Dynamic Alignment in MHD turbulence: Computations

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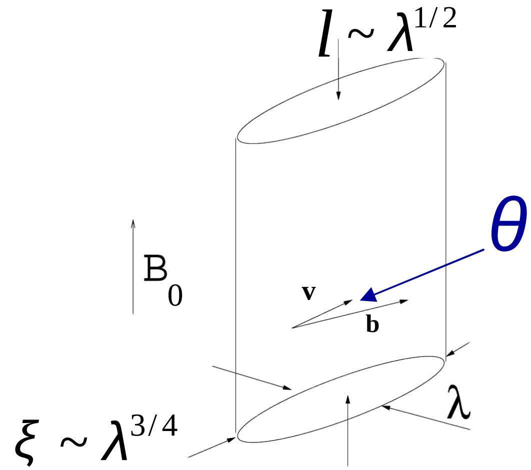
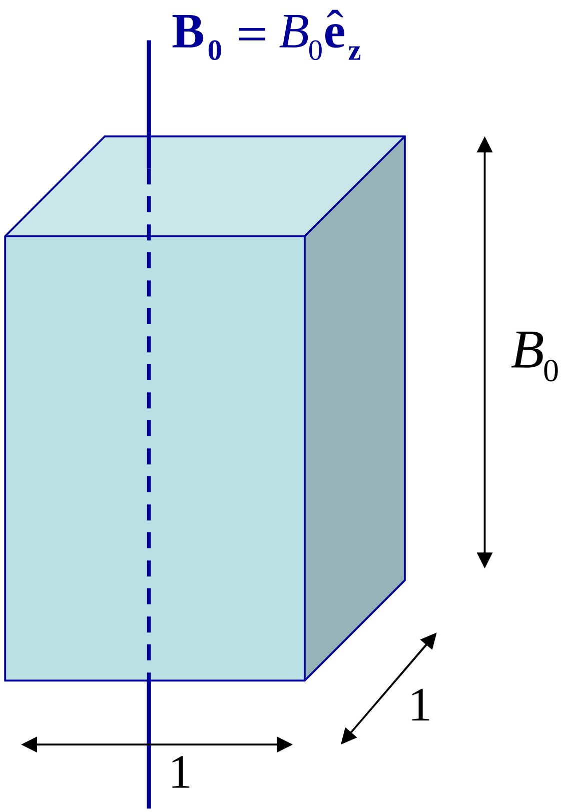
Alignment in Driven MHD turbulence



$$\theta_\lambda \sim \lambda^{1/4}, \quad E(k_\perp) \propto k_\perp^{-3/2}, \quad l \sim \lambda^{1/2} \sim \xi^{2/3}$$

1. **Measure alignment angle**
2. **Measure field-perpendicular spectrum**
3. **Illustrate consistency with the exact scaling relations in MHD turbulence (analogous to Kolmogorov's 4/5-law)**

Computation details



$$R_e = \frac{uL}{\nu} \approx \theta$$

$$P_m = \frac{\nu}{\eta} = 1$$

$$\partial_t \mathbf{u} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \mathbf{j} \times \mathbf{B} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

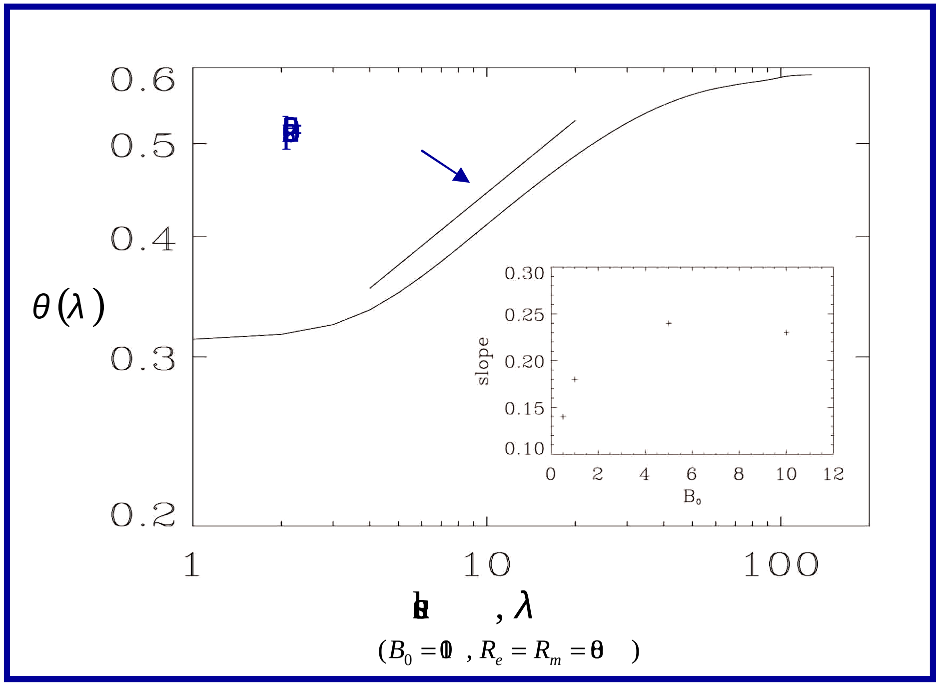
$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 B$$

