Cosmic Ray Scattering In Compressible MHD

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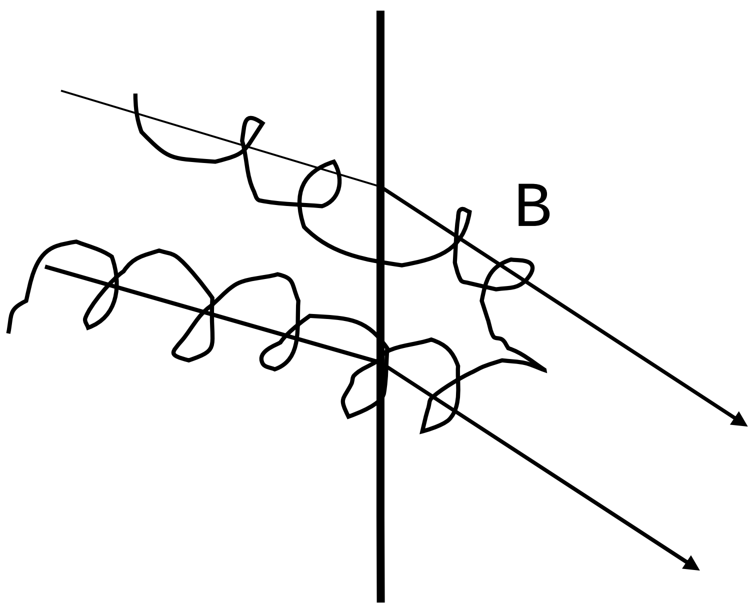
Early ideas

- A concept of "ideal scattering centers". Usually prescribes a certain isotropic diffusion coefficient, *D*, and was used for early shock acceleration models. Usually *D* does not depend on energy. This model is simple and easy to use, but lacks physical ground.
- Prescribed scattering parallel to the magnetic field.



Scattering by magnetic perturbations

First models:

Isotropic, delta-correlated Alfven mode perturbations with Kolmogorov (-5/3) or I-K (-3/2) spectrum.

More realistic models, based on turbulence:

1. Gyro-resonance with Alfven mode in anisotropic Goldreich-Sridhar turbulence (Chandran 2000, Yan, Lazarian 2002). Basically showed that low-energy CR's do not scatter at all (inverse scattering frequencies are larger than the age of the Universe).
2. Gyro-resonance with isotropic-like fast mode (Yan, Lazarian 2004). The result strongly depends on the damping of this mode. Predicts pretty large paths for Galactic ISM (tens of parsecs)
3. Scattering by TTD (or n=0, or Cherenkov). Important when gyro-resonance is small.

Problems:

In ICM, where collisionless damping of compressive modes is large, mean free paths turn out to be large.

This work: including self-action of CR's

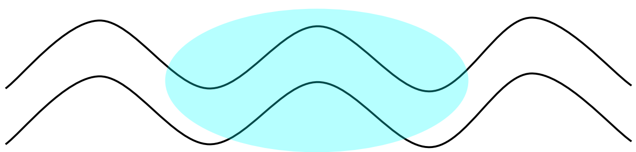
Self-action could generally be important: CR pressure is often comparable with magnetic and thermal pressure.

Standard treatment of self-action: find the instability. Particles create a wave and are being scattered by the wave. The perturbations are either stable or unstable.

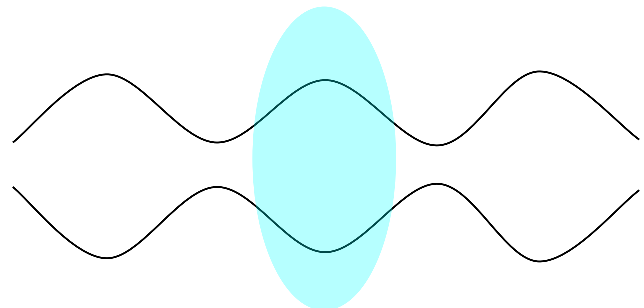
Instabilities were used for quite a while in CR physics, e.g. to justify pitch-angle scattering needed for many acceleration models.

Hydrodynamic instabilities:

fire-hose



mirror



Require rather large anisotropies.
Does turbulence produce such CR anisotropies? We argue it doesn't.

Key components of the model:

1. Properties of turbulence: both Alfvenic and compressible modes.
2. Instability rate.
3. Nonlinear suppression: we take into account scattering provided by instability and assume that only those perturbations that are on the scale of the mean free path provide most scattering.
4. Saturation: in the presence of nonlinear effects such as steepening the instability is saturated.
5. Damping by turbulence: we have to account for it, whether we like it or not.

