Strong magnetohydrodynamic turbulence with cross helicity

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- The degree of imbalance is measured by the cross helicity.
- Most turbulence theories and simulations assume the turbulence is *balanced*, i.e., the average cross-helicity is zero.
- Recent works support the emerging picture that turbulence is locally imbalanced even when the turbulence is globally balanced.

Recent advances on imbalanced MHD turbulence

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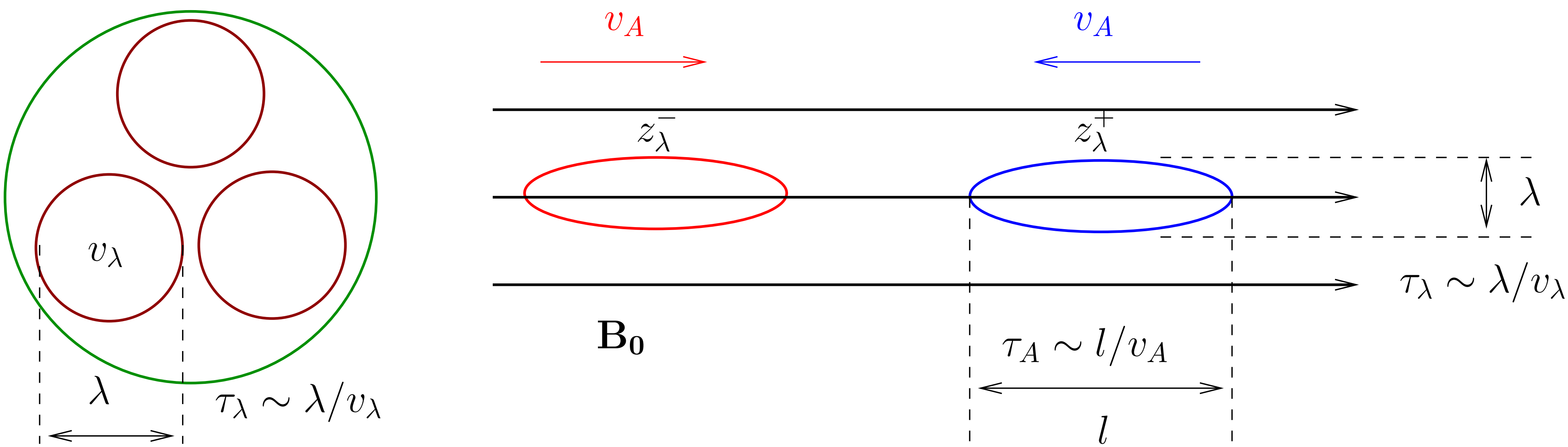
- Observations and numerical simulations indicate that MHD turbulence is constituted by regions of both positive and negative cross-helicity. In simulations this holds even in the balanced case.

HD vs MHD turbulence pictures



Hydrodynamic turbulence:

- Interaction of eddies.
- Isotropic.
- Strong interactions.
- Energy Cascade.

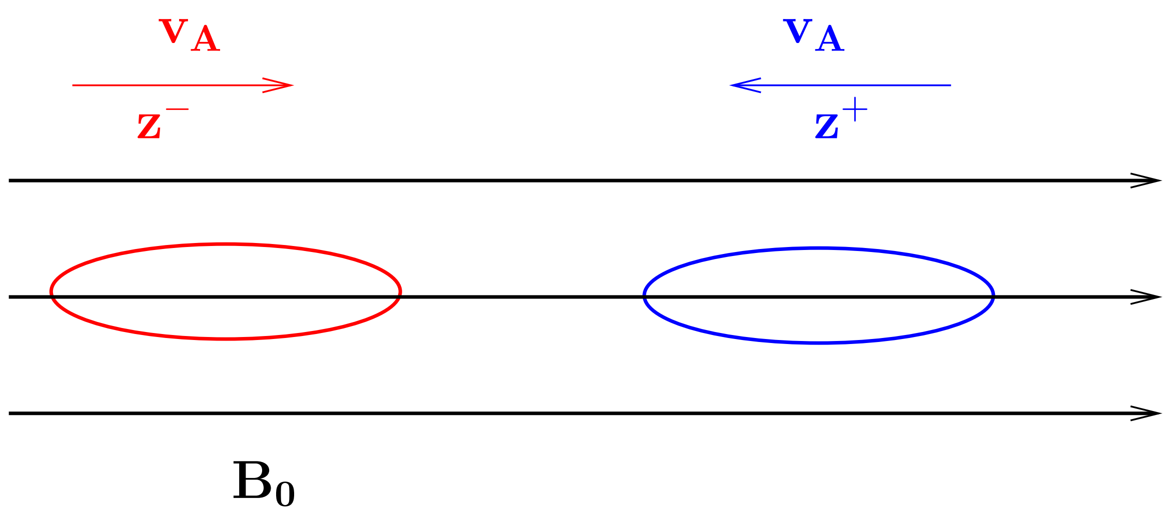
Magnetohydrodynamics:

- Interaction of wave packets.
- Anisotropic.
- Weak or Strong interactions.
- Energy and Cross-Helicity cascades.
- Balanced or imbalanced.

- MHD turbulence is best studied in the Elsässer formulation

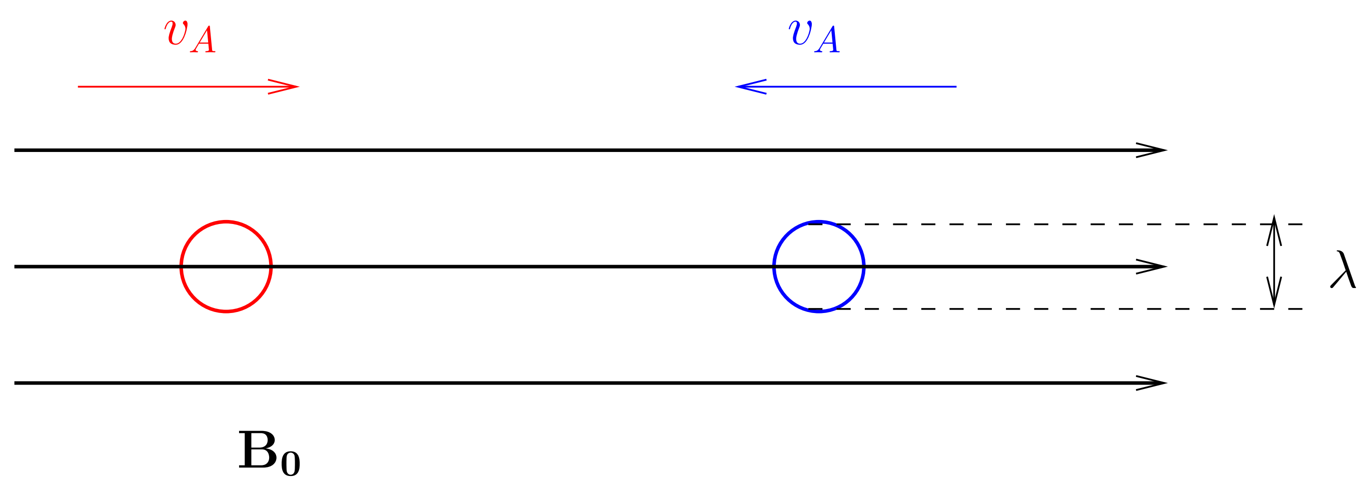
$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp \underbrace{(\mathbf{v}_A \cdot \nabla)}_{\sim \ 1/\tau_A} \mathbf{z}^{\pm} + \underbrace{(\mathbf{z}^{\mp} \cdot \nabla)}_{\sim \ 1/\tau_{NL}} \mathbf{z}^{\pm} = -\nabla P + \frac{1}{R} \nabla^2 \mathbf{z}^{\pm} + \mathbf{f}^{\pm}$$

where $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$, and $\nabla \cdot \mathbf{z}^{\pm} = 0$.



- Two competing time scales in the problem: wave time scale τ_A and interaction time scale τ_{NL}.

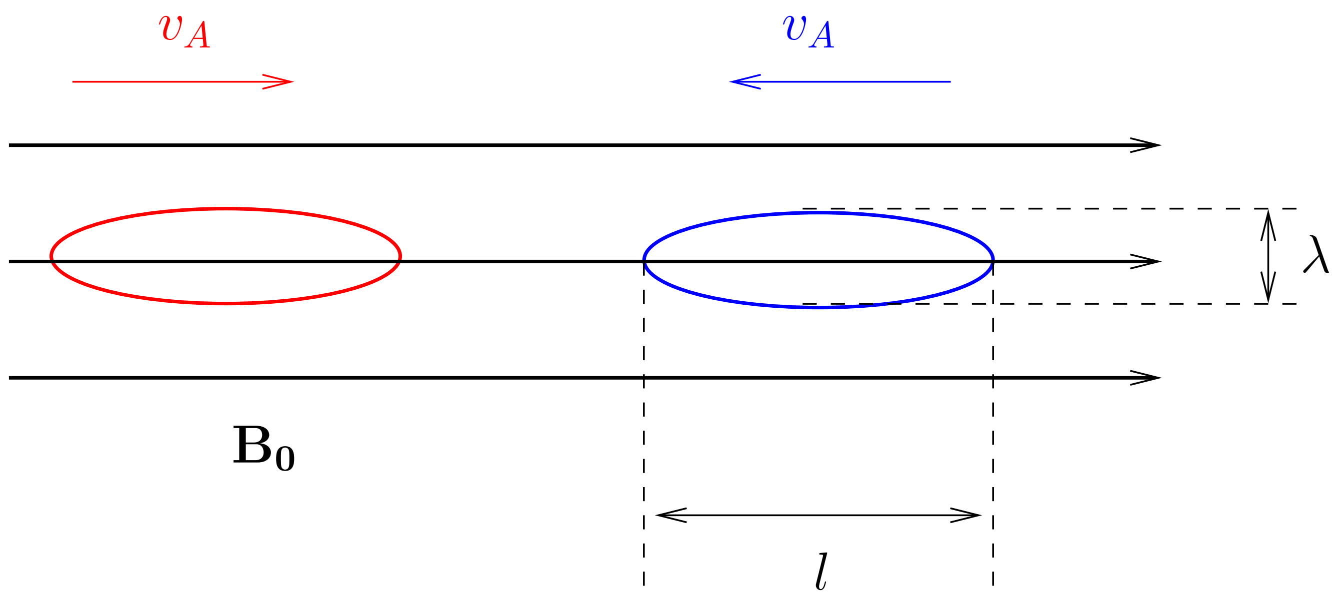
- Iroshnikov and Kraichnann (IK): isotropic eddies