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{B} = 0$$

1. Angular Alignment

- M ... l
- ... -
$$\delta \mathbf{v}_\lambda \equiv \mathbf{v}(\mathbf{x}+\lambda) - \mathbf{v}(\mathbf{x}), \qquad \delta \tilde{\mathbf{v}}_\lambda = \delta \mathbf{v}_\lambda - (\delta \mathbf{v}_\lambda \cdot \mathbf{n})\mathbf{n}, \quad \mathbf{n} = \frac{\mathbf{B}(x)}{|\mathbf{B}(x)|}$$
- M ... m ...

$$\theta_\lambda \approx \sin(\theta_\lambda) = \frac{\left\langle \left|\delta \tilde{\mathbf{v}}_\lambda \times \delta \tilde{\mathbf{b}}_\lambda\right| \right\rangle}{\left\langle \left|\delta \tilde{\mathbf{v}}_\lambda\right| \left|\delta \tilde{\mathbf{b}}_\lambda\right| \right\rangle} \sim \lambda^{1/4}$$



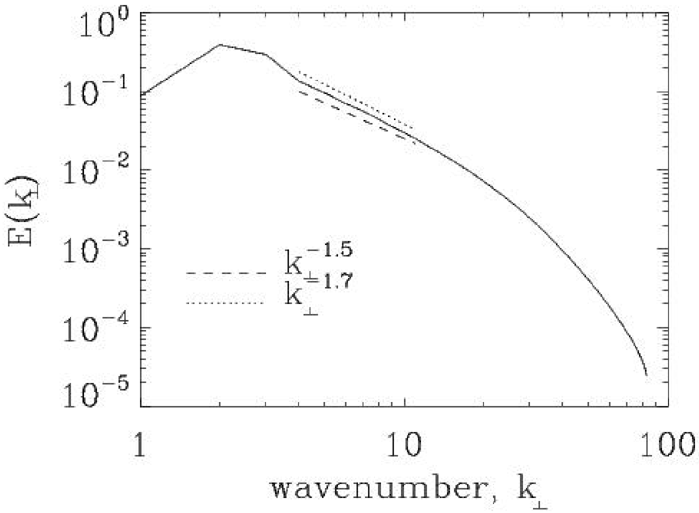
Regions of Alignment

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{b}}{|\mathbf{u}||\mathbf{b}|}$$