The instability between circularly polarized Alfven wave and the anisotropic distribution of fast particles (CRs)

$$\gamma_{\rm CR} = \pi^2 e^2 v_A \int \frac{v_\perp^2}{c^2} \left(\frac{\partial F}{\partial p_\parallel} - \frac{v_\parallel}{v_\perp} \frac{\partial F}{\partial p_\perp} \right) \delta(k_\parallel v_\parallel \pm \omega_C)\, d^3\mathbf{p}$$

$$\gamma_{\rm CR}(k_\parallel) = \omega_{pi} \frac{n_{\rm CR}(p > m\omega_B/k_\parallel)}{n} AQ,$$

$$Q = \frac{\pi^{3/2}}{32}(\alpha + 2)(\alpha - 1)\frac{\Gamma(\alpha/2)}{\Gamma(\alpha/2 + 3/2))}.$$

The anisotropy factor A is determined by the amplitude of the mode that changes the absolute value of B most of all. It is slow wave in high-β plasma and fast wave in low-β plasma. The anisotropy factor, sinθ is unimportant in generic MHD turbulence (slow modes have it ≈ 1 and fast modes are mostly isotropic, so its around unity).

$$A \sim 2\delta v / v_A$$

$$\gamma_{\rm CR}(r_p) = \frac{\delta v}{L_i} \left(\frac{r_p}{r_0} \right)^{-\alpha+1},$$

$\alpha \sim 2.6$ in the Galaxy

$$L_i = 3.7 \cdot 10^{-7} \frac{1}{Q} \left(\frac{B}{5 \cdot 10^{-6}\ {\rm G}} \right) \left(\frac{4 \cdot 10^{-10}\ {\rm cm}^{-3}}{n_{\rm CR}(r_p > r_0)} \right)\ {\rm pc}.$$

Mean free path:

$$\lambda \sim N r_p \sim r_p/\phi^2 \sim r_p B^2/(\delta B)^2,$$

(wandering in pitch-angle)

“Reduced” instability rate.

$$\gamma_{\rm CR} \approx \frac{v_A}{L_i}\left(\frac{r_p}{L}\right)^{\mu}\left(\frac{\delta B}{B}\right)^{-2\mu}\left(\frac{r_p}{r_0}\right)^{-\alpha+1},$$

$$\gamma_{\rm steep} \approx -(\delta B/B)^2 k_{\|} v_A,$$

Saturation…

Saturated spectrum:

$$\frac{\delta B}{B} \approx \frac{r_0^{1/2}}{L_i^{1/(2\mu+2)} L^{\mu/(2\mu+2)}} \left(\frac{r_p}{r_0}\right)^{(\mu-\alpha+2)/(2\mu+2)},$$

Mean free path:

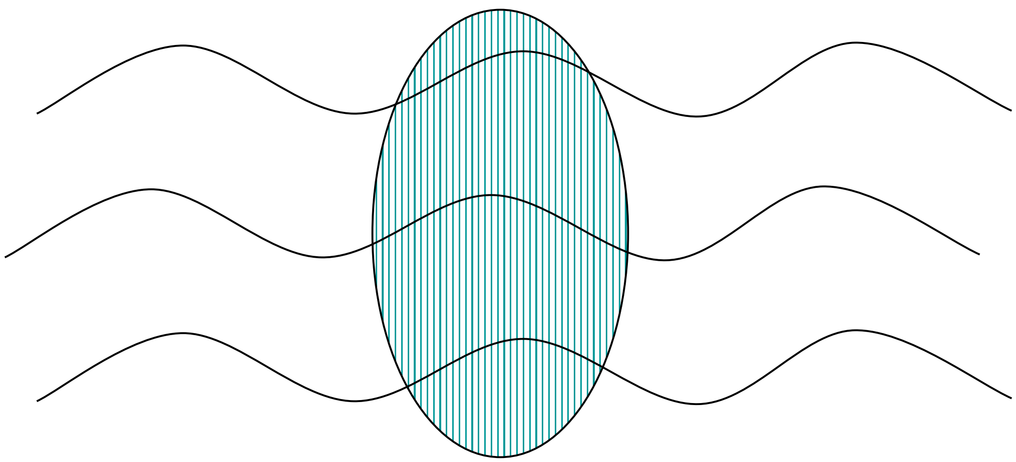
$$\lambda \approx L_i^{1/(\mu+1)} L^{\mu/(\mu+1)} \left(\frac{r_p}{r_0}\right)^{(\alpha-1)/(\mu+1)}.$$