- Energy spectrum:

$$E(k) \sim k^{-3/2}$$

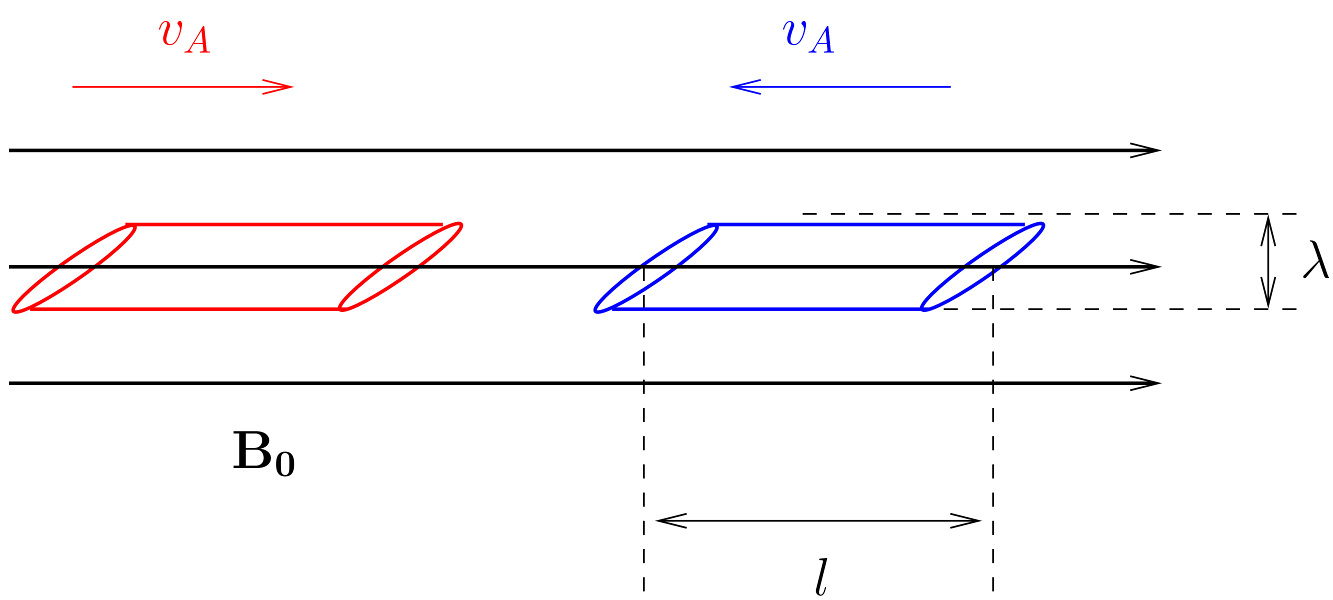
- Goldreich & Sridhar (GS): elongated eddies, *Critical Balance*: ($\tau_A \sim \tau_{NL}$)



- Energy spectrum:

$$\underbrace{\frac{v_A}{l} \sim \frac{v_\lambda}{\lambda}}_{\text{Critical Balance}} \Rightarrow E(k_\perp) \sim k_\perp^{-5/3}$$

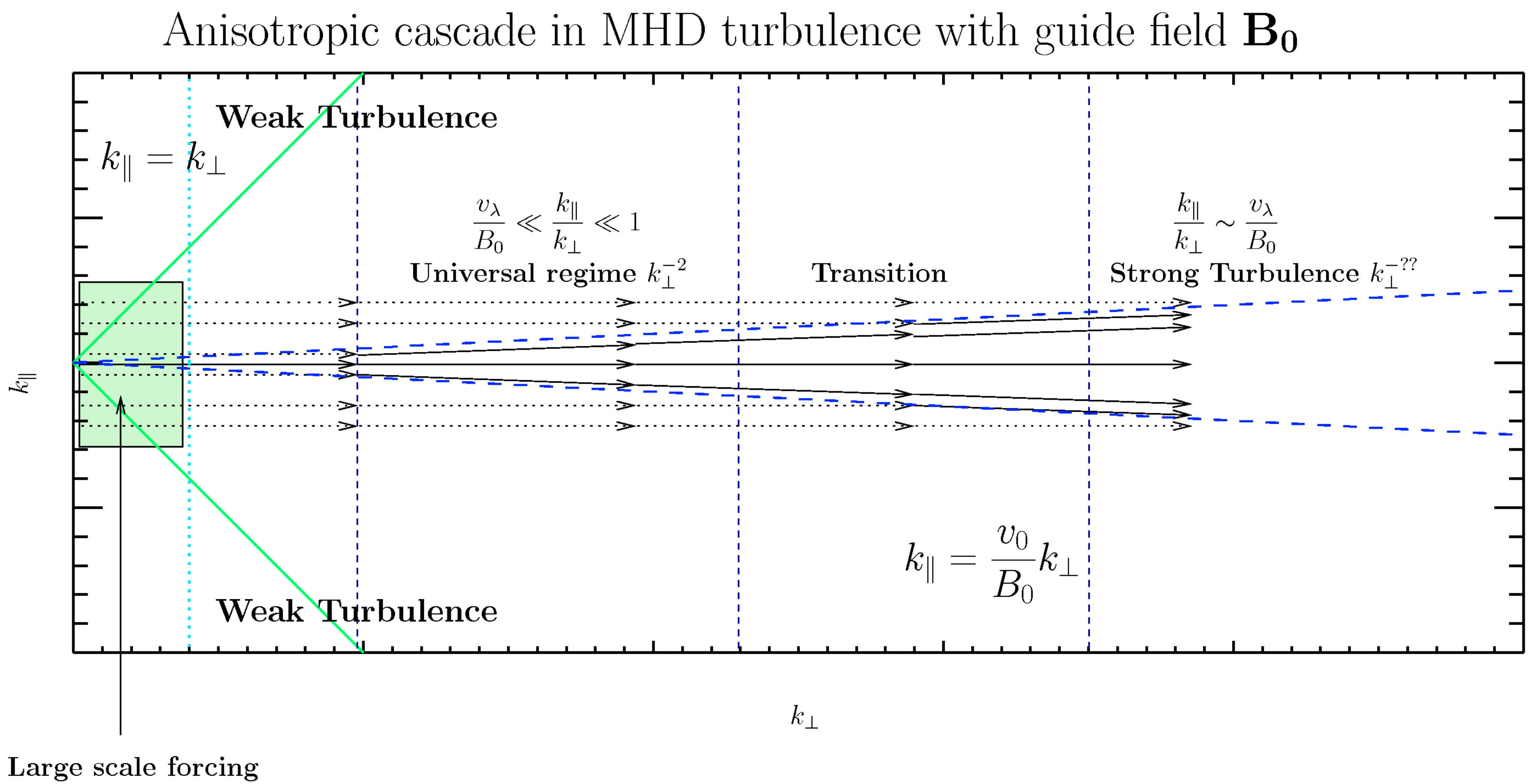
- S. Boldyrev (SB): critical balance, dynamic alignment magnetic field fluctuations



- Energy spectrum:

$$\frac{v_A}{\lambda_\|} \sim \frac{v_\lambda}{\lambda_\perp} \underbrace{\frac{v_\lambda}{v_A}}_{\theta_\lambda} \Rightarrow E(k_\perp) \sim k_\perp^{-3/2}$$

General picture of anisotropic Balanced MHD turbulence



Energy and Cross-Helicity cascades

- Energy and Cross-helicity are defined as

$$E = \frac{1}{2}\left\langle \mathbf{v}^2 \right\rangle + \frac{1}{2}\left\langle \mathbf{b}^2 \right\rangle , \quad H_c = \langle \mathbf{v} \cdot \mathbf{b} \rangle$$

- H_c measures the degree of correlation between velocity and magnetic fluctuations in the turbulent state.

- In terms of Elsasser variables energy and cross-helicity invariants take the form

$$E = E^{+} + E^{-}, \quad H_c = E^{+} - E^{-}, \qquad E^{\pm} \equiv \frac{1}{4}\left\langle |\mathbf{z}^{\pm}|^2 \right\rangle$$

- Cross-helicity measures the imbalance between counter-propagating waves.

- Together with energy, cross-helicity undergoes a turbulent cascade from large to small scales. The same holds for $E^{\pm}$.

Simulations of driven HD vs MHD

- Simulations of driven Hydrodynamic turbulence are performed on a cubic box L^3.

- Forcing is applied at the lowest wave numbers

$$\mathbf{k} = \frac{2\pi}{L}(m_x, m_y, m_z), \qquad m_x, m_y, m_z = \text{small integers.}$$

- HD Turbulence is isotropic and always strong.

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- HD Turbulence is isotropic and always strong.

- MHD turbulence can be weak or strong, depending on the ratio between the characteristic linear and nonlinear time scales.

- Whatever forcing method is used, the critical balance condition should be controlled at the forcing scale.

- Numerical settings of HD have to be modified to capture the different aspects of MHD turbulence.

Challenges in MHD numerical simulations

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For the limited inertial ranges obtained in simulations, large scale forcing should excite eddies that match the turbulence anisotropy as much as possible.

This will avoid long transitions to the strong turbulence regime where a universal power law scaling is expected.

