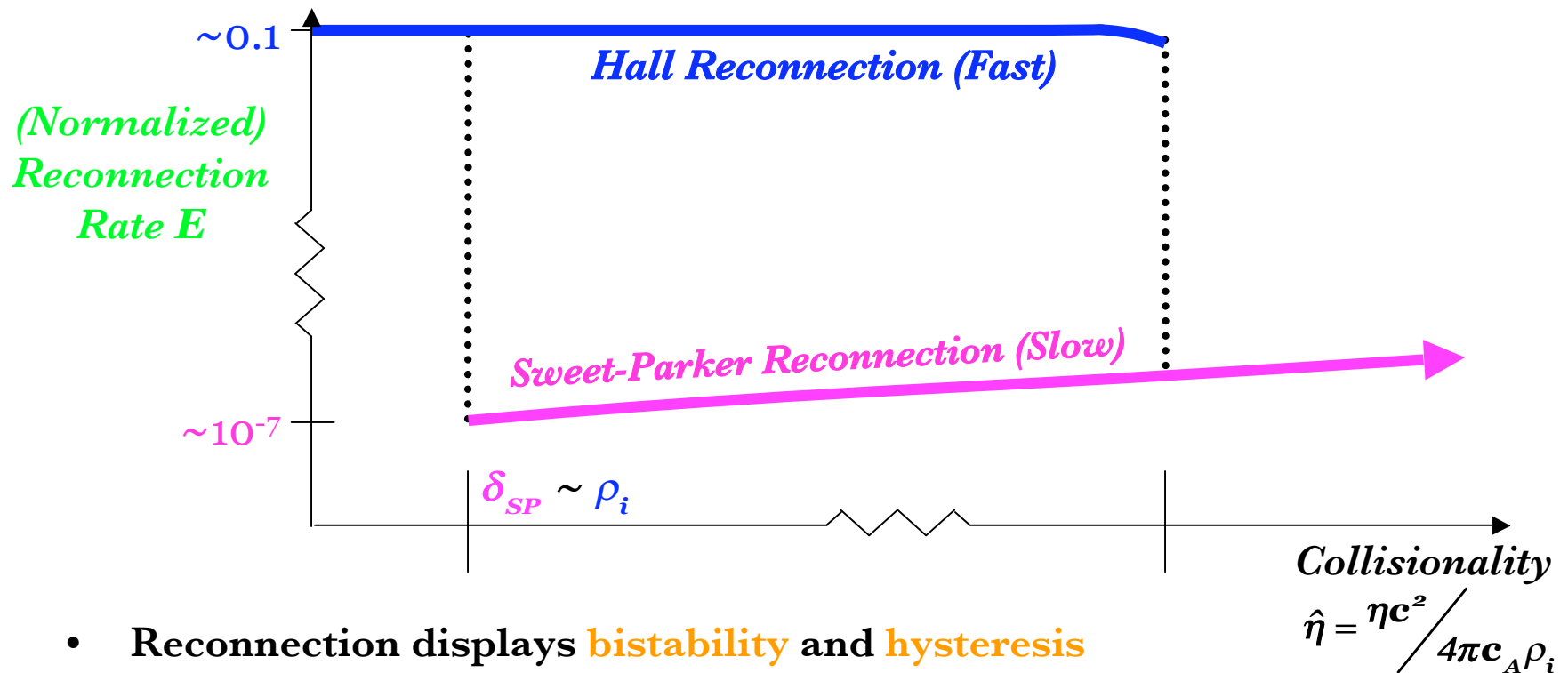
$$(\vec{v}_e \cdot \vec{\nabla}) B_x > \frac{\eta c^2}{4\pi} \nabla^2 B_x$$

$$\hat{\eta} < 0.1$$

- These are very different conditions!
  - This is because convection is much larger ( $10^6$  times for solar flares!) during Hall reconnection than it is during Sweet-Parker reconnection.

$$\delta_{SP} \sim \sqrt{\frac{\eta c^2}{4\pi c_A}} L_{SP}, \quad \delta = \text{thickness}, \quad L = \text{length}, \quad \hat{\eta} = \frac{\eta c^2}{4\pi c_A \rho_i}$$