What if compressions are suppressed at some scale?

$$\frac{\delta B}{B} \approx \left(\frac{r_0^{1/2}}{L_i^{1/4} L^{\mu/4} l_{\rm cut}^{(1-\mu)/4}}\right)^{1/4} \left(\frac{r_p}{r_0}\right)^{(3-\alpha)/4},$$

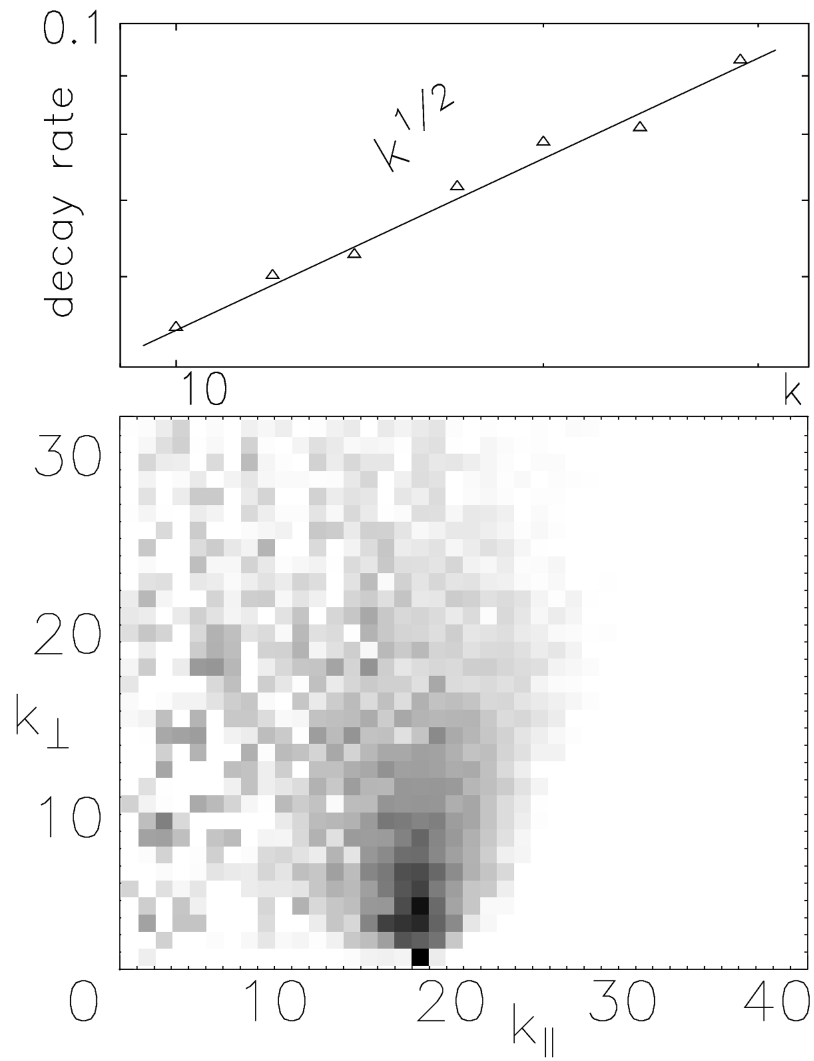
Turbulent damping of the wave

Hydrodynamic: γ~kc_s



$$k_\perp \sim \delta k_\perp \sim r_p^{-1}(r_p/L)^{1/4}$$

(Farmer, Goldreich, 2004)