- Turbulence simulations generally require high resolutions to resolve the large difference of scales between forcing and dissipation characteristic of a turbulent system.

- In MHD, the turbulence spectrum declines quite slowly as a power law in $k_\perp$ and drops sharply in $k_{||}$ (Cho & Vishniac 2000; Maron & Goldreich 2001; Perez & Boldyrev 2008,2009).

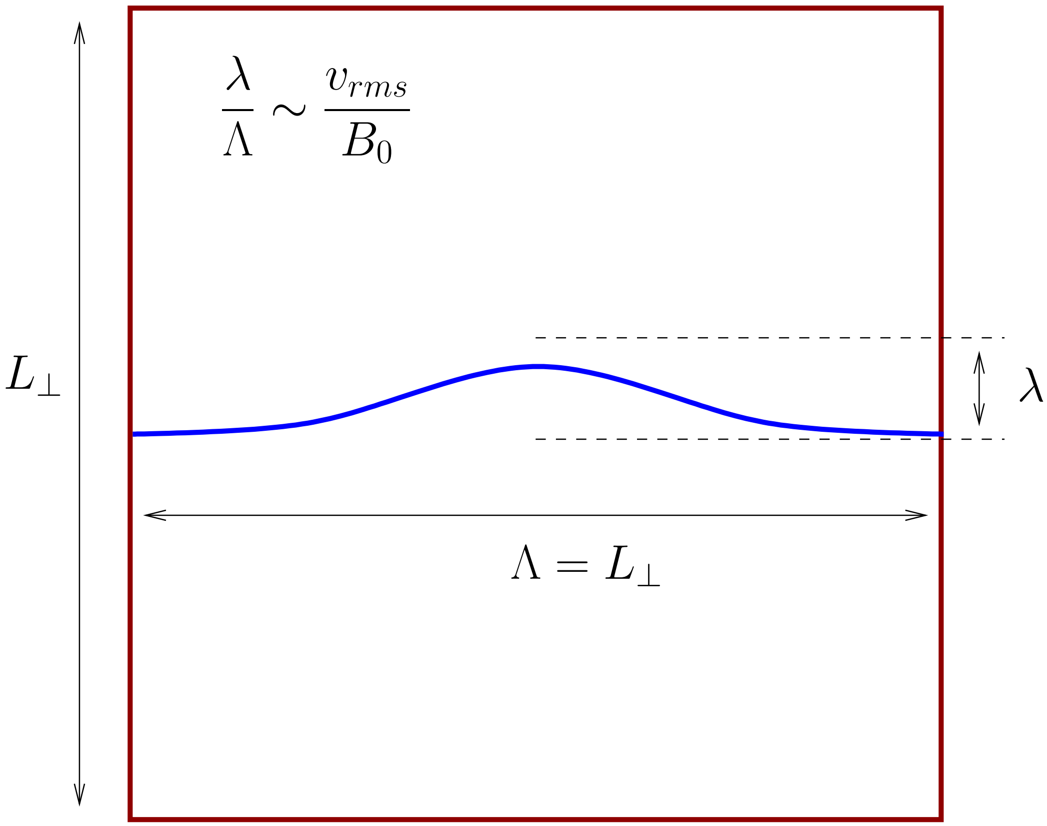
- This different spectral behaviour in $k_\perp$ and $k_{||}$ allows us to reduce the numerical resolution in the field parallel direction.

- Simulations show that restoring full resolution in the field parallel direction does not change the results, while significantly increases the computing costs.

Strength of the guide field

- In the inertial range, the guide field B_0 should be strong compared to turbulent fluctuations $b_\lambda \ll B_0$.
- Numerical tests found: $b_\lambda/B_0 > 1/3$ energy spectrum scaling is closer to Kolmogorov $-5/3$, and changes to $-3/2$ for $b_\lambda/B_0 < 1/3$. Mason, Cattaneo & Boldyrev, PRL, 97, 255002 (2006).
- The ratio $b_\lambda/B_0 \sim 1/5$ is a good "sweet spot" for balanced turbulence.
- Smaller ratios do not lead to different results in the simulations.

Critical balance at the forcing scale



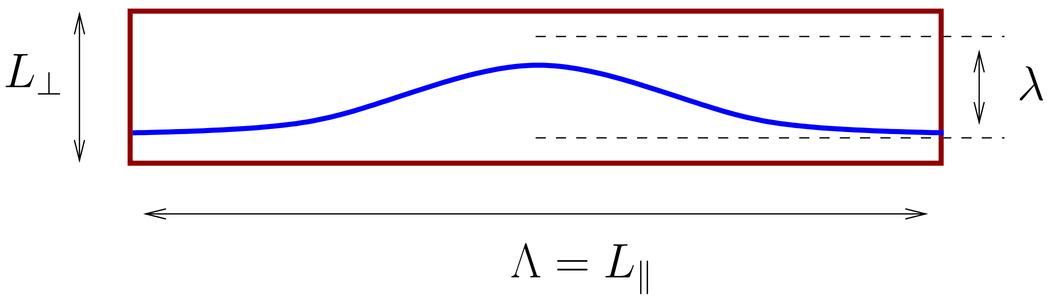
- MHD turbulence is anisotropic: eddies elongated along $\mathbf{B}_0$.
- Cubic simulation box is not optimal: a significant number of perpendicular scales wasted.
- Can lead to inaccurate measurements of spectral index.
- The box aspect ratio is crucial to maximize the range of useful perpendicular scales.

- Ratio between linear and nonlinear time scales is controlled at the forcing scale

$$\chi = \frac{k_\| v_A}{k_\perp v_0}$$

Critical balance at the forcing scale

$$\frac{\lambda}{\Lambda} \sim \frac{v_{rms}}{B_0}$$



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Impact of cross helicity in numerical resolution

- The formal Reynolds numbers corresponding to z^+ and z^- fluctuations are essentially different $\Rightarrow$ resolution requirements increase with the amount of cross helicity.

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- Number of required grid points in the field-perpendicular direction scales as

$$N \sim Re^{\beta},$$

  where $\beta = 2/3$ or $3/4$ depending on the the spectral slope $3/2$ or $5/3$ respectively.

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- Increasing the imbalance by a factor of 3 requires at least doubling the resolution, increasing computational costs by a factor of 16!.

- We used a 2/3-dealiased pseudo-spectral method to solve RMHD equations

$$\left(\frac{\partial}{\partial t} \mp v_A \partial_{\|}\right) \mathbf{z}_S^{\pm} + \left(\mathbf{z}_S^{\mp} \cdot \nabla_{\perp}\right) \mathbf{z}_S^{\pm} = -\nabla_{\perp} P + \frac{1}{R} \nabla^2 \mathbf{z}_S^{\pm} + \mathbf{f}_{\perp}^{\pm}$$

introducing the Elsässer potentials $\mathbf{z}_S^{\pm} = \mathbf{e}_z \times \nabla \phi^{\pm}$.

- Fluctuating $\mathbf{z}_S^{\pm}$ are perpendicular to the guide field, taken along the $\mathbf{e}_z$ direction.

- No artificial viscosity is used (hyper-viscosity), to minimize undesired bottleneck effects.

- Code has good strong and weak scaling up 16,384 cores on ranger and 131,072 cores on IBM Blue Gene/P at Argonne NL.

Random forcing (balanced)

- We use a random force $\mathbf{f}$: where $\mathbf{e}_z \cdot \mathbf{f} = 0$ and $\nabla \cdot \mathbf{f} = 0$
- The only non vanishing Fourier coefficients of $\mathbf{f}$ in the range

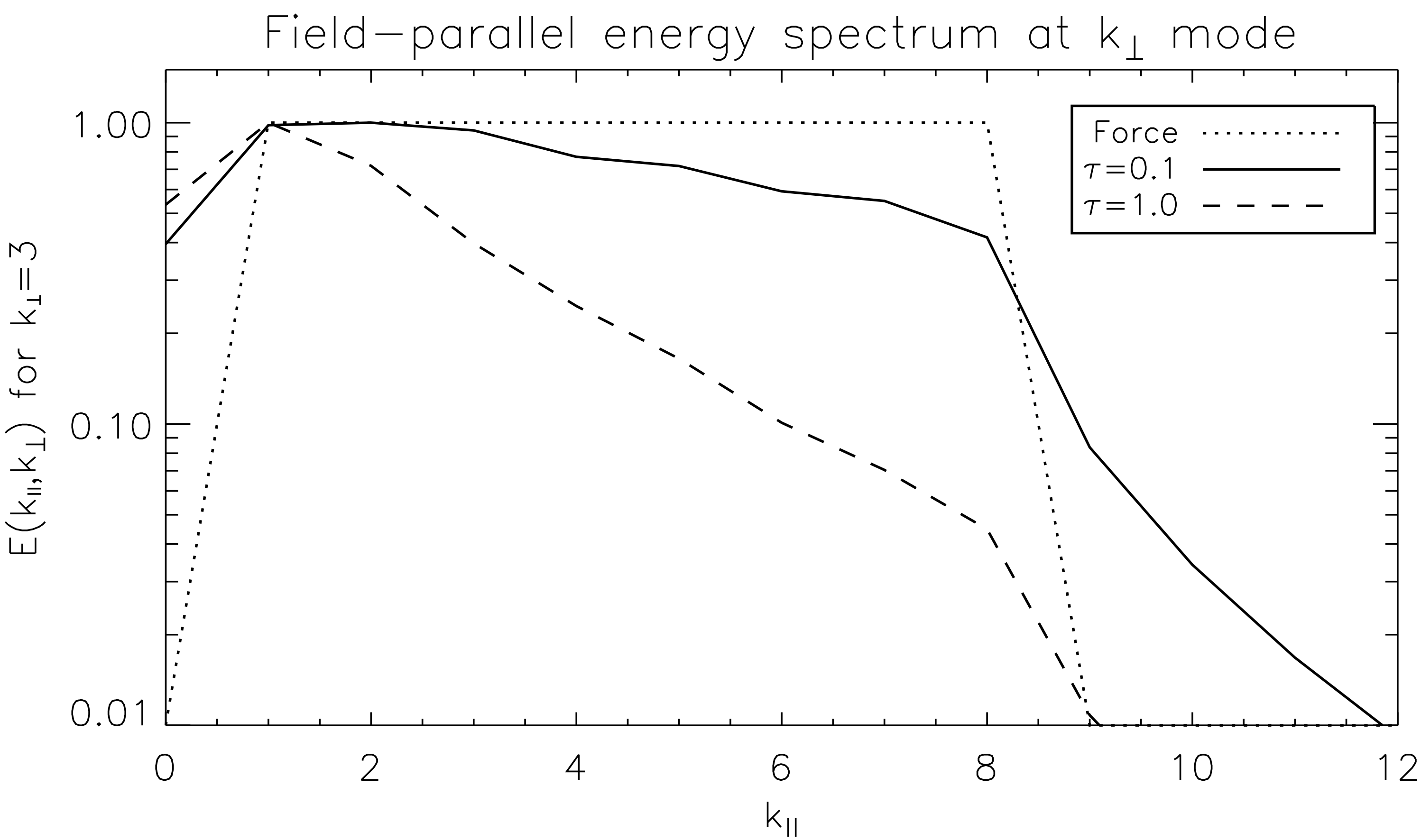
$$1 \leq k_\perp \leq 2, \quad \frac{L_\perp}{L_\|} \leq k_\| \leq \kappa \equiv \frac{L_\perp}{L_\|} n$$