# Reconnection is **Bistable**

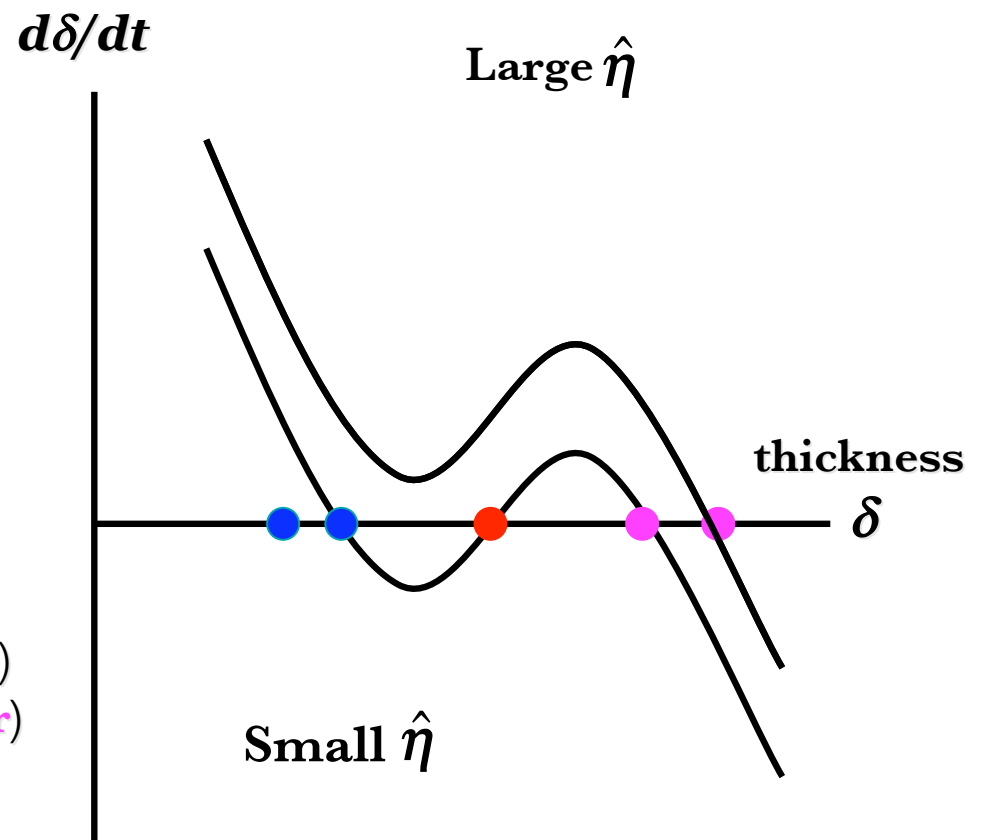


- Reconnection displays **bistability** and **hysteresis**
  - Verified with simulations (Cassak, Shay and Drake, 2005)
- Hall reconnection onsets **catastrophically**
  - Occurs when **thickness  $\delta_{SP}$  of Sweet-Parker layer** reaches **ion gyroradius  $\rho_i$**
  - Observed in reconnection experiments (Ren et al., 2005; Yamada et al., 2006; Egedal et al., 2007)
  - Potentially explains **onset** of solar eruptions (Cassak, Shay and Drake, 2005 & 2006)

# Interpretation with **Bifurcation Theory**

- **Bistability/hysteresis** is well known in nonlinear dynamics
  - “Saddle-node bifurcations” bring fixed points into/out of existence as a control parameter ( $\hat{\eta}$ ) is varied (Cassak et al., in prep.)

- = Stable fixed point (**Hall**)
- = Unstable fixed point (**Unknown!**)
- = Stable fixed point (**Sweet-Parker**)

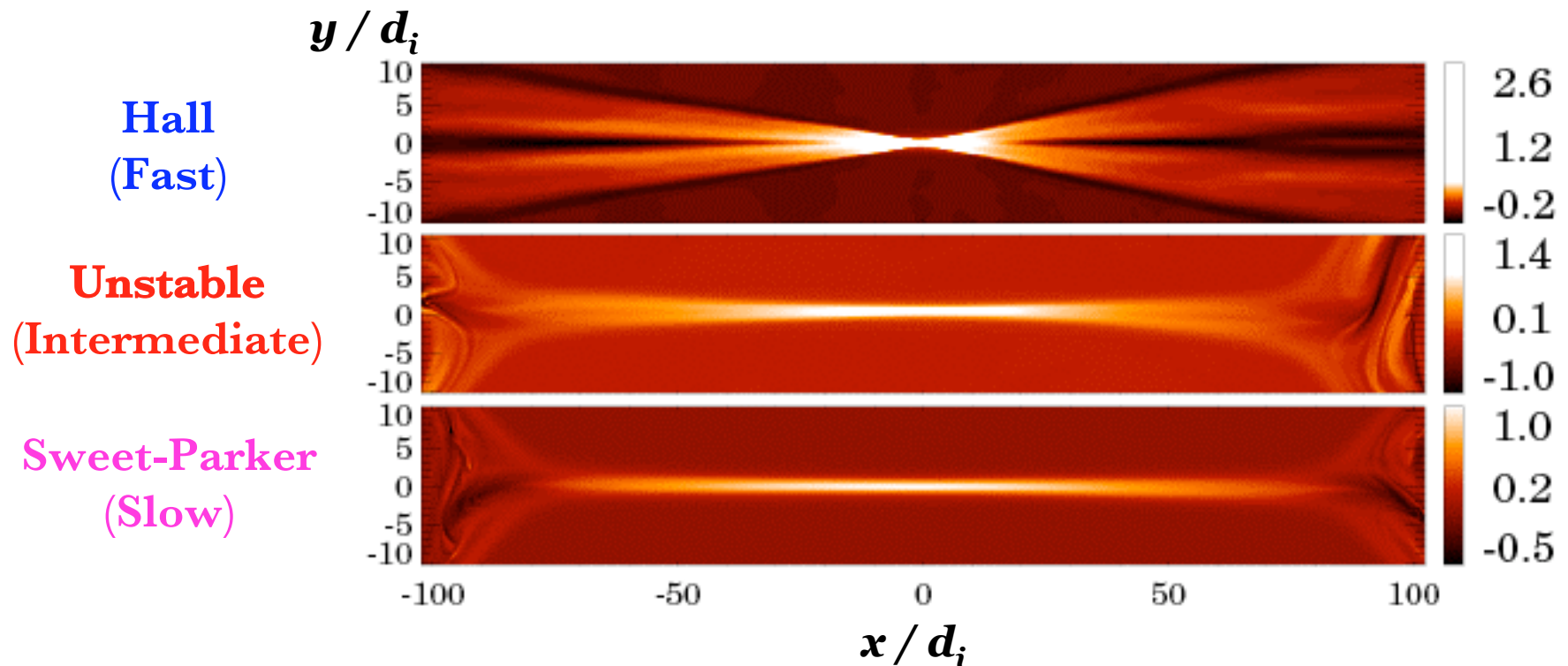


Predicts the temporal evolution of thickness  $\delta$ .

