Imbalanced MHD turbulence

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What is turbulence?

No exact definition. Loosely, random motion of fluid where many nonlinearly interacting modes are involved.

Plasma in astrophysical systems (solar wind, interstellar medium, galaxy clusters, etc) is typically magnetized and turbulent.

What are signatures of turbulence?

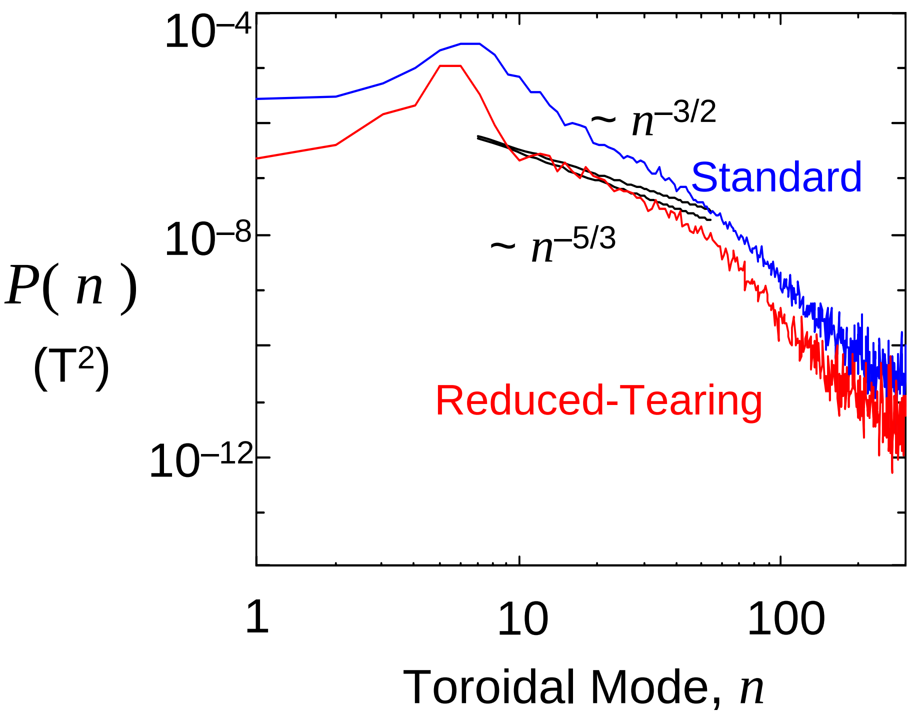
Scaling relations: spectra, structure functions, etc.

Structures: filaments, current sheets, etc.

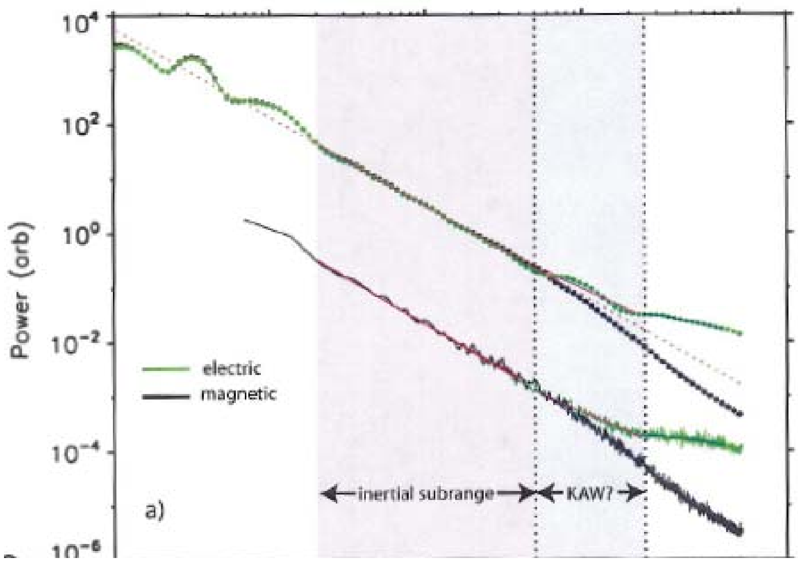
Different turbulent systems have similar signatures…