  are Gaussian random numbers.

- κ controls the parallel width of the forcing spectrum. $L_\perp / L_\| = 1/5, 1/10$.

- The individual random values are independently refreshed, on average, every $\tau = 0.1 L_\perp / v_{rms}$.

Impact of forcing time correlation



Field-parallel energy spectrum at the dominant mode $k_\perp = 3$ for different forcing correlation times. The dotted line represents the normalized spectrum of the forcing. We observe that as the correlation time increases, large $k_\parallel$ modes get suppressed despite the fact that force amplitude is the same for all modes.

Random forcing (imbalanced)

- Correlation is introduced at the forcing scales by controlling the correlation between the velocity and magnetic field forces, $\mathbf{f}_v = \frac{1}{2}\left(\mathbf{f}^+ + \mathbf{f}^-\right)$ and $\mathbf{f}_b = \frac{1}{2}\left(\mathbf{f}^+ - \mathbf{f}^-\right)$.

- This is achieved by taking $\mathbf{f}^\pm$ as uncorrelated Gaussian random forces, so that cross-helicity is controlled by the difference in the variances of the random forces:

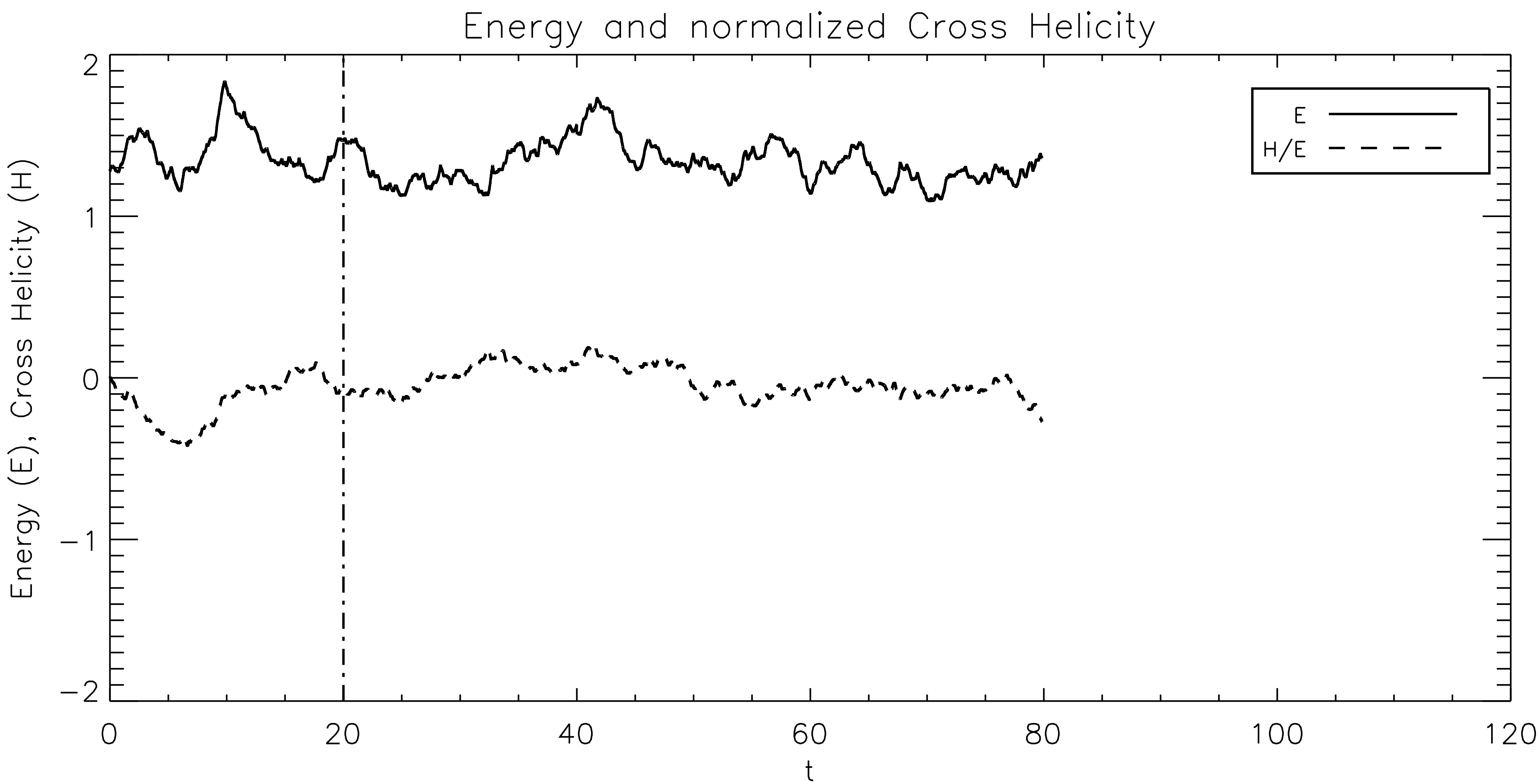
$$\langle \mathbf{f}_v \cdot \mathbf{f}_b \rangle = \frac{1}{4}\left(\sigma_+^2 - \sigma_-^2\right), \qquad \sigma_\pm^2 \equiv \langle |\mathbf{f}^\pm|^2 \rangle$$

- We set the variances to satisfy $\sigma_+^2 + \sigma_-^2 = 2$.

- It is convenient to define the parameters α:

$$\begin{aligned} \alpha &\equiv \frac{\sigma_+^2 - \sigma_-^2}{\sigma_+^2 + \sigma_-^2} \Rightarrow \langle \mathbf{f}_v \cdot \mathbf{f}_b \rangle = \frac{\alpha}{2}, \\ \rho_c &\equiv \frac{H_c}{E} = \frac{E^+ - E^-}{E^+ + E^-} \end{aligned}$$

Balanced strong turbulence RMHD simulations



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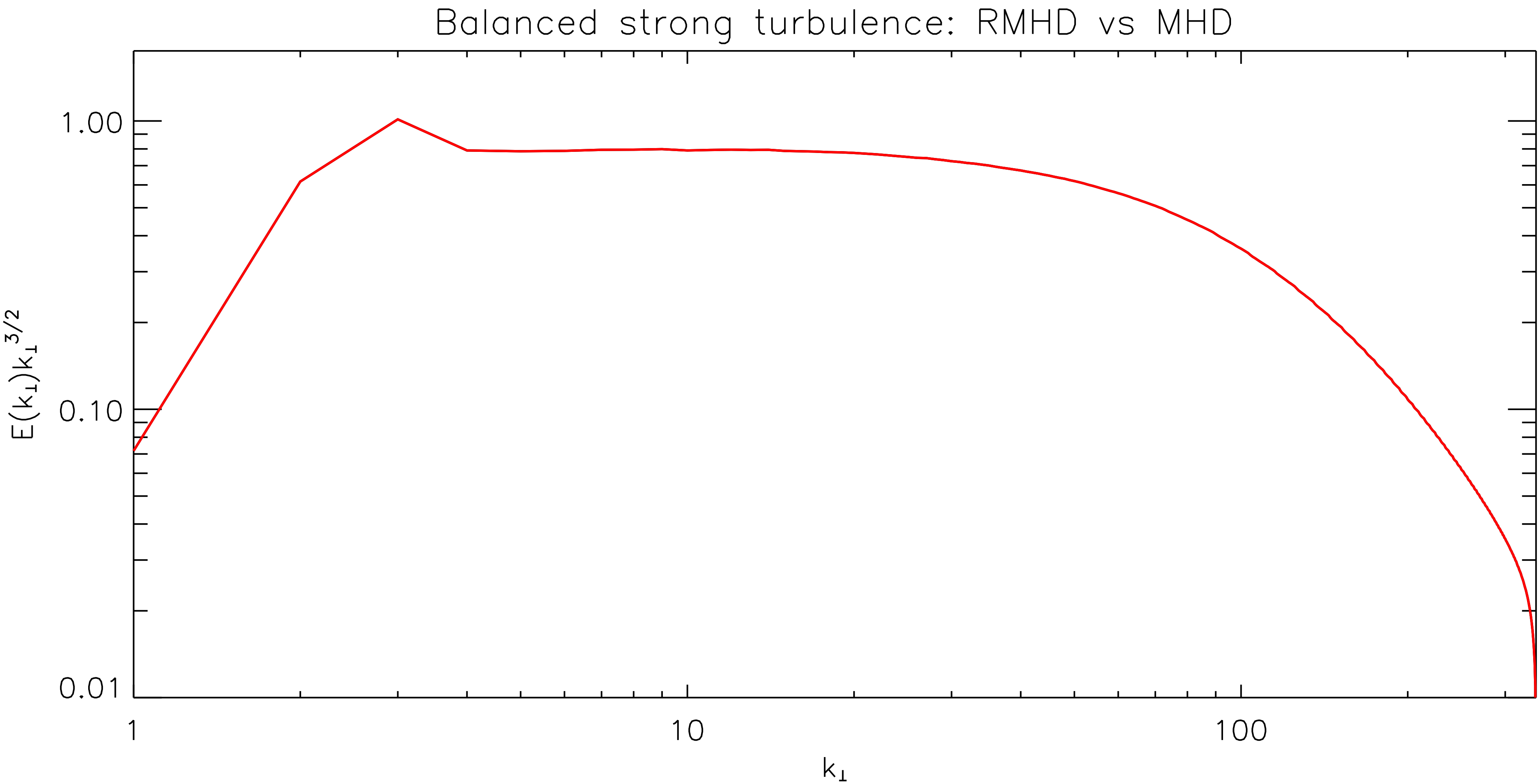

- RMHD data published in Perez & Boldyrev, ApJ, 710, L63 (2010).