Predicts the existence of an **unstable** reconnection solution!

# The Unstable Equilibrium

- We used a novel iterative technique (Skufca et al., 2006) to find the **unstable solution** (Cassak et al., 2007).
  - Out of plane current density  $J_z$  for the **unstable** solution for  $\eta = 0.015$



The **unstable** solution is a hybrid of the **Sweet-Parker** and **Hall** solutions

# Physics of the **Unstable** Solution

Reconnection Mode:

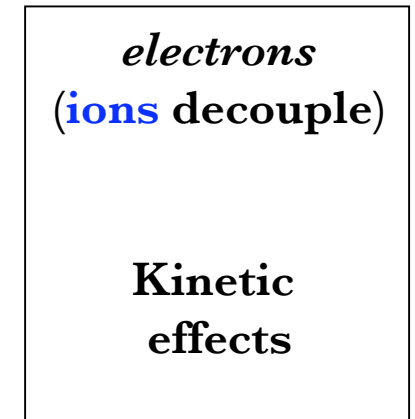
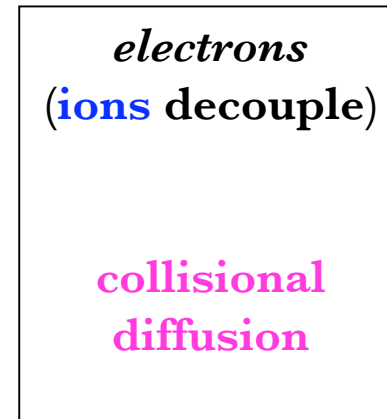
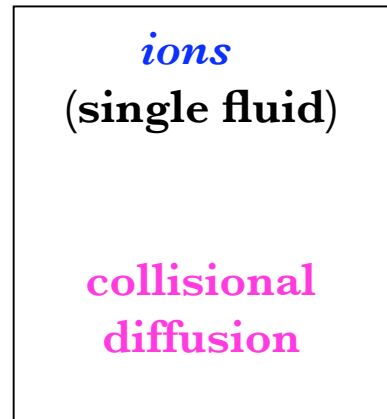
Sweet-Parker

Unstable

Hall

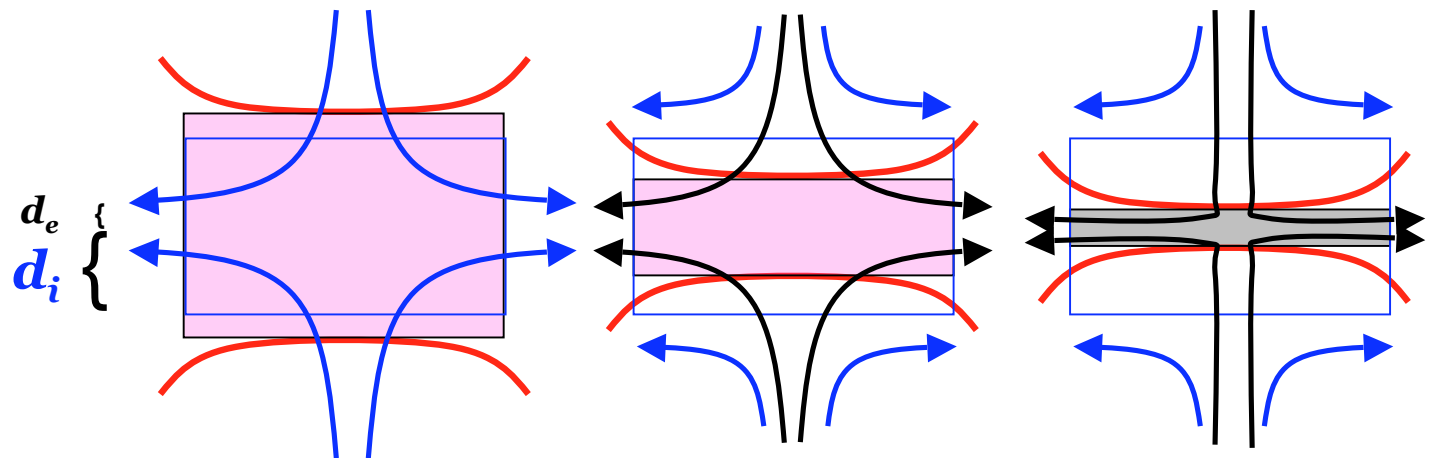
Magnetic fields  
convected into  
dissipation region by:

Convection  
balanced by:



Key:

- Ion flow
- Electron flow
- Magnetic field
- Collisional dissipation
- Electron inertia dissipation