Magnetic turbulence in nature

Energy spectra

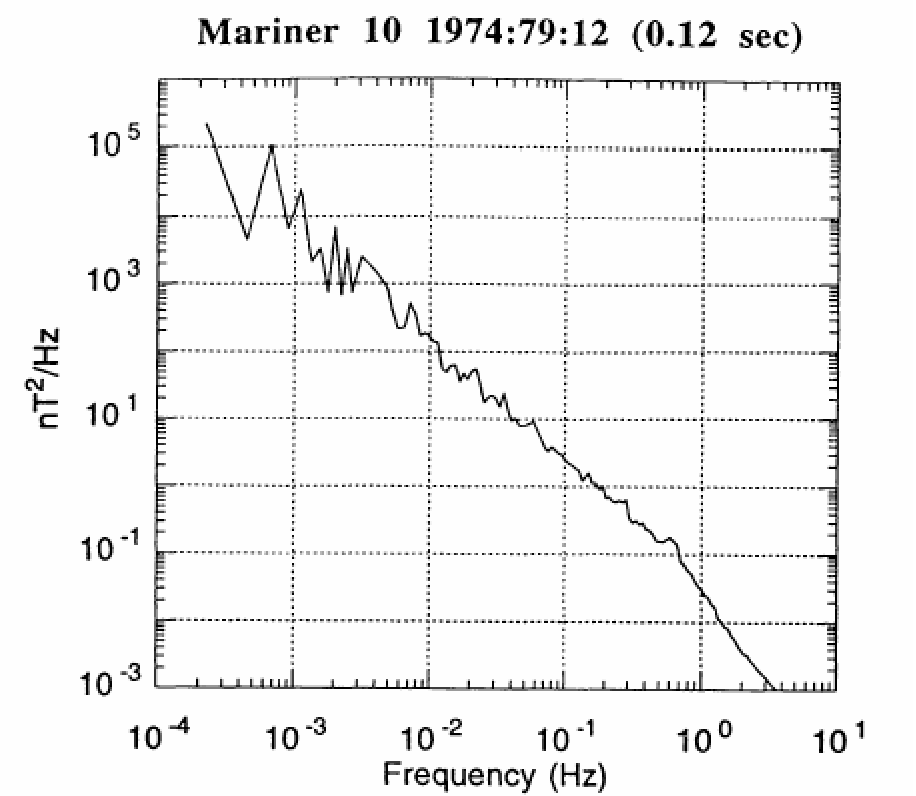


(MST experiment)

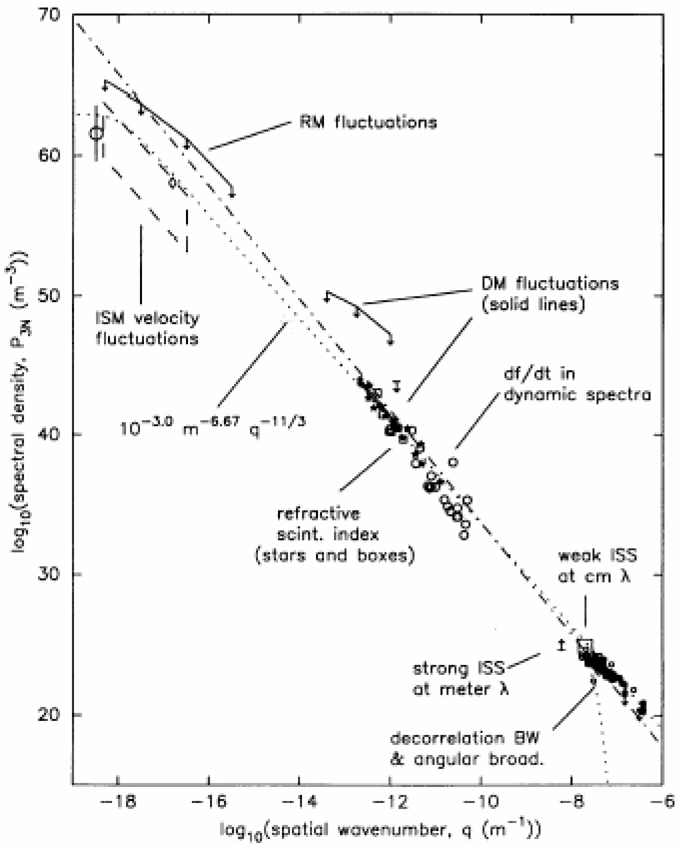


Solar wind (V, B)
Bale et al 2005

Magnetic turbulence in nature



Solar wind



ISM

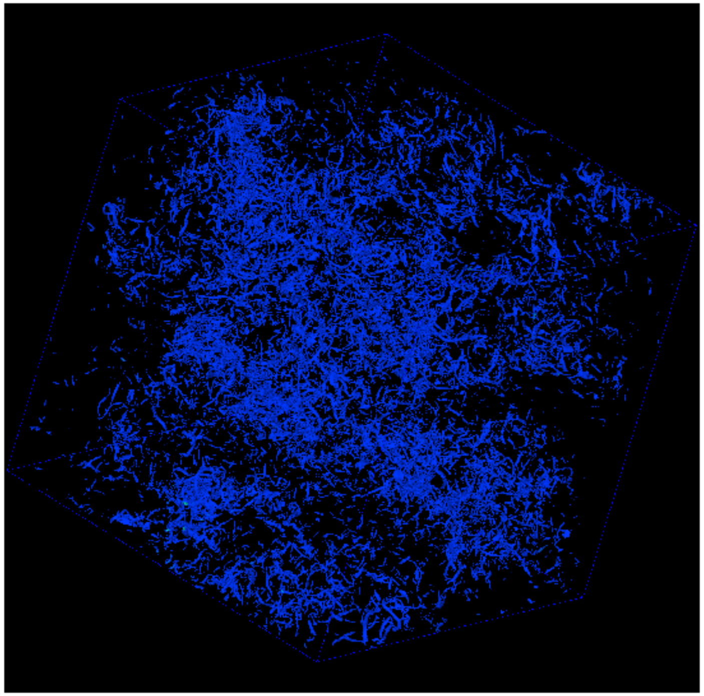
[Goldstein, Roberts, Matthaeus (1995)]

[Armstrong, Rickett, Spangler (1995)]