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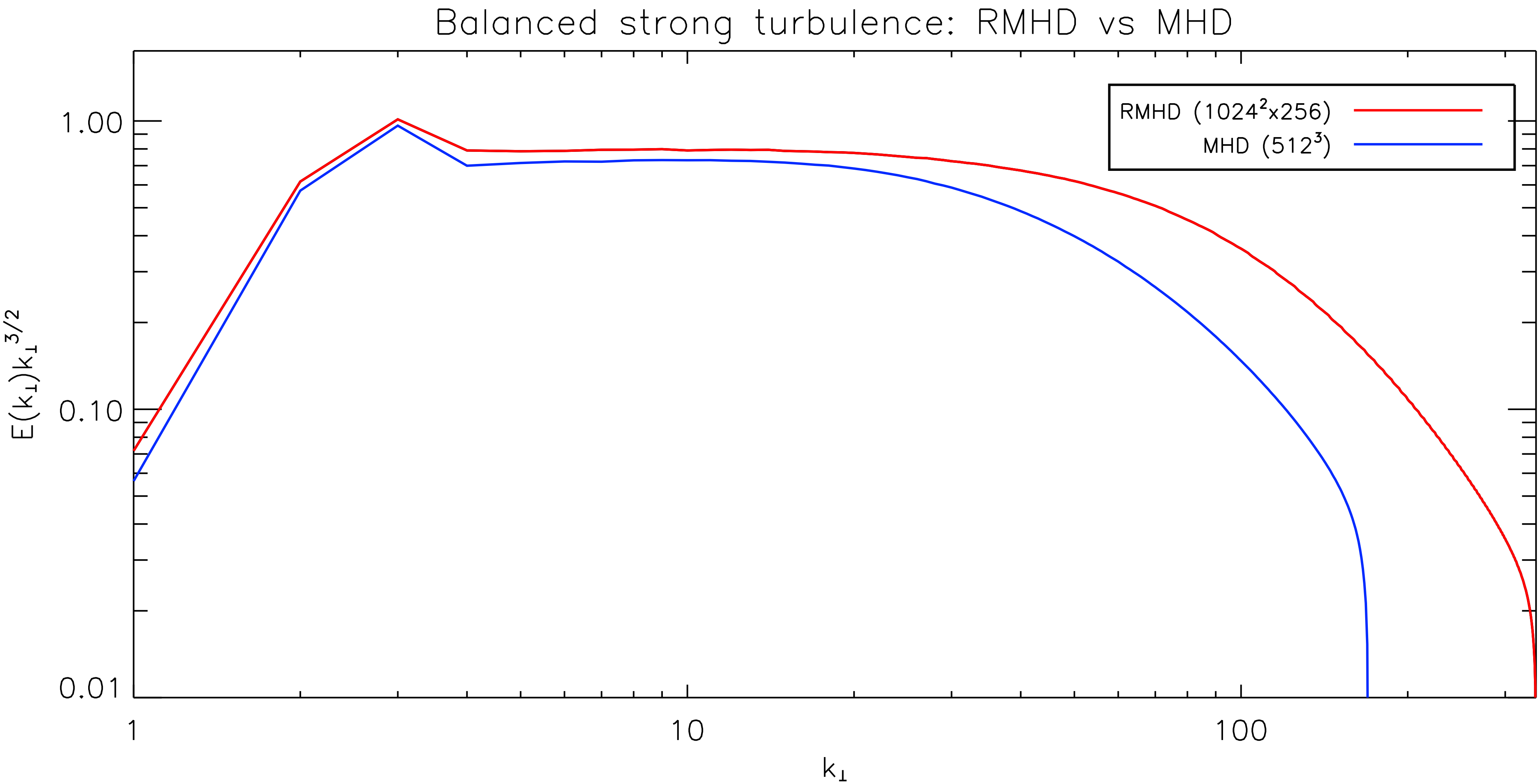

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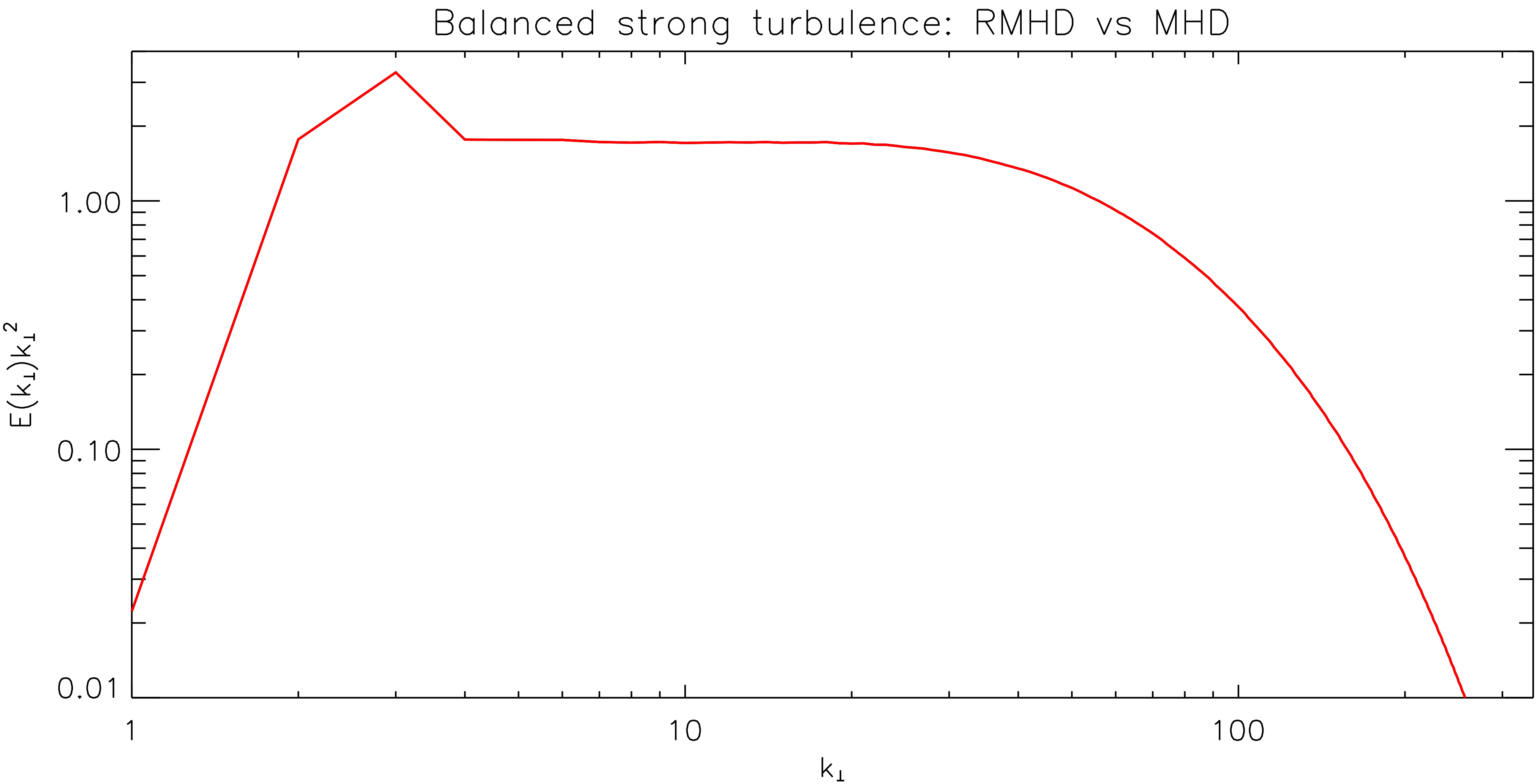
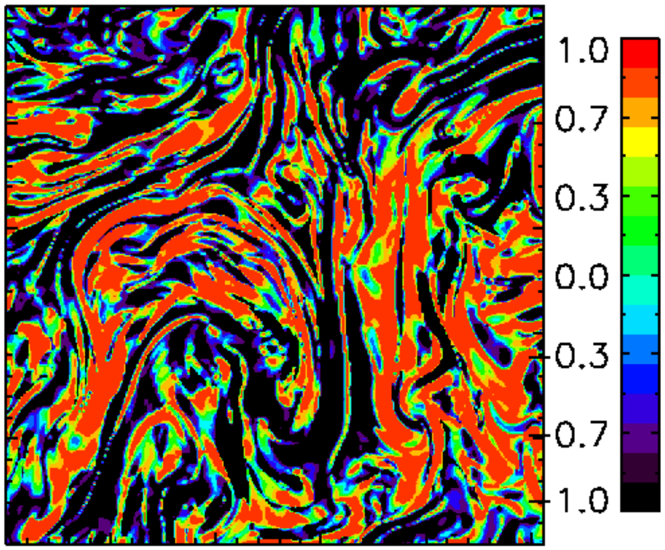
- MHD data courtesy of Mason, Cattaneo and Boldyrev, published in Mason, Cattaneo & Boldyrev, PRE, 77, 036403 (2008).