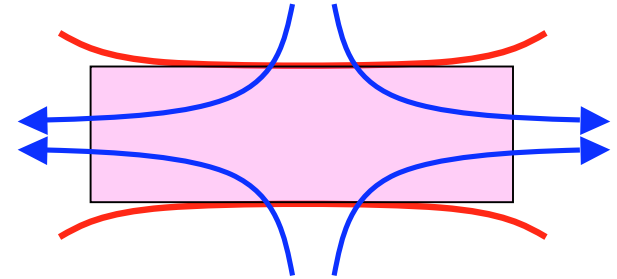
$$\vec{E} + \frac{\vec{v}_i \times \vec{B}}{c} = \eta \vec{J} + \frac{1}{nec} \vec{J} \times \vec{B} - \frac{1}{ne} \vec{\nabla} \cdot \vec{p}_e - \frac{m_e}{e} \frac{d\vec{v}_e}{dt}$$

# What determines stability?

- Consider a **Sweet-Parker** layer
  - In a steady-state, **diffusion** (broadens) balances **convection** (contracts)

$$\frac{v_{in}}{\delta} \sim \frac{\eta c^2}{4\pi\delta^2}$$

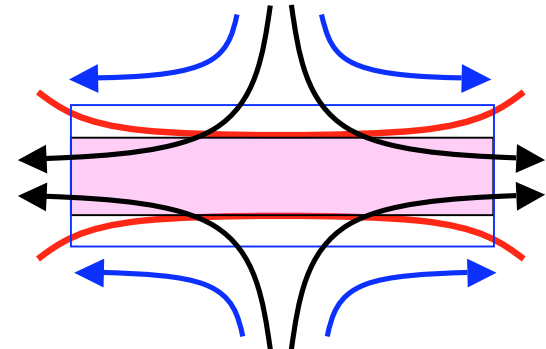
- If you squeeze the layer, **diffusion** goes up more than **convection**
  - **Diffusion** tends to broaden the layer  $\Rightarrow$  stability!
- Similar argument at electron layer shows **Hall solution** is stable



- Consider the **unstable** layer

$$\frac{(v_{in})_e}{\delta} \sim \frac{\eta c^2}{4\pi\delta^2}$$

- If you squeeze the layer, **diffusion** goes up, but electron convection goes up even more (whistler physics!)  $\Rightarrow$  *instability*!

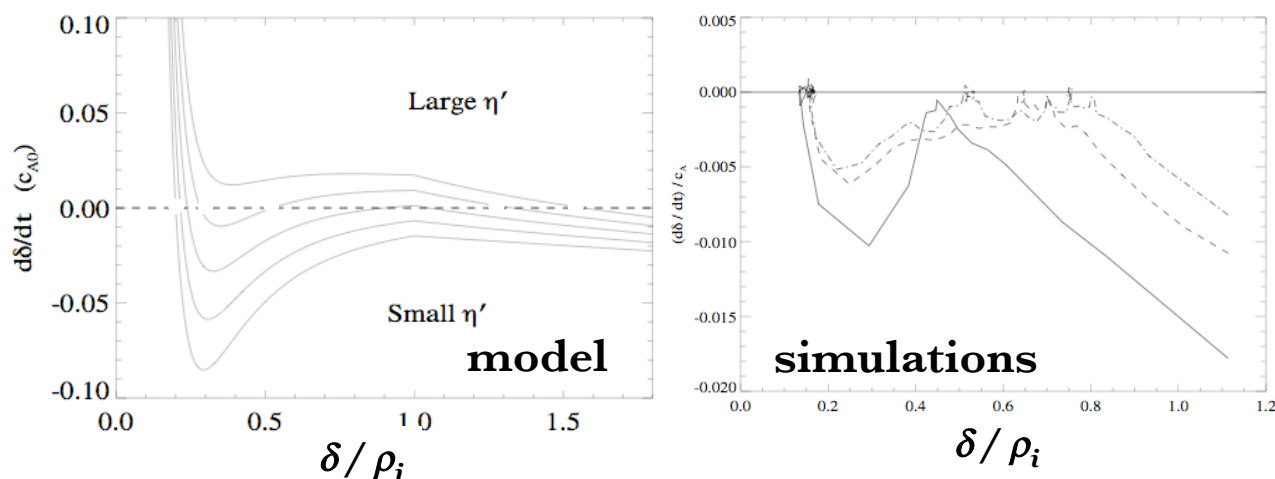


- The transition from **Sweet-Parker** to **Hall**

- **Sweet-Parker** is valid for  $\delta > \rho_i$
- When  $\delta < \rho_i$ , the ions decouple and electron convection balances diffusion
- This is unstable, and the current layer collapses to **Hall configuration**