Magnetic turbulence in nature

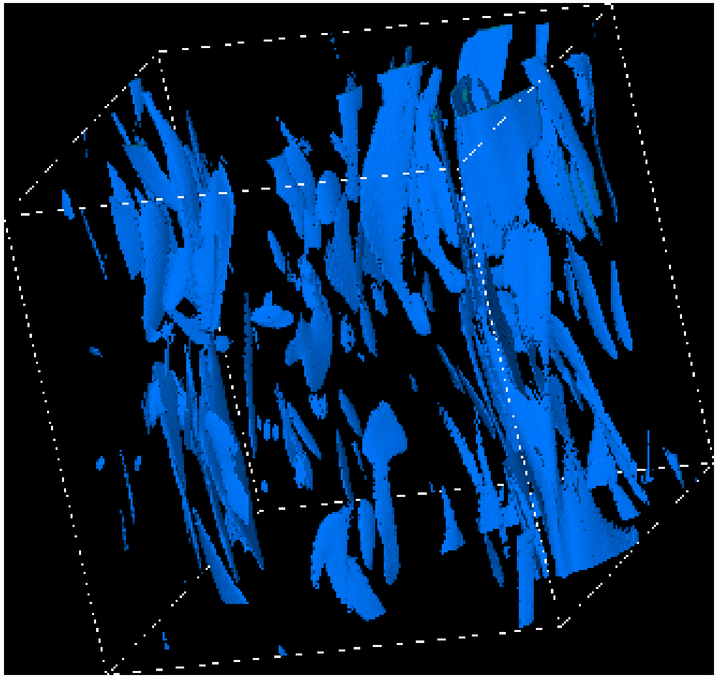
structures

Neutral fluid, B=0



Filaments

MHD, B>0



“Ribbons” stretched along B

Biskamp & Muller (2000)

Magnetic turbulence in nature: problems

1. Do observed power-law spectra imply that turbulence is present?
-- not always... E.g., discontinuous, shock-dominated non-turbulent fields produce energy spectrum $E(k) \sim k^{-2}$, same as weak MHD turbulence.

2. Do turbulent systems observed in nature exhibit universal behavior?
-- not necessarily. E.g., magnetic and velocity fluctuations in the solar wind have different spectra at different distances from the Sun. Two-fluid effects, compressibility effects, kinetic effects, etc. typically invalidate one-fluid incompressible MHD description.

3. Does MHD turbulence have universal regimes (similar to hydrodynamic turbulence, that is, independent of driving, dissipation, etc)?
-- possibly yes, as is seen from recent high-resolution numerical simulations.

Observations, laboratory, analytical, and numerical studies are required to answer these questions.

MHD turbulence cascades: search for universality

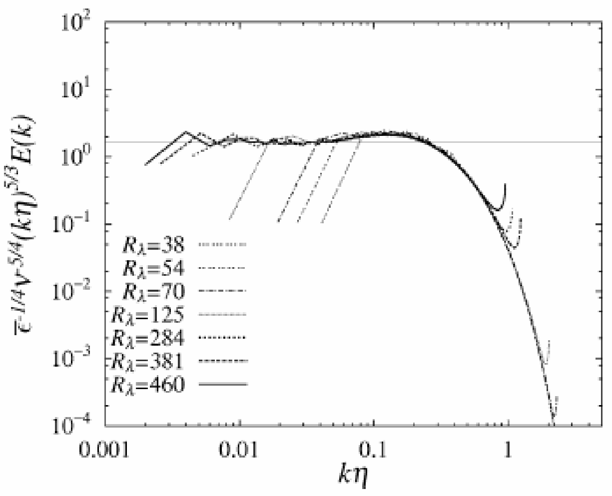


Fig. 1. Scaled energy spectra, $\bar{\epsilon}^{-1/4}\nu^{-5/4}(k\eta)^{5/3}E(k)$. The inertial range is between $0.007 \leqslant k\eta \leqslant 0.04$. $K = 1.64 \pm 0.04$. The horizontal line indicates $K = 1.64$. Gotoh 2002

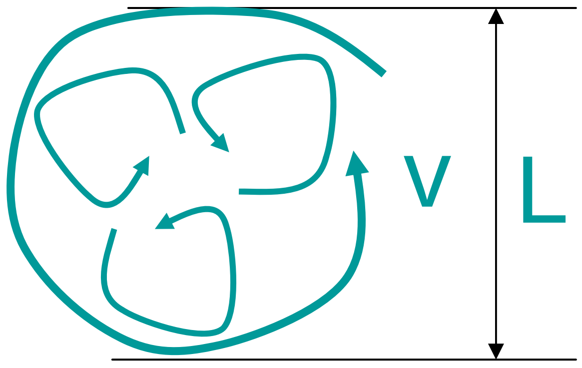
Hydrodynamic turbulence is universal. Has same spectrum (close to -5/3) independent of large-scale driving and small-scale dissipation