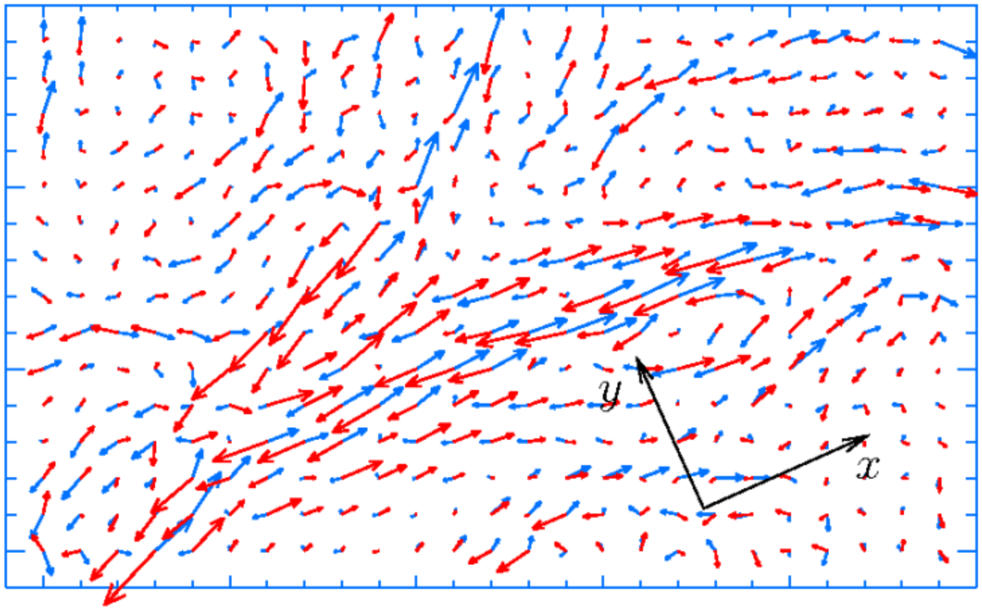
Balanced weak turbulence RMHD simulations



- The spectra of balanced weak MHD turbulence. The solid line is $E^+(k_\perp)$, the dashed line is $E^-(k_\perp)$; $Re = 6000$, resolution $1024^2 \times 256$ points.
- Consistent phenomenological (Ng & Bhattacharjee 1996 and Golreich & Sridhar 1997) and weak turbulence theory predictions (Galtier et. al. 2000).

The role of cross helicity $H_c = E^+ - E^-$



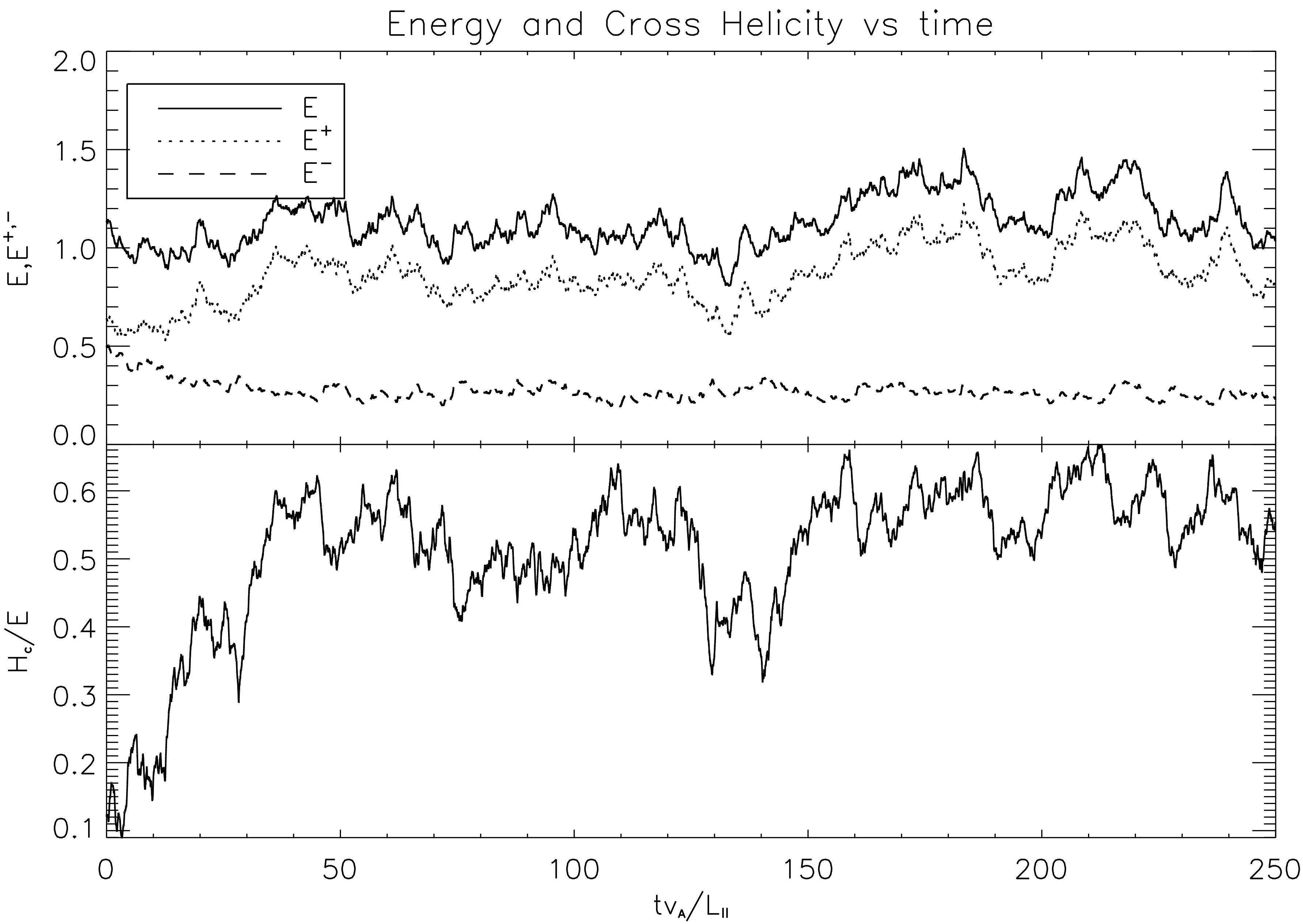


- Left panel shows cosine of the alignment angle between $\mathbf{v}_\lambda$ and $\mathbf{b}_\lambda$ fluctuations in the guide-field perpendicular plane at scales $\lambda = L_\perp/12$ for balanced turbulence. Right panel shows velocity and magnetic fields at one alignment region.

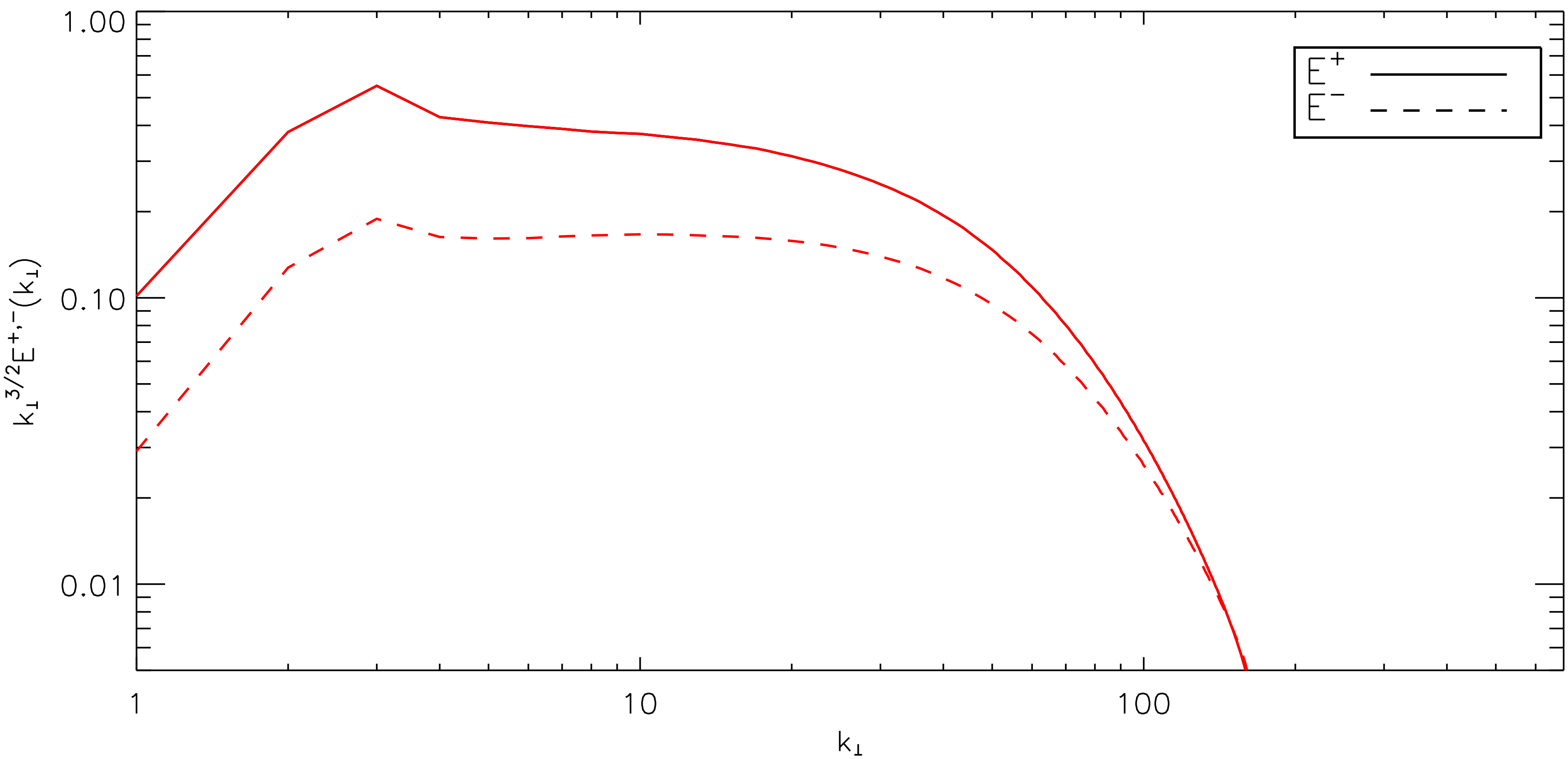
- Domains are split in patches of highly aligned and anti-aligned regions.