# A Minimal Physics Model

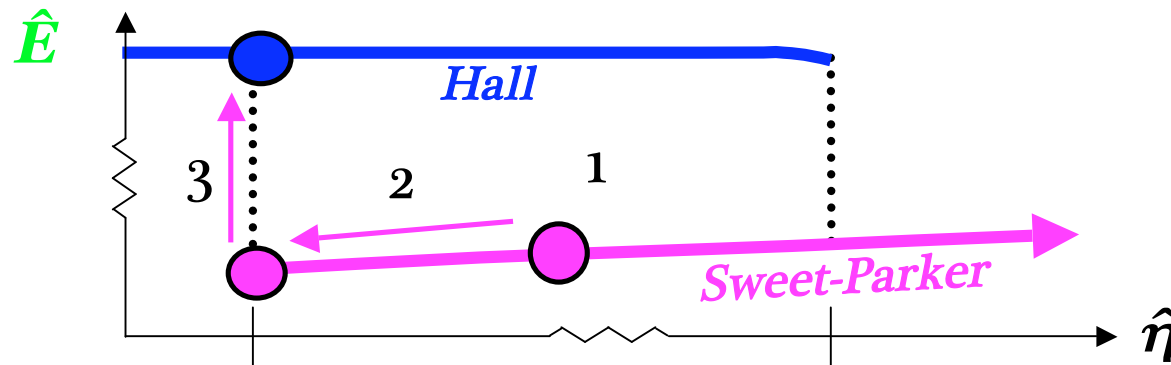
- Try evolution equation of 
$$\frac{d\delta}{dt} = -v_{in} + \frac{\eta c^2}{4\pi\delta} + v_{in} \frac{\rho_e^2}{\delta^2}$$
$$v_{in}(\delta) \sim \frac{\delta}{L(\delta)} v_{phase}(\delta)$$
  - Contains minimal physics: **continuity** and **Hall physics**
  - Compare: recent analysis by Chacón, Simakov and Zocco, 2007
- Using standard ODE techniques, we find this system has **saddle-node bifurcations!** (Cassak et al., in prep.)



**This minimal physics model reproduces complex nonlinear dynamics of magnetic reconnection.**

# Onset Model and Predictions

- Onset Model
  - 1) Before an eruption, the corona must be **collisional**
    - If it isn't, the stored energy would be explosively released!
    - Reconnection is slow (**Sweet-Parker**), so magnetic energy accumulates
  - 2) Something drives the corona to a state of lower collisionality (will discuss mechanism later)
    - **Hall reconnection** does not begin because reconnection is **bistable**
  - 3) When  $\delta_{SP} \sim \rho_i$ , reconnection undergoes a **bifurcation**
    - **Hall reconnection** onsets **catastrophically**
    - The stored energy is rapidly released, manifested as an eruption
  - 4) The post-eruption corona is again **collisional**, and the process repeats



- Prediction: At reconnection (or flare) onset, the thickness of a **Sweet-Parker** layer is comparable to the appropriate **ion gyroradius**

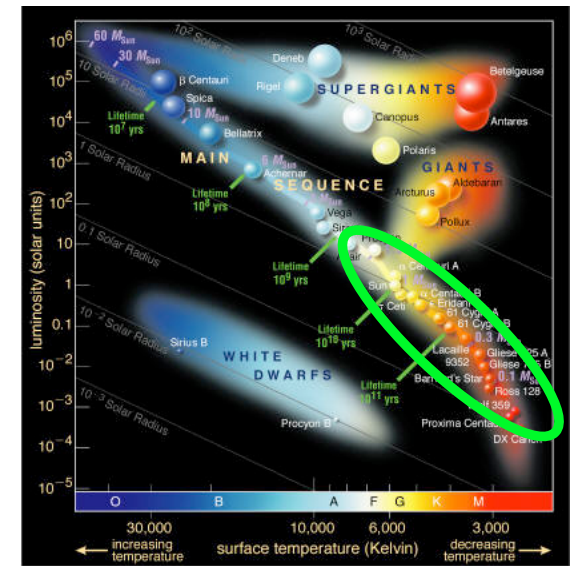
$$\delta_{SP} \sim \rho_i [= c/\omega_{pi} \text{ or } c_s/\Omega_{ci}]$$

# Is there Evidence that $\delta_{SP} \sim \rho_i$ ?

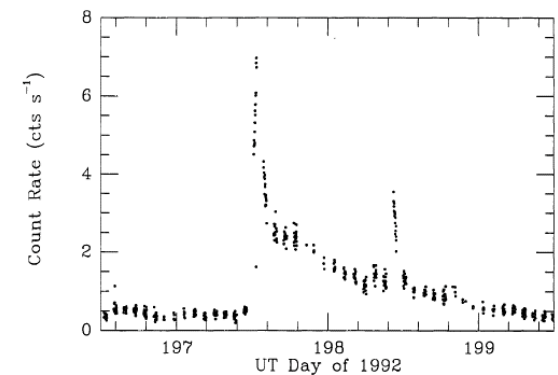
- The length scales are too small to measure directly, but can be estimated indirectly by relating to plasma parameters  $T$ ,  $n$ ,  $L$ , and  $B$ .
  - For simplicity, consider anti-parallel reconnection:  $\rho_i \sim c / \omega_{pi} = d_i$ . This differs from  $\rho_s = c_s / \Omega_{ci}$  by  $\beta^{1/2}$ .
- Can estimate lengths in the corona (taking  $L$  as the radius of a flux tube)
  - For an active region,  $T \sim 1$  MK,  $n \sim 3 \times 10^9 \text{ cm}^{-3}$ ,  $L \sim 10^9 \text{ cm}$ ,  $B \sim 100 \text{ G}$   
 $\Rightarrow d_i \sim 400 \text{ cm}$ ,  $\delta_{SP} \sim 200 \text{ cm}$ 
    - Known previously (Priest and Forbes, 2000; Uzdensky, 2003; Bhattacharjee, 2004; ...)
  - In the quiet solar corona,  $T \sim 1$  MK,  $n \sim 10^9 \text{ cm}^{-3}$ ,  $L \sim 10^{10} \text{ cm}$ ,  $B \sim 5 \text{ G}$   
 $\Rightarrow d_i \sim 720 \text{ cm}$ ,  $\delta_{SP} \sim 770 \text{ cm}$
  - How do we know this isn't a coincidence?
- Look at flares on other sun-like stars!
  - Light curves for stellar flares have same structure as solar flares (Gershberg, 2005)
  - Statistics of various quantities for both solar (e.g., Charbonneau et al., 2001) and stellar (Shakhovskaya, 1989) flares have similar power law spectra over *many* orders of magnitude