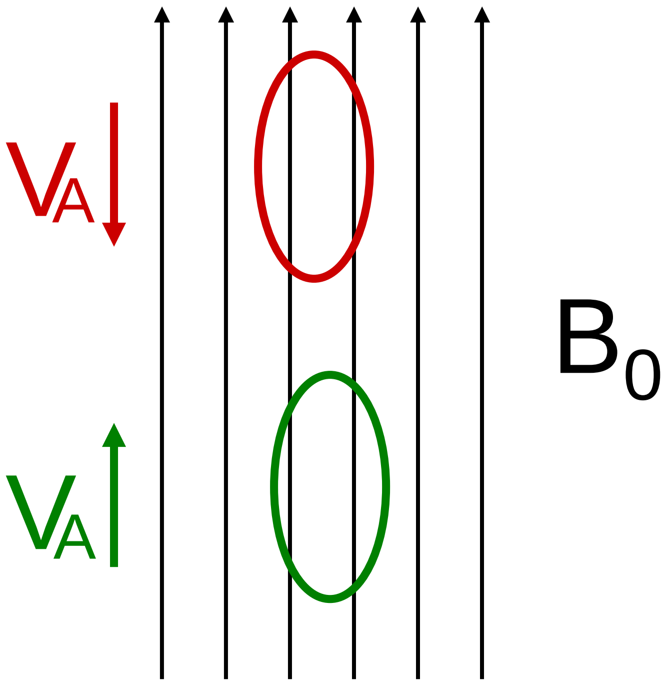
Are there universal regimes of magnetized turbulence? What regimes can be efficiently studied (analytically, numerically, etc)?

Nature of MHD turbulence cascades

HD turbulence:
interaction of eddies

MHD turbulence:
interaction of wave packets
moving with Alfven velocities





Nature of MHD turbulence: Alfvenic cascade

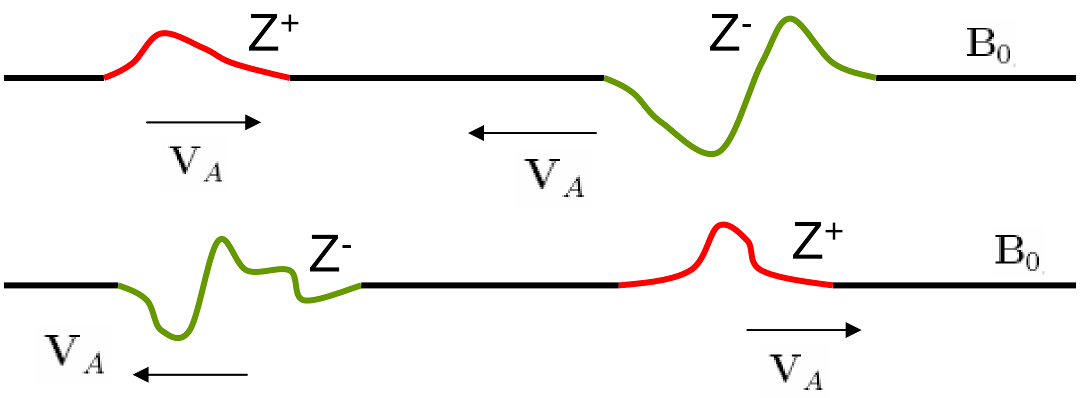
$$\partial \mathbf{z}^{\pm} \mp (\mathbf{v}_A \cdot \nabla)\mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla)\mathbf{z}^{\pm} = -\nabla P + \frac{1}{Re}\nabla^2 \mathbf{z}^{\pm} + \mathbf{f}^{\pm}$$

Ideal system conserves the Elsasser energies

$$E^{+} = \int (\mathbf{z}^{+})^2 \, d^3x \qquad E^{-} = \int (\mathbf{z}^{-})^2 \, d^3x$$

$$\equiv$$

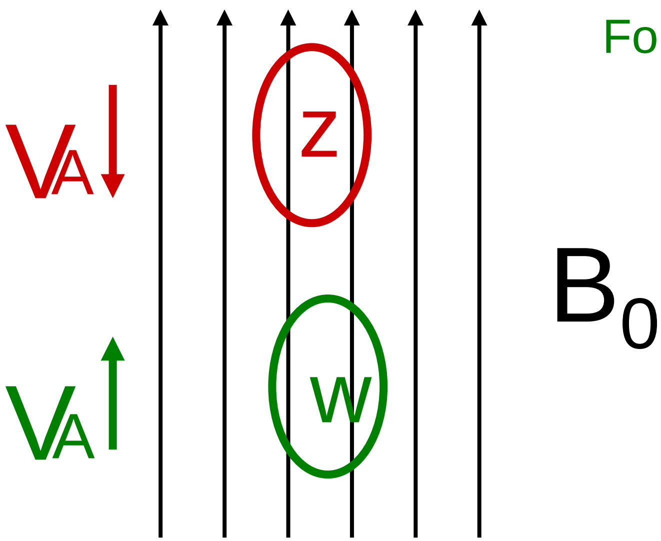
$$E = \frac{1}{2}\int (v^2 + b^2) d^3x \qquad H^C = \int (\mathbf{v} \cdot \mathbf{b}) d^3x$$



E^+ and E^- are independently conserved in ideal MHD.