- A theory of imbalanced turbulence should be consistent with the fact that balanced turbulence is composed of imbalanced regions.

Long Imbalanced turbulence simulation $(1024^2 \times 256)$

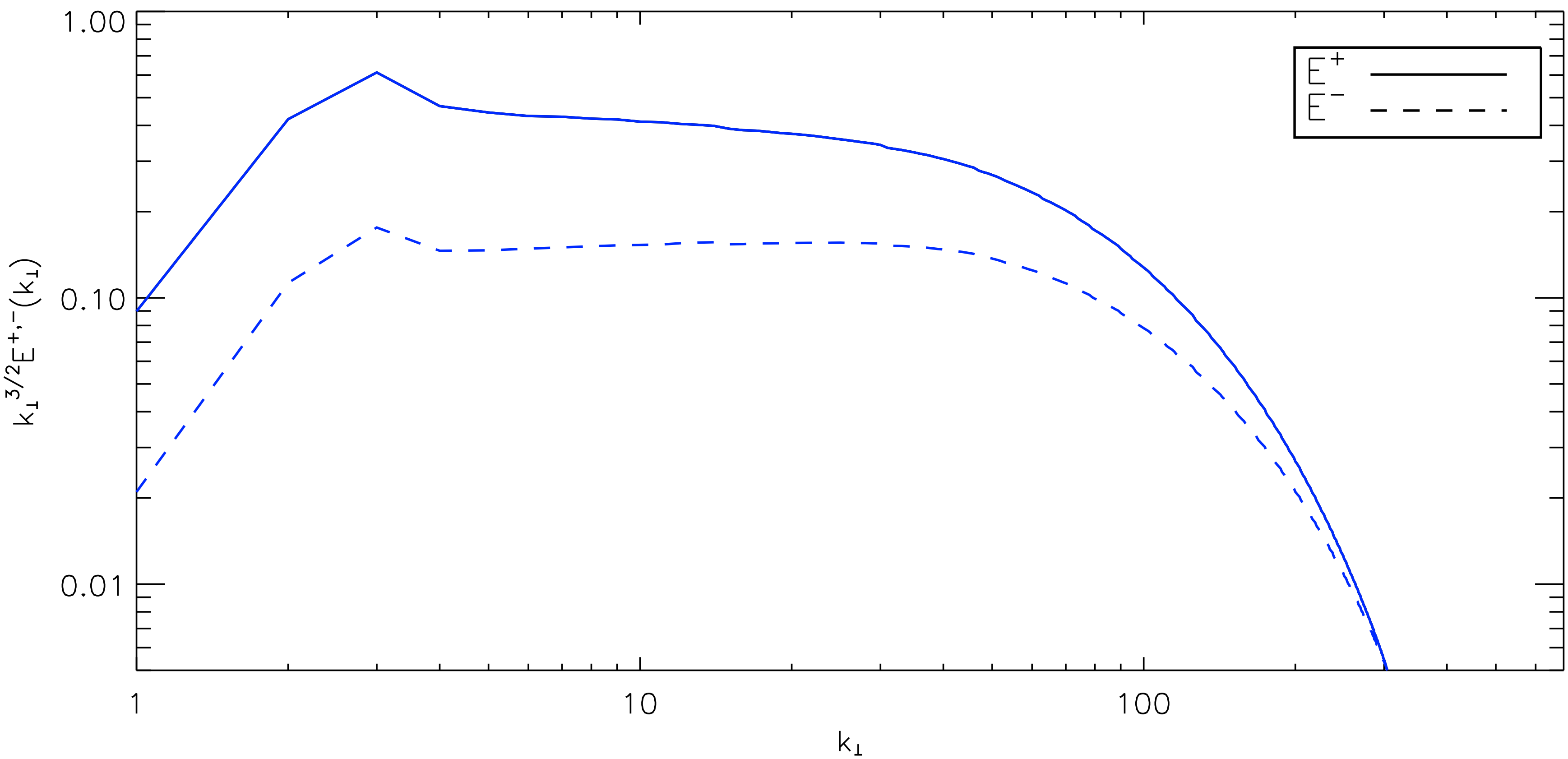


Resolution: $512^2 \times 256$



- Energy spectra $E^{\pm}$ for $\gamma \sim 2$ at various $Re = 2200, 5600, 14000.$
- Largest grid size: $2048^2 \times 512.$

Resolution: $1024^2 \times 256$



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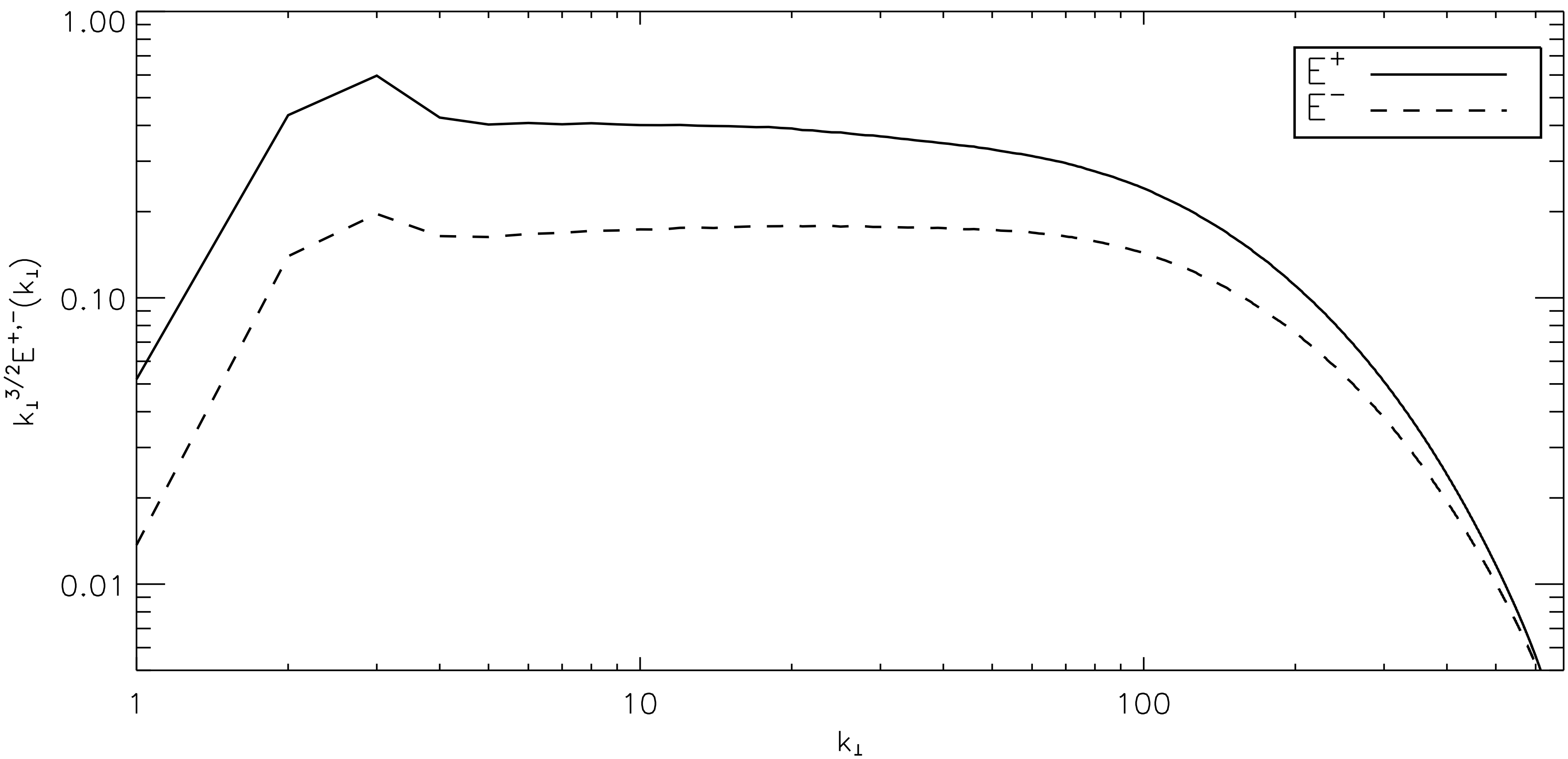