# Plasma Parameters in Stellar Flares

- Mullan et al., 2006 calculated parameters during 134 flares in 44 F, G, K, and M type stars
  - All data from *EUVE* satellite
    - No variations from different detectors
- From light curves to plasma parameters
  - Extract  $\tau_d$  and emission measure  $EM$ 
    - Decay time  $\tau_d$  of signal
    - Power radiated  $P = n^2 \Lambda(T) V = EM \Lambda(T)$
  - Use Haisch model (1983)
    - Assume  $\tau_{rad} \sim \tau_{cond} \sim \tau_d$ 
      - $\tau_{rad} = E / P \sim 3 n T / n^2 \Lambda(T)$
      - $\tau_{cond} = E / (\nabla \cdot \text{heat flux}) \sim 3 n T / (\kappa_{th} T / L^2)$
    - Three equations, three unknowns for  $T, n, L$ 
      - At flare temperatures ( $T \geq 10^7$  K),  $\Lambda(T) \sim 10^{-26.2} T^{0.5}$
      - $\kappa_{th} \sim 10^{-6} T^{2.5}$  (Spitzer),  $EM \sim n^2 V, V \sim L^3 / 100$
      - $T \sim \alpha_T (EM)^{0.25} \tau_d^{-0.25}$
      - $n \sim \alpha_n (EM)^{0.125} \tau_d^{-1.125}$
      - $L \sim \alpha_L (EM)^{0.25} \tau_d^{0.75}$
    - Lower limit on  $B$  is  $\beta < 1$  to preserve loop structure
 
$$B_{min} \sim (8\pi n_e T)^{0.5}$$



www.nau.edu



Mullan et al. (2006)

# Observational Results

- Plot  $\delta_{SP}$  and  $d_i$  inferred from stellar data

$$\delta_{SP} = (\eta c^2 L / 4\pi c_A)^{1/2}$$

$$d_i = (m_i c^2 / 4\pi m e^2)^{1/2}$$

- 107 flares from 37 stars

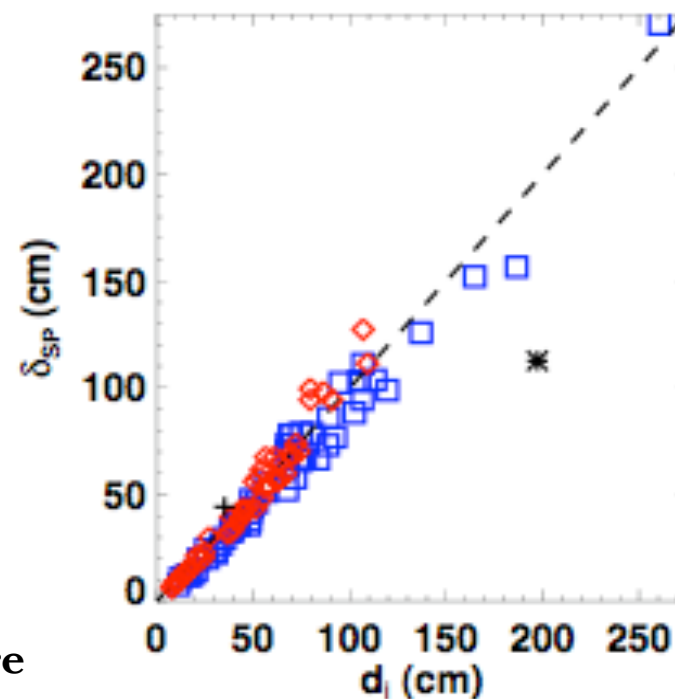
Legend -

**Diamonds** - M dwarfs

**Boxes** - F, G, K stars

Asterisk (\*) - avg eruptive solar flare

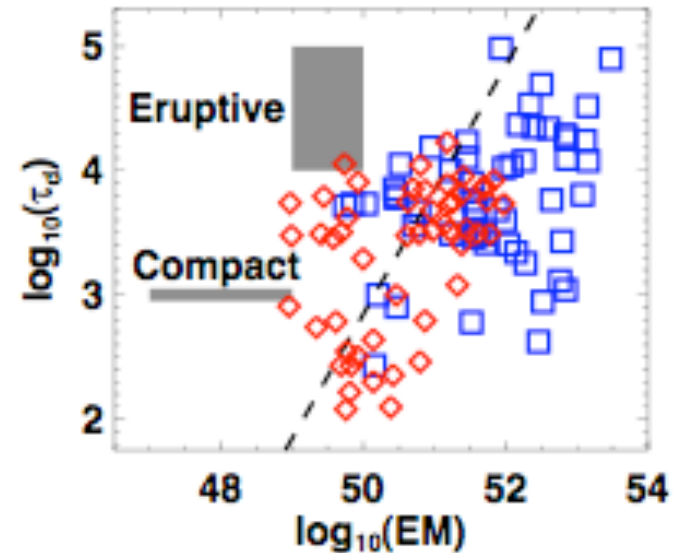
Plus (+) - avg compact solar flare



Agreement of  $\delta_{SP}$  and  $d_i$  is *very good!*  
(Cassak, Mullan, and Shay, 2008)