Nature of MHD turbulence: Search for universality

$$\partial \mathbf{z}^{\pm} \mp (\mathbf{v}_A \cdot \nabla)\mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla)\mathbf{z}^{\pm} = -\nabla P$$



Limiting cases of MHD turbulence – Four universal regimes	$z^+ \sim z^-$ balanced	$z^+ \gg z^-$ Imbalanced, cross-helical
$k_{\parallel} V_A \gg k_{\perp} z^{\pm}$ weak turb	√	√ ?
$k_{\parallel} V_A \sim k_{\perp} z^{\pm}$ strong turb	√	√ ?

Universal regimes: Computational challenges

$$\partial \mathbf{z}^{\pm} \mp (\mathbf{v}_A \cdot \nabla)\mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla)\mathbf{z}^{\pm} = -\nabla P$$

Incompressible MHD	$z^+ \sim z^-$ balanced	$z^+ \gg z^-$ imbalanced, cross-helical
$k_{\|} V_A \gg k_{\perp} z^{\pm}$ weak turb	√	√ ?
$k_{\|} V_A \sim k_{\perp} z^{\pm}$ strong turb	√	√ ?

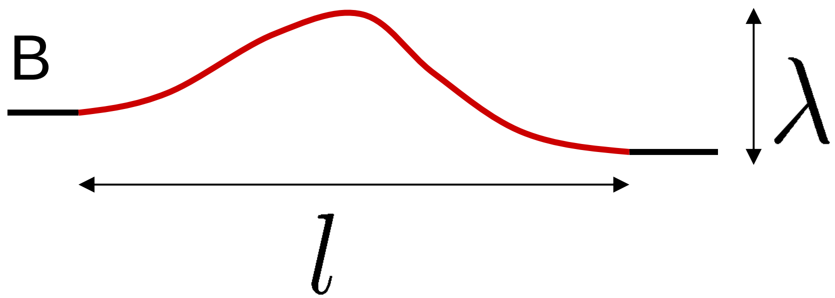
Pseudo-spectral MHD codes, reduced MHD codes. Parallelized codes, run on supercomputers

Numerical study of these regimes is more challenging than the study of hydrodynamic turbulence.

State-of-the-art simulations can have resolution 2048^2 in the guide-field-perpendicular direction. Require thousands of processors, and millions of CPU hours.

Strong MHD turbulence

Anisotropy of “eddies”



$$\partial \mathbf{z}^{\pm} \mp (\mathbf{v}_A \cdot \nabla)\mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla)\mathbf{z}^{\pm} = -\nabla P$$

$$V_A/l \sim \delta b_\lambda/\lambda$$

Critical Balance

$$l \gg \lambda$$

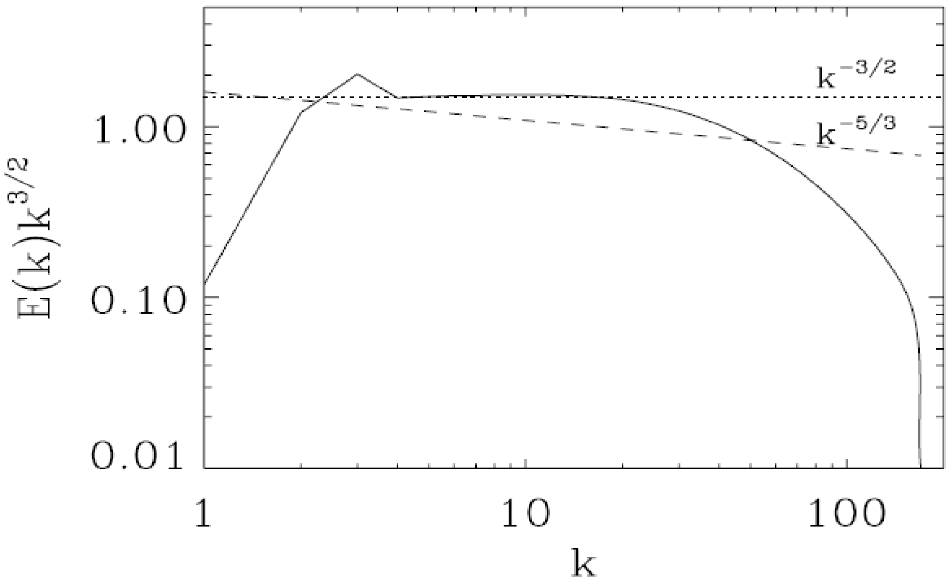
[Goldreich & Sridhar 1995]