$k > k_f,\ B_0 = 5,\ R_{e,m} = \theta$

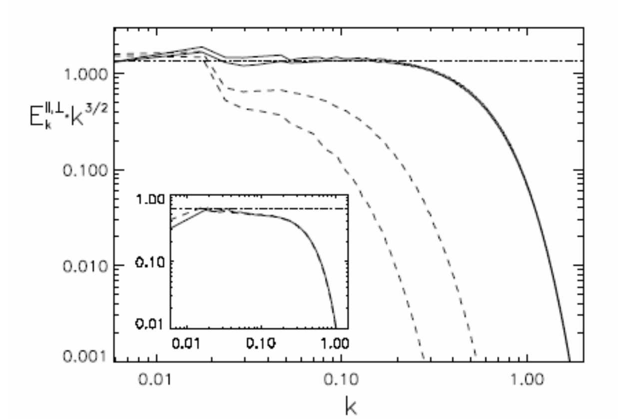
2. Energy Spectrum

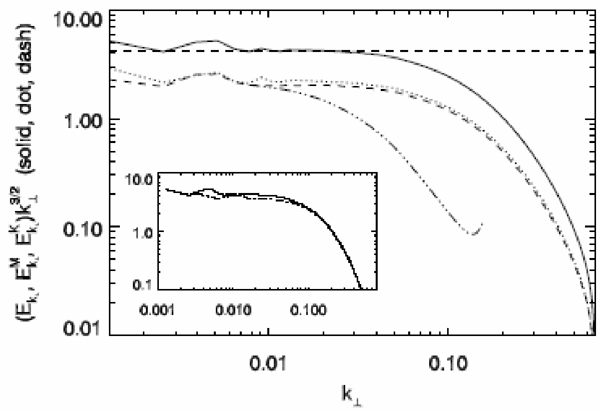
- **Our moderate spatial resolution makes identification of the scaling law for the energy spectrum difficult**



$$E(k_\perp) = \left\langle \left|\mathbf{v}(k_\perp)\right|^2 \right\rangle k_\perp + \left\langle \left|\mathbf{b}(k_\perp)\right|^2 \right\rangle k_\perp$$

$$k_\perp = \sqrt{k_x^2 + k_y^2}$$





3. Exact relations (Kolmogorov's 4/5-law)

• **Hydrodynamic turbulence** $\left\langle \delta v_L^3(\mathbf{r}) \right\rangle = -\frac{4}{5}\varepsilon r \;\rightarrow\; E(k) \propto k^{-5/3}$,

where $\delta v_L(\mathbf{r}) \equiv [\mathbf{v}(\mathbf{x}+\mathbf{r}) - \mathbf{v}(\mathbf{x})]\cdot \mathbf{r}$

• **Isotropic MHD turbulence (Politano & Pouquet, 1998)** $\delta \mathbf{z}^{\pm} = \delta \mathbf{v} \pm \delta \mathbf{b}$

$$\left\langle \delta z_L^- (\delta \mathbf{z}^+)^2 \right\rangle = -\frac{4}{3}\varepsilon^+ r \quad \text{and} \quad \left\langle \delta z_L^+ (\delta \mathbf{z}^-)^2 \right\rangle = -\frac{4}{3}\varepsilon^- r$$

• **For a strong guiding field**

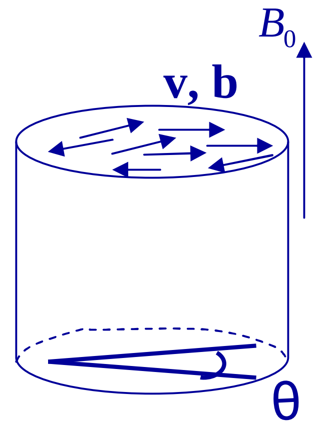
$$\left\langle \delta z_L^- (\delta \mathbf{z}^+)^2 \right\rangle = -2\varepsilon^+ r_\perp \quad \text{and} \quad \left\langle \delta z_L^+ (\delta \mathbf{z}^-)^2 \right\rangle = -2\varepsilon^- r_\perp$$

~> inconsistent with $E(k_\perp) \propto k_\perp^{-3/2}$ **?**

• Check the theory of angular alignment through exact relations; theory predicts

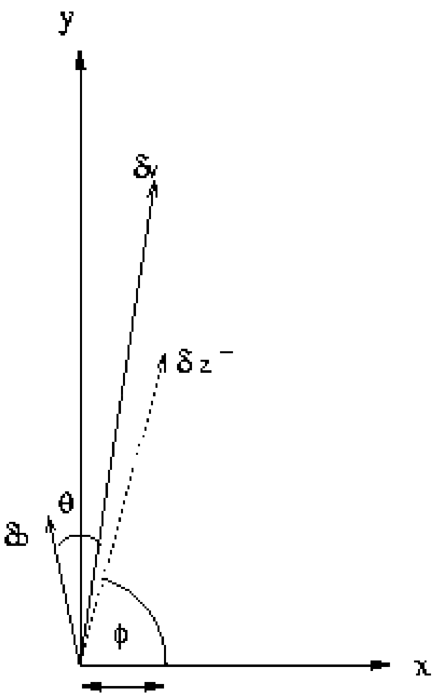
$$\left\langle \delta z_L^- (\delta \mathbf{z}^+)^2 \right\rangle = \theta_r \delta v_r^3 \sim r$$