# Too good to be true? Yes and No

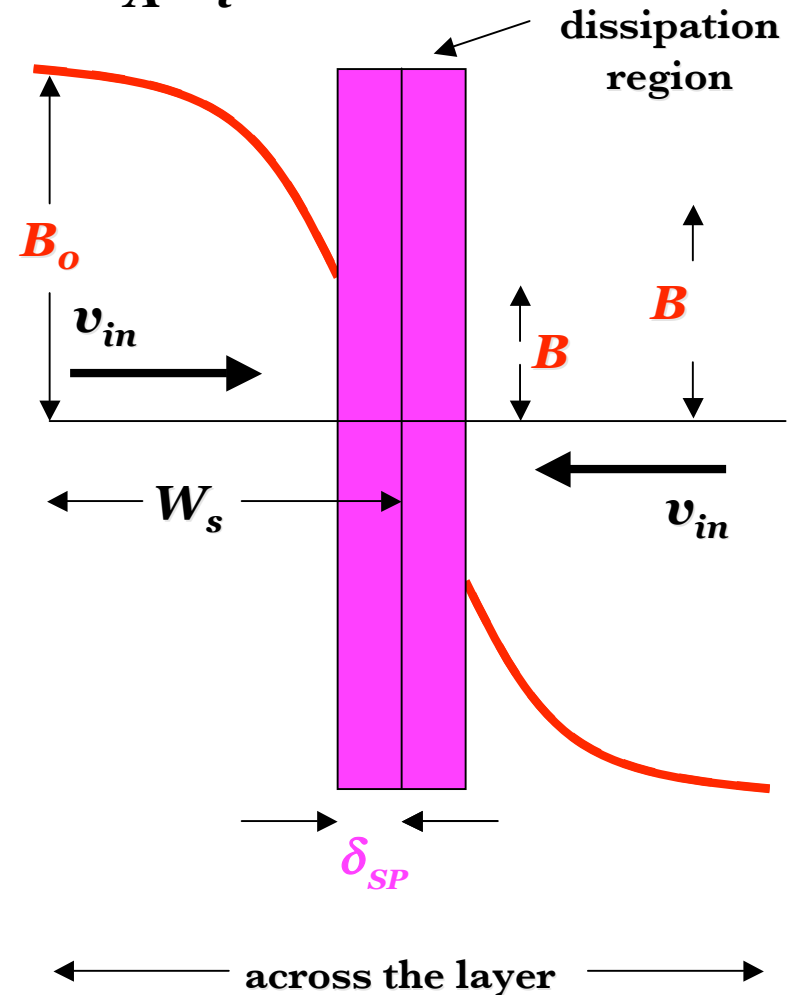
- Can derive  $(\delta_{SP} / d_i) = \text{Constant} (\tau_d / EM)^{1/16}$ 
  - Scatter is suppressed, so data falls on a line. However, the slope is unconstrained by the Haisch model, so the agreement is compelling!
- The Haisch model derives 4 parameters ( $T$ ,  $n$ ,  $L$ , and  $B_{min}$ ) from 2 measurements ( $\tau_d$  and  $EM$ ), so the data are not independent.
  - However, the data derived from the Haisch model compares well with data determined using other methods (Mullan et al., 2006).
- Alternately, we can write  $\tau_d \sim \text{Constant} (\delta_{SP} / d_i)^{16} EM$ .
  - The dashed line is the prediction assuming  $\delta_{SP} \sim d_i$ . If  $\delta_{SP}$  was even 10  $d_i$  (still microscopic!), the predicted line would be 16 units away from the data!



First (indirect) observational evidence that the **Hall effect** plays an important role in coronal dynamics

# What makes $\hat{\eta} = \frac{\eta c^2}{4\pi c_A d_i}$ decrease?

- $\hat{\eta}$  decreases if  $B$  increases
  - Onset occurs when
 
$$\delta_{SP} < d_i \Rightarrow B > \text{Const.} \left( \frac{n^{3/2}}{T^{3/2}} L \right)$$
  - This occurs **spontaneously!**
- Macroscopic current layer
  - Dissipation region “embedded”  
 $W_s \gg \delta_{SP} > d_i$
  - $B$  less than ambient field  $B_o$
- Inflow **convects** in stronger  $B$ 
  - Sweet-Parker sheet thins
  - Transition when  $B$  reaches critical strength where  $\delta_{SP} \sim d_i$



Verified numerically (Cassak, Drake, and Shay, 2006)