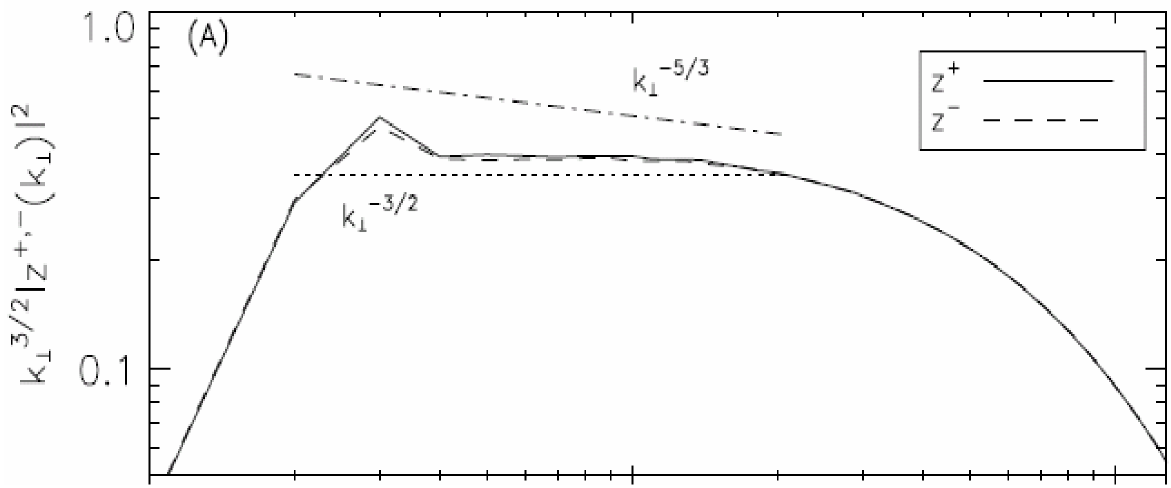
Shear Alfvén waves dominate the cascade:

$$\delta \mathbf{z}_\lambda^{\pm} \perp \mathbf{B}_0$$

Energy spectrum: $E(k_\perp) \propto k_\perp^{-5/3}$

Spectrum of strong MHD turbulence

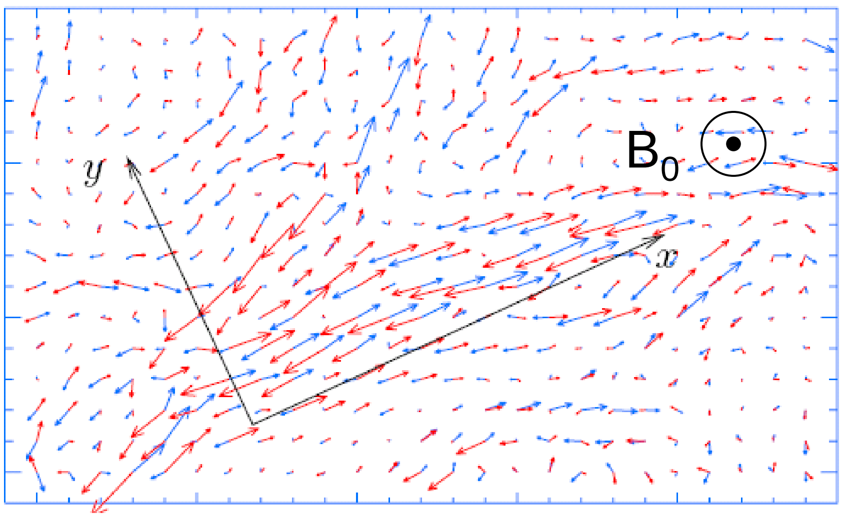


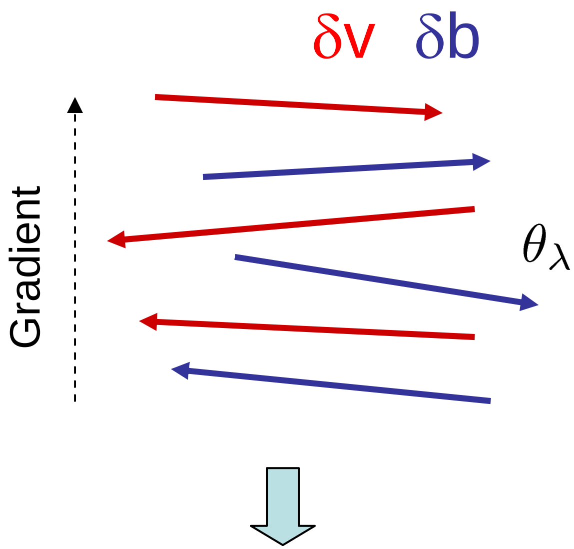


Possible explanation of the -3/2 spectrum

Dynamic Alignment theory

Fluctuations $\delta\mathbf{v}_\lambda$ and $\delta\mathbf{b}_\lambda$ become spontaneously aligned in the field-perpendicular plane within angle θ_λ





Nonlinear interaction is depleted ⇐ $(\mathbf{z}\cdot\nabla)\mathbf{w} \sim \theta_\lambda \delta v_\lambda^2/\lambda$

S.B. ApJ 626 (2005) L37, Perez & S.B. PRL (2009)

Unbalanced MHD turbulence

(non-balanced, imbalanced, cross-helical…)

Imbalance means that cross-helicity is nonzero:

$$H^C = \int (\mathbf{v} \cdot \mathbf{b}) d^3x = \frac{1}{4}(E^+ - E^-) \neq 0$$

Or, equivalently, the energies of waves traveling in opposite directions along the guide field are not equal. This is a very common situation in nature:

• Solar wind: more Alfven waves travel out of the sun than toward the sun

• Interstellar medium: MHD turbulence is driven by spatially localized sources

• Even when balanced overall, MHD turbulence is always locally unbalanced– it creates patches of positive and negative cross-helicity.