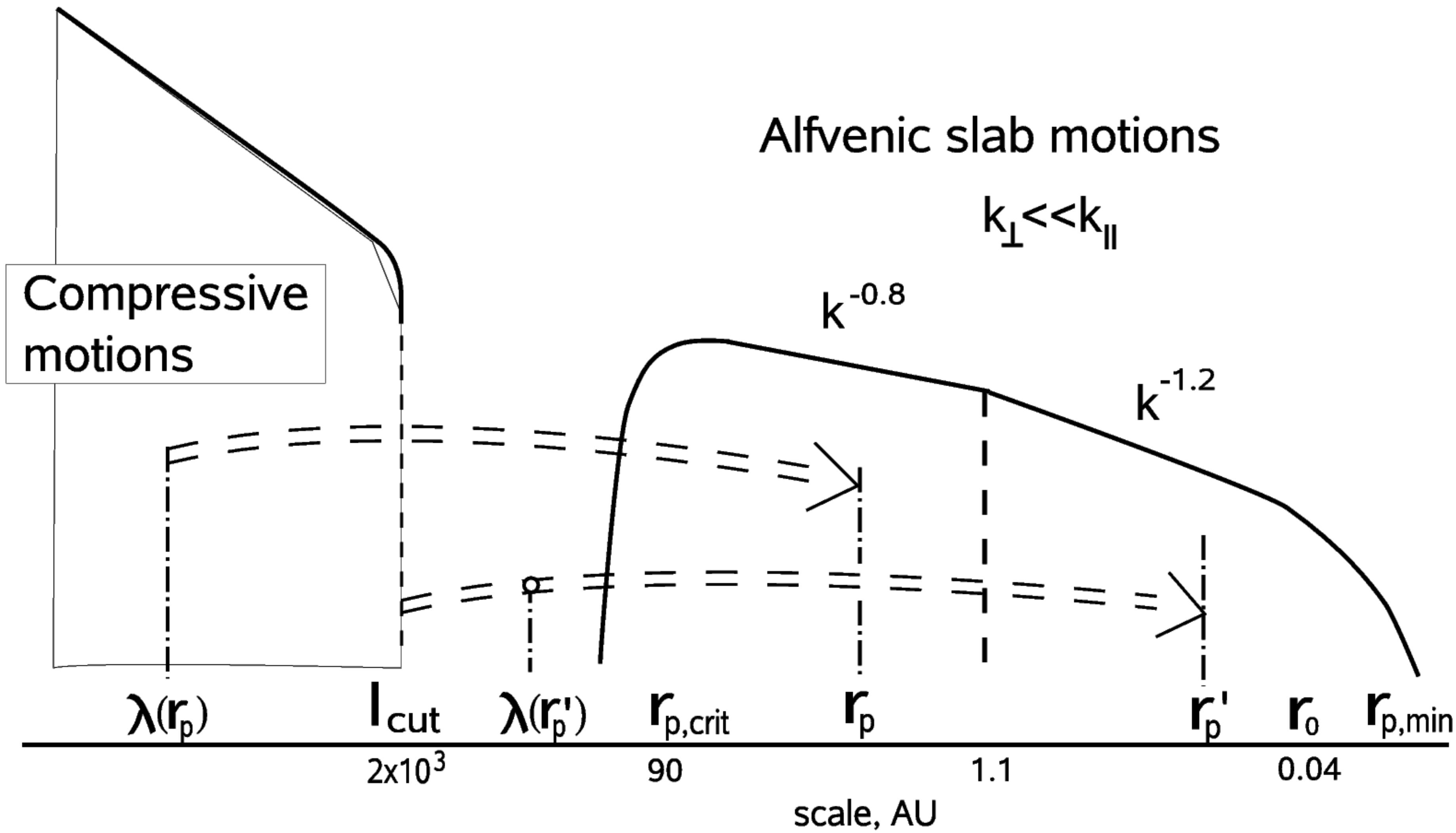


(Beresnyak, Lazarian, 2006)