# Implications for Coronal Conditions

- The conditions in the corona never stray far from the critical condition where the bifurcation occurs ( $\delta_{SP} \sim \rho_i$ ) (Cassak, Mullan, and Shay, 2008; Uzdensky, 2007)
  - The corona “self-organizes” to this critical state
  - The critical state is *actively* set by the dynamics of reconnection
  - This model applies *both* to solar eruptions in active regions *and* coronal heating (Parker’s nano-flares)
- Cassak, Mullan, and Shay, 2008:  $\delta_{SP} < d_i \Rightarrow B > \text{Const.} \left( \frac{n^{3/2}}{T^{3/2}} L \right)$ 
  - Possible explanation of why corona is as hot as it is!
    - If coronal temperature increases, the critical magnetic field decreases  $\Rightarrow$  less magnetic energy released in eruption  $\Rightarrow$  temperature decreases
    - If coronal temperature decreases, the critical magnetic field increases  $\Rightarrow$  more magnetic energy released in eruption  $\Rightarrow$  temperature increases
- Uzdensky, 2007:
  - Coronal heating event occurs, chromospheric evaporation makes density increase, subsequent events will not occur, high density  $\Rightarrow$  more radiation loss, plasma cools and falls back to chromosphere, density lowers, process repeats
  - Similar affect may be occurring in accretion disk coronae (Goodman and Uzdensky, submitted)

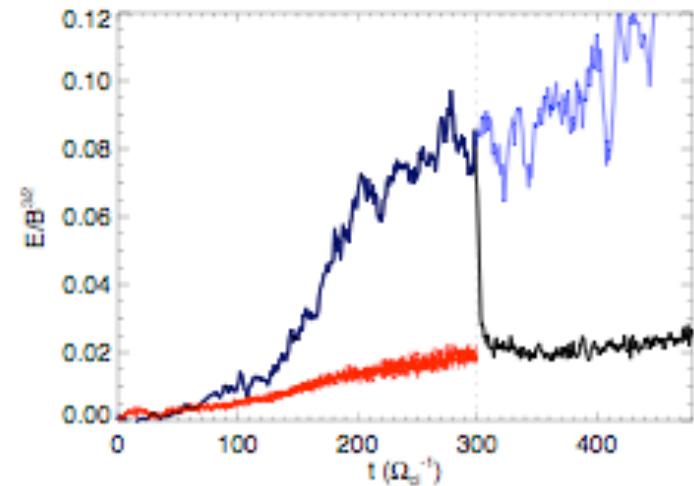
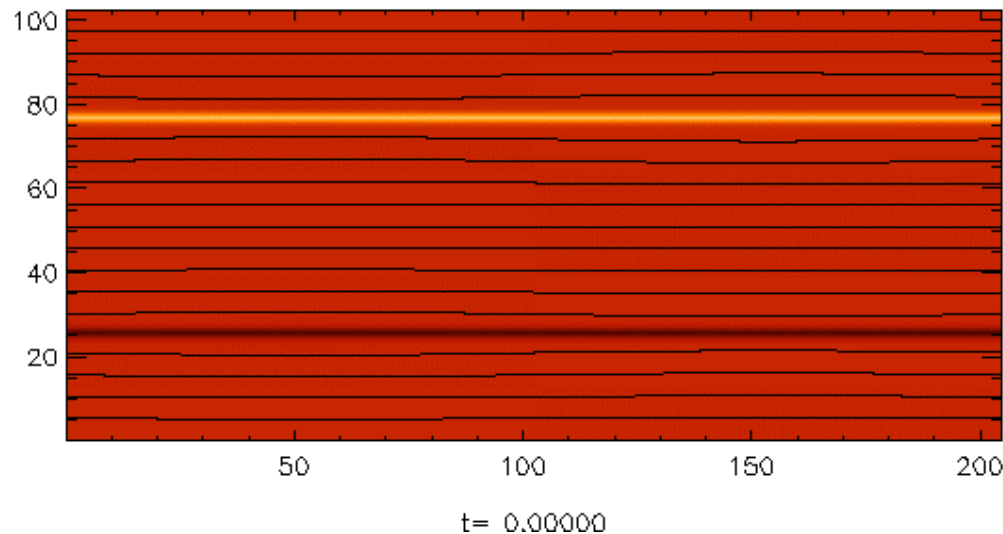
# Conclusion / Open Issues

- **Conclusions:**  
The nonlinear dynamics of magnetic reconnection is very rich
  - **Saddle-node bifurcations** underlie the dynamics
  - Motivates why **Hall reconnection** onsets suddenly (**catastrophically**) in a weakly collisional plasma
  - Observations from laboratory and solar/stellar flares agree with predictions
- **Open Issues:**  
We have assumed Sweet-Parker scaling persists for *very* high aspect ratio current sheets ( $L / \delta \sim 10^7$ )
  - These current sheets are susceptible to secondary tearing and island formation
  - It has been proposed that turbulence (Matthaeus and Lamkin, 1986; Lazarian and Vishniac, 1999) and/or secondary islands (Kliem, 1995; Shibata and Tanuma, 2001; Daughton et al., 2006) is sufficient to make resistive reconnection fast
    - This needs to be studied further. If true, this model would be in peril.
- The onset mechanism presented here requires that **Sweet-Parker** reconnection occurs before **Hall** reconnection, so it does not obviously apply to the magnetosphere.

Thank you and  
Happy Birthday, Russell!

# Importance of Hall Effect

- Hybrid simulations recently performed with and without the Hall term (Malakit et al., in prep), similar to Karimabadi et al., 2004
  - Hall term turned off at  $t = 300$



- The Hall term is clearly important for maintaining an open outflow configuration