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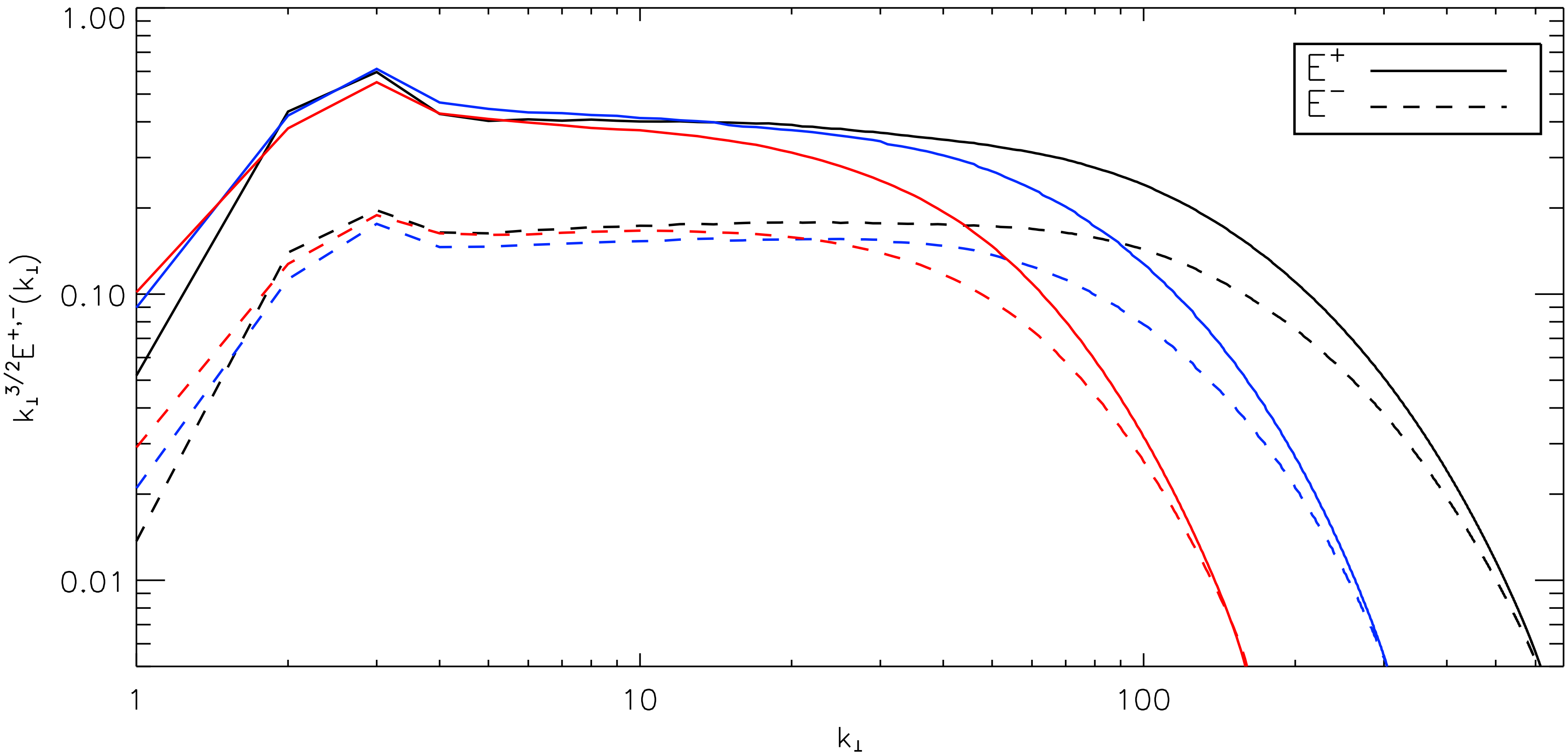
All resolutions



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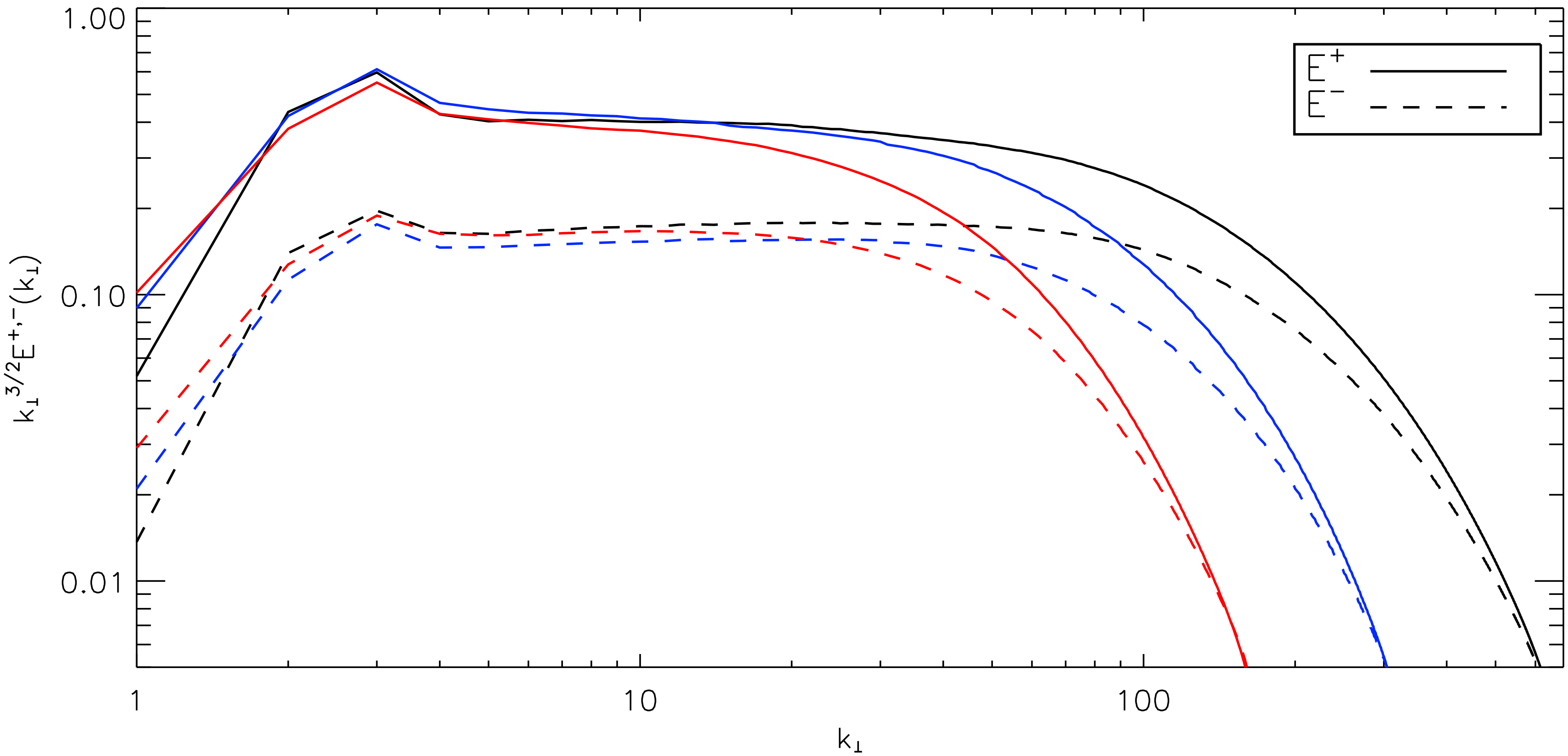
Pinning and bottleneck



- Spectra of $E^{\pm}$ are pinned in the dissipation range.
- Large scale parts of spectra are insensitive to Re.
- As Re increases, spectra $E^{\pm}$ become more parallel.
- No evidence of bottleneck is observed up to $2048^2 \times 512$ resolution.

Comparison with previous models

Our simulations are in agreement with models predicting same scaling -3/2 for both Elsasser fields.

- Lithwick et al 2007: Simulations indicate $E^{\pm}$ have same scaling, however scaling is more consistent with $k_{\perp}^{-3/2}$ than with $k_{\perp}^{-5/3}$. Dynamic alignment is not considered.

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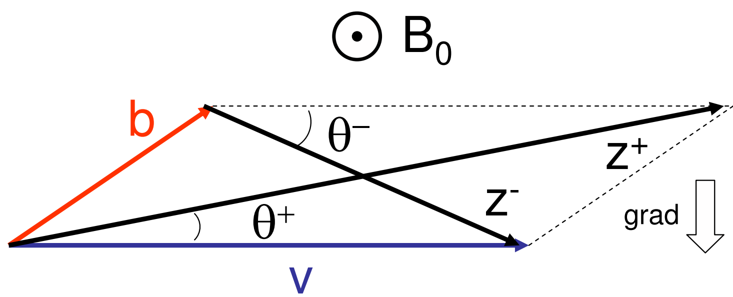
- Beresnyak & Lazarian 2009: Puzzling findings from simulations might be due to extremely large imbalance, up to $\gamma \sim 31$. It might need significantly more resolution to reach the universal regime of strong turbulence.

Dynamic alignment model for the observed spectra: imbalanced case

- We assume that alignment is also present in the imbalanced case.
- However, alignment angle is different for z^+ and z^-, with the geometric constraint $z^+\theta^+ \sim z^-\theta^-$.



- Nonlinear interaction time is the same for both, $\tau^+ = \tau^-$.

$$\frac{\partial}{\partial t}\mathbf{z}^\pm \mp v_A \partial_\| \mathbf{z}^\pm + \underbrace{(\mathbf{z}^\mp \cdot \nabla)}_{z_\lambda^\mp \theta_\lambda^\mp k_\perp \sim 1/\tau^\pm} \mathbf{z}^\pm = 0$$

- Constant flux assumption $(z^\pm)^2/\tau_\lambda^\pm$ =const leads to same scaling $k_\perp^{-3/2}$, although $E^\pm$ have different amplitudes.

Concluding Remarks

- RMHD provides an effective and accurate way of simulating strong MHD turbulence, with and without cross helicity.
- For balanced turbulence, RMHD simulations show scaling consistent with results by several other groups.
- MHD turbulence, both balanced and imbalanced, is characterized by regions of positive and negative cross-helicity.
- Numerical simulations show that the spectra of E^+ and E^- approach the scaling $k_\perp^{-3/2}$ with different amplitudes.
- Dynamic alignment theory provide a coherent picture of balanced and imbalanced MHD turbulence.