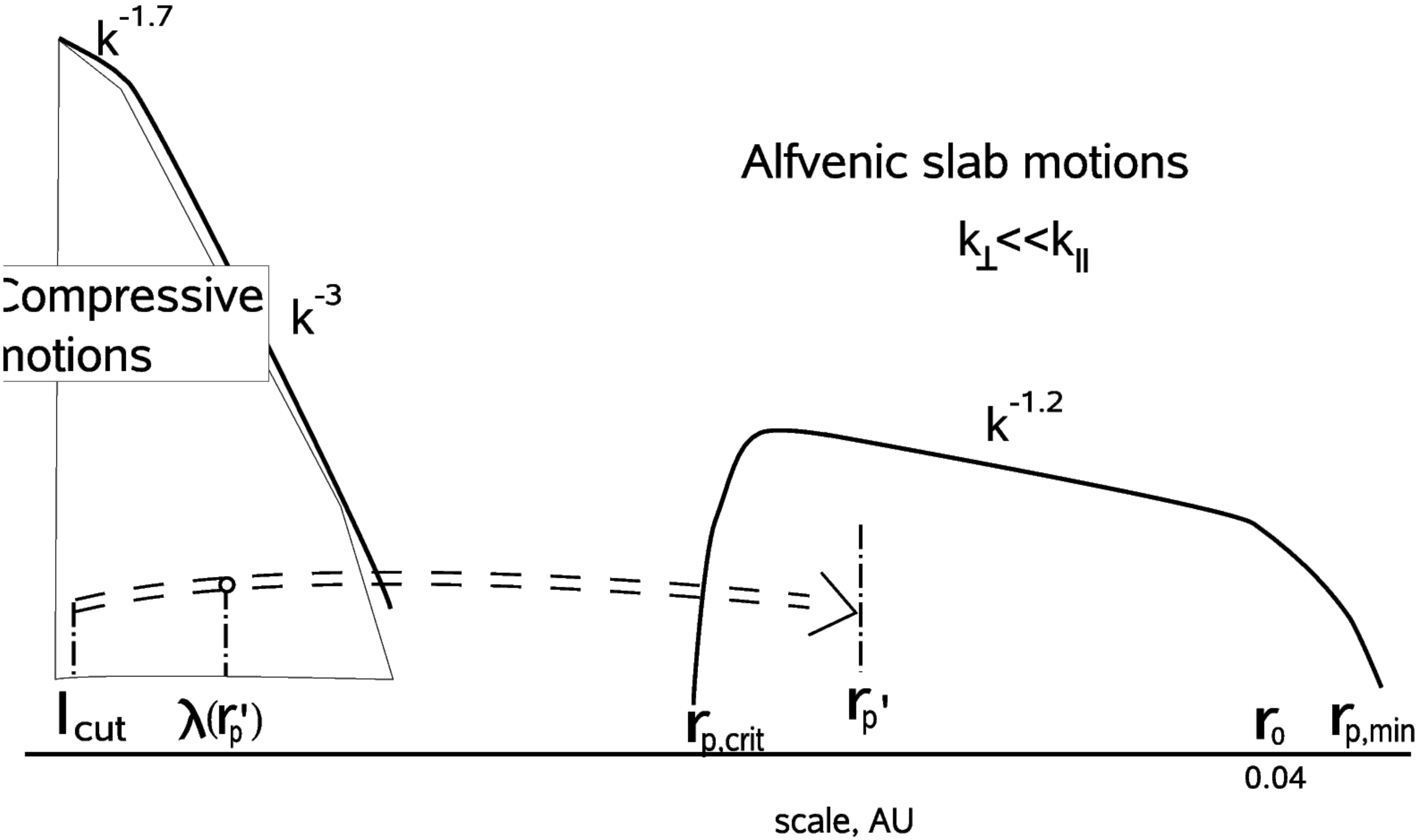
$$\gamma_{\rm turb} \sim -k_\perp v_\perp \sim -v_A k_\perp^{2/3} L^{-1/3} \sim -v_A r_p^{-1/2} L^{-1/2},$$

The effect exists only for low-energy CRs, with Larmor radius smaller than

$$r_{p,\mathrm{crit}} \approx r_0 \left(L^{1-\mu} r_0^{\mu+1} L_i^{-2} \right)^{1/(2\alpha-\mu-3)} \quad .$$



Alfvenic slab motions
$k_\perp << k_\parallel$
Compressive motions
$k^{-0.8}$
$k^{-1.2}$
$\lambda(r_p)$
l_{cut}
$\lambda(r_p')$
$r_{p,crit}$
r_p
r_p'
r_0
$r_{p,min}$
$2x10^3$
90
1.1
0.04
scale, AU



k^-1.7
Compressive motions
k^-3
Alfvenic slab motions
k⊥<<k||
k^-1.2
l_cut
λ(r_p')
r_p,crit
r_p'
r_0
r_p,min
0.04
scale, AU

Mean free paths of thermal particles.

Coulomb mean free path of a thermal particle in a cluster (T=10^8K, n=10^{-3}cm^{-3}) is about 4 kpc.
Can this particle be thermal???

Yes, it can, because its real mean free path is small!

Using $A = 2(\lambda_0/L)^\mu = v_A/c$ we get $\lambda_0 \approx 10^{-6}$pc in galaxy clusters. This value is 9 orders of magnitude smaller than the Coulomb mean free path.

Summary

1. Turbulent compressions of magnetic field result in the kinetic instability of CRs that drives Alfvénic perturbations of much higher frequency with wave vectors almost parallel to the magnetic field. These Alfvénic perturbations efficiently scatter and isotropise CRs.

2. The above effect is present over the limited energy range of the CRs. The high energy cut-off is determined by the ambient Alfvénic turbulence. The non-linear backreaction via limiting the CR mean free path and the steepening of the generated waves control the intensity of the new slab-type Alfvénic component. This intensity depends on both the amplitude of the compressible perturbations and CR pressure.

3. The presence of linear damping of compressible motions modifies the instability and results in a slightly steeper spectrum at higher frequencies of generated Alfvénic perturbations.