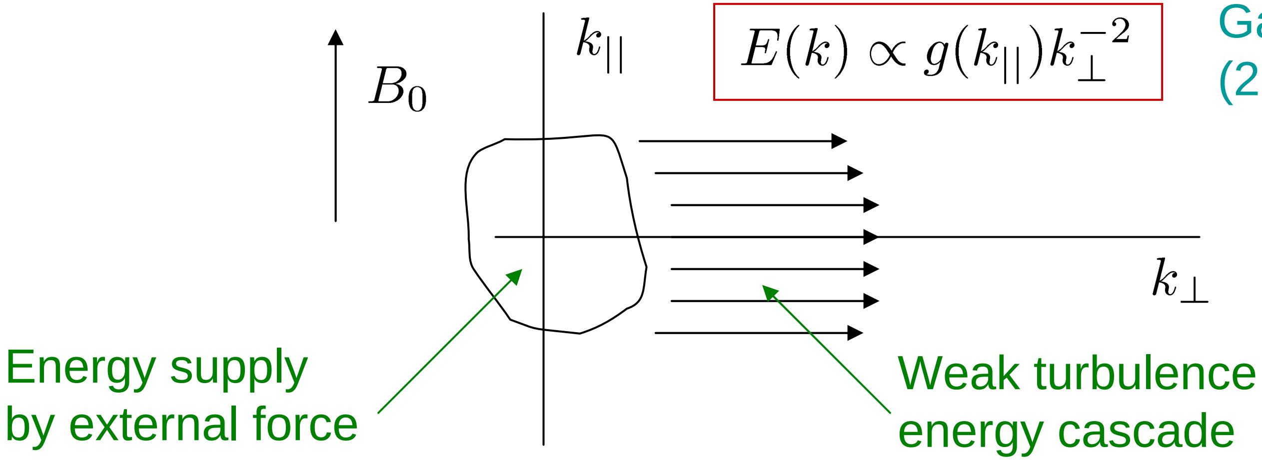
[Lithwick & Goldreich (2003); Ng et al (2003); Rappazzo et al (2007); Chandran (2008); Beresnyak & Lazarian (2008); Matthaeus et al (2008); Perez & SB (2009)]

Weak balanced turbulence spectrum

In weak MHD turbulence, energy is transferred to small scales in the field-perpendicular direction:

$$E(k) \propto g(k_{||}) k_{\perp}^{-2}$$

Galtier et al (2002)



$$\partial_t \mathbf{z} + (V_A \cdot \nabla)\mathbf{z} + (\mathbf{w} \cdot \nabla)\mathbf{z} = -\nabla p + f$$
$$\partial_t \mathbf{w} - (V_A \cdot \nabla)\mathbf{w} + (\mathbf{z} \cdot \nabla)\mathbf{w} = -\nabla p + f$$

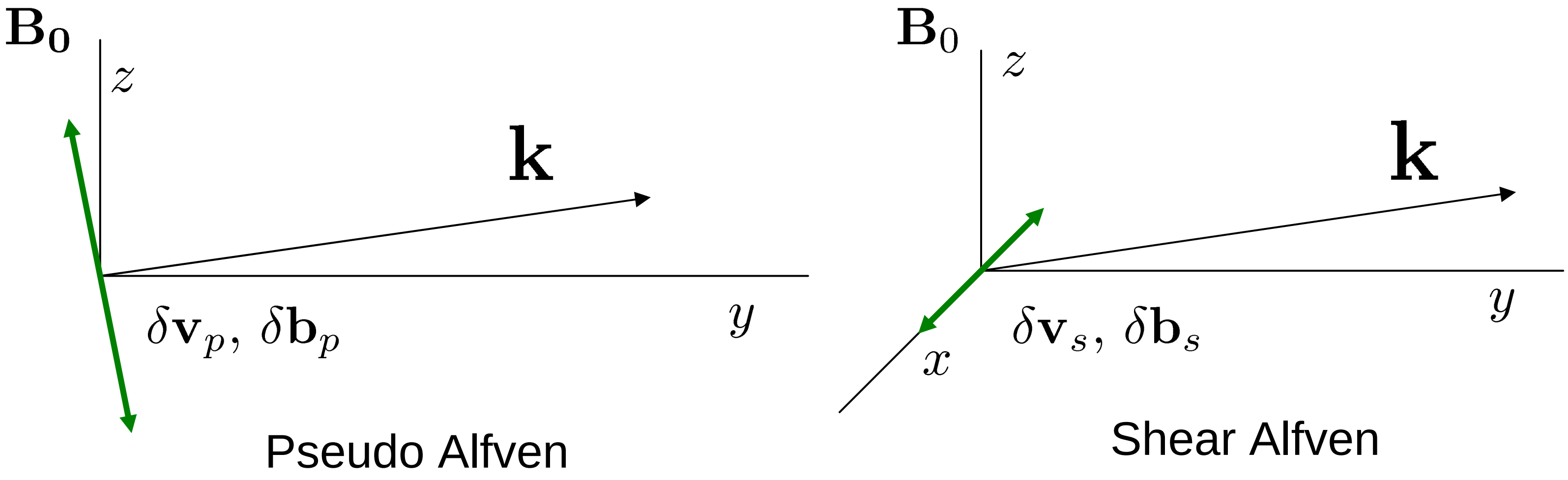
Linear terms do not change

Nonlinear terms increase

Eventually, turbulence becomes strong!