3. Exact relations



- **Consider δv aligned with +δb and the prediction** $\left\langle \delta z_L^- \left(\delta z^+\right)^2 \right\rangle = \theta_r \delta v_r^3 \sim r$

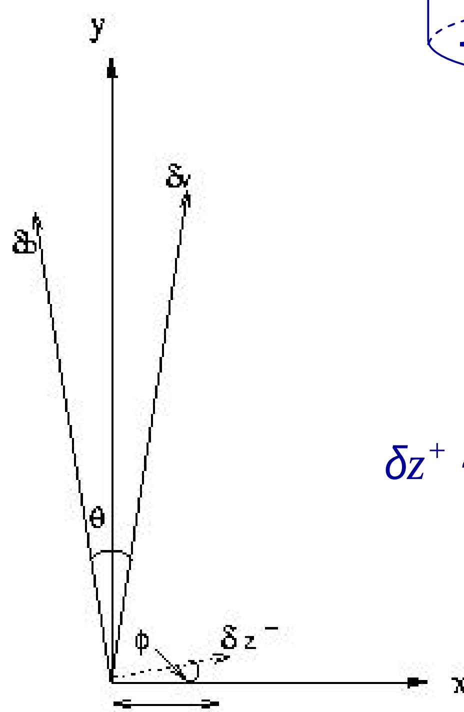
$|\delta v| >> |\delta b|$



$\delta z^+ \sim \delta v$

$$\delta z_L^- = \delta z^- \cos(\varphi) \approx \delta z^- \cos\left(\theta - \frac{\pi}{2}\right) = \delta z^- \sin(\theta) \approx \delta z^- \theta \approx \delta v \theta$$

$|\delta v| \approx |\delta b|$

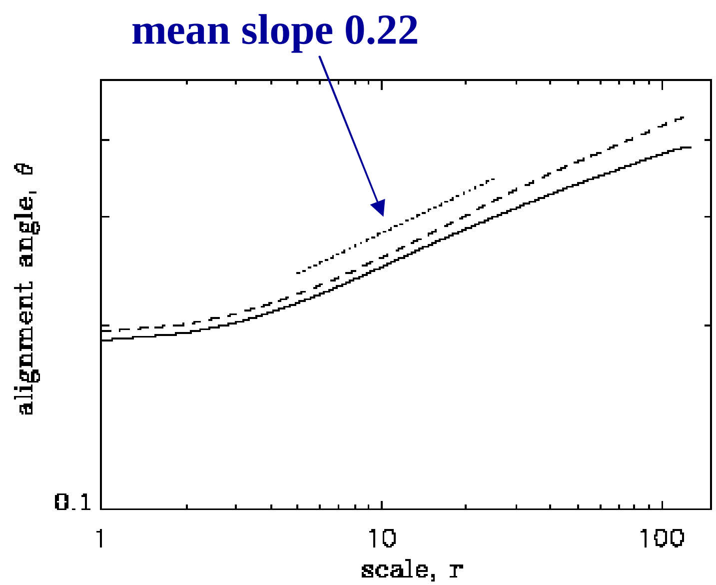


$\delta z^+ \sim \delta v$

$$\delta z_L^- = \delta z^- \cos(\varphi) \approx \delta z^- \approx \delta v \sin(\theta) \approx \delta v \theta,$$

3. Exact relations

- **Measure** $\frac{\left\langle |\delta z_L^-| (\delta \mathbf{z}^+)^2 \right\rangle}{\left\langle (\delta \mathbf{z}^+)^3 \right\rangle}$ **and** $\frac{\left\langle |\delta z_L^+| (\delta \mathbf{z}^-)^2 \right\rangle}{\left\langle (\delta \mathbf{z}^-)^3 \right\rangle}$ **(expect** $\sim \theta \sim \lambda^{1/4}$ **)**



- **'Exact' relations provide an important analytic verification of dynamic alignment in MHD turbulence**

Conclusions

- **Good agreement between numerical results and theory: Magnetic and velocity field fluctuations become dynamically aligned**

  à **Eddies are three-dimensionally anisotropic: ribbon-like dissipative structures rather than filaments**

  à **Perpendicular energy spectrum** $E\left(k_{\perp}\right) \propto k_{\perp}^{-3/2}$

- **Theory is consistent with the exact Politano & Pouquet relations**

References

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