Anisotropy of MHD turbulence

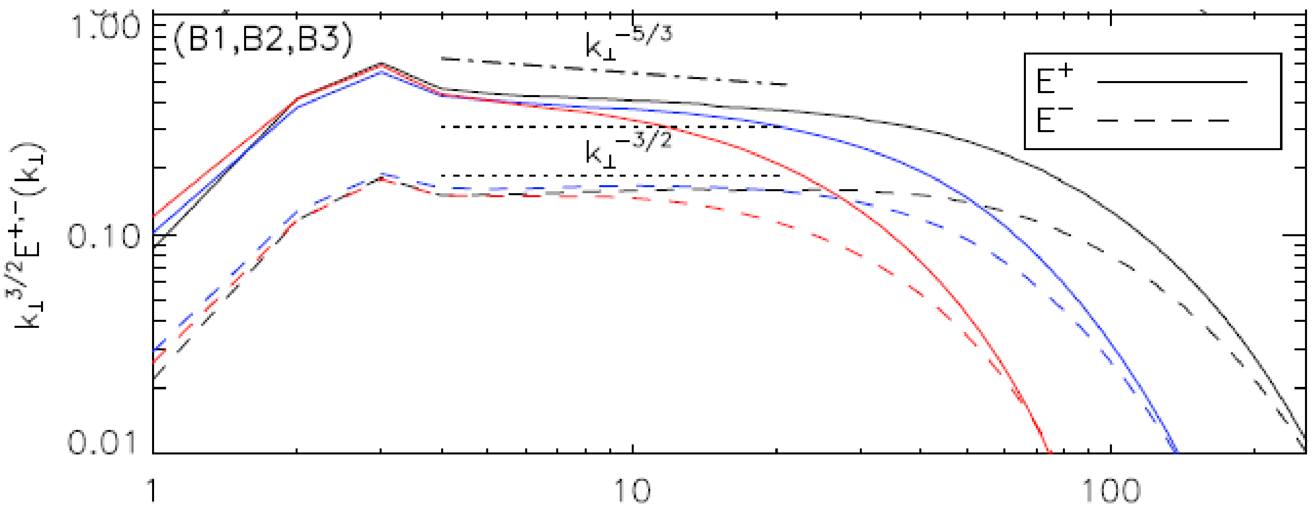
Polarization of shear Alfven and pseudo Alfven waves



Cascade is dominated by shear Alfven modes, e.g.,

$$(\mathbf{z}_s \cdot \nabla)\mathbf{w}_s \gg (\mathbf{z}_p \cdot \nabla)\mathbf{w}_s$$

Strong Imbalanced MHD turbulence: Numerics



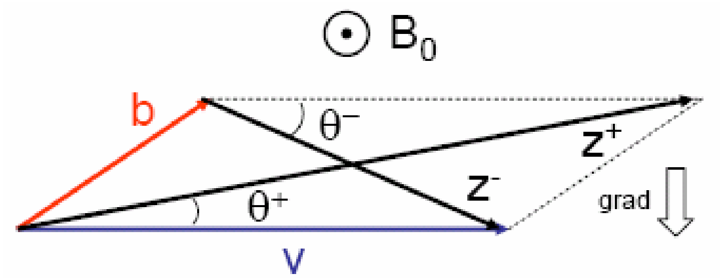
Spectra of E^+ and E^- for different magnetic Reynolds numbers, Rm=900, 2200, 5600 [Perez & Boldyrev 2009].

A model for imbalanced MHD turbulence

$$\left(\frac{\partial}{\partial t} \mp \mathbf{v}_A \cdot \nabla_{\parallel}\right) \mathbf{z}^{\pm} + \left(\mathbf{z}^{\mp} \cdot \nabla_{\perp}\right) \mathbf{z}^{\pm} = -\nabla_{\perp} P$$

$$\tau_\lambda \sim 1/(\mathbf{z}_\lambda^{\pm} \cdot \mathbf{k}_\perp) \sim 1/(z_\lambda^{\pm} k_\perp \theta_\lambda)$$

Take into account dynamic alignment -> geometric constraint:



$$\theta_\lambda^+ z_\lambda^+ \sim \theta_\lambda^- z_\lambda^-$$

The interaction times have to be the same: $\tau_\lambda^{\mp} \sim 1/(z_\lambda^{\pm} k_\perp \theta_\lambda^{\pm})$

$$(z_\lambda^{\pm})^2/\tau_\lambda^{\pm} \sim \epsilon^{\pm} = \text{const} \quad \Rightarrow \quad z_\lambda^+/z_\lambda^- \sim \sqrt{\epsilon^+/\epsilon^-}$$

Spectra E^+ and E^- have different amplitudes, but same scaling

Strong MHD turbulence: summary of numerical results

- In balanced MHD turbulence, both spectra have same amplitudes and spectra

$$E^+(k_\perp) \sim E^-(k_\perp) \sim k_\perp^{-3/2}$$

- In imbalanced MHD turbulence, the spectra E^+ and E^- have different amplitudes, but same scaling:

$$E^+(k_\perp) \propto E^-(k_\perp) \propto k_\perp^{-3/2}$$

- MHD turbulence is always locally imbalanced, at all scales. This hierarchical structure is a fundamental property of MHD turbulence, which results from cross-helicity conservation.