Weak Imbalanced Turbulence

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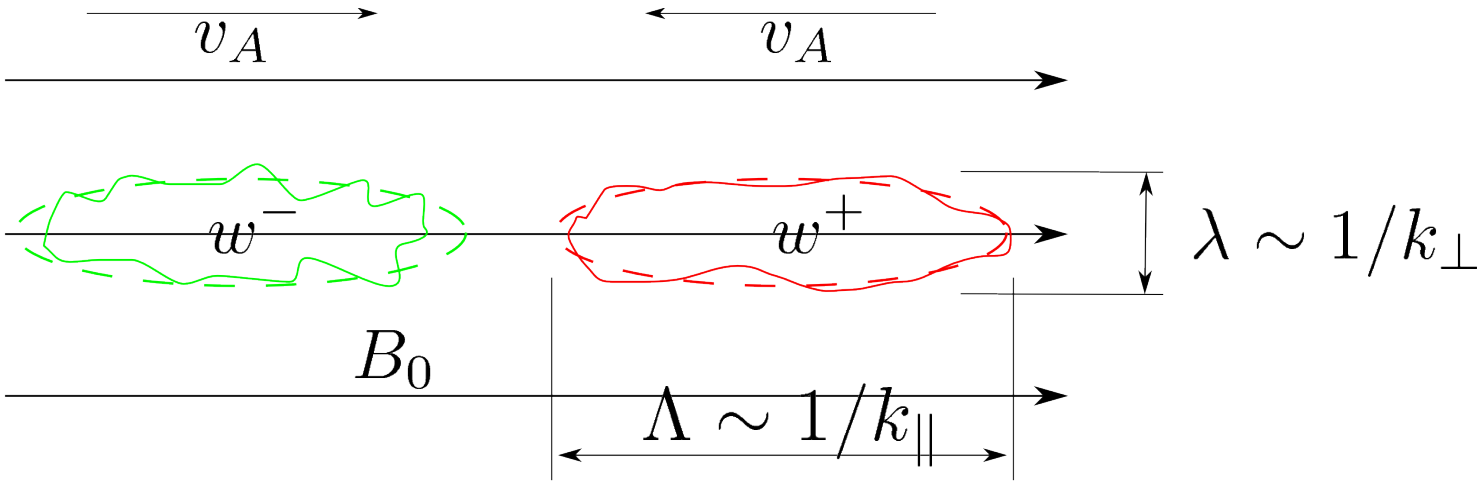
MHD Turbulence

linear *nonlinear*

$$\partial_t \delta\mathbf{w}^{\pm} \mp (\mathbf{v_A}\cdot\nabla)\delta\mathbf{w}^{\pm} + \hat{S}(\delta\mathbf{w}^{\mp}\cdot\nabla)\delta\mathbf{w}^{\pm} = 0$$

$$\sim v_A k_{\|} \qquad \sim \delta w k_{\perp}$$



Weak turbulence

$$v_A k_{\|}/\delta w k_{\perp} \gg 1$$

Weak Alfvenic turbulence

Was studied in Ng & Bhattacharjee 1996, 1997, Goldrech & Sridhar 1997, Lazarian&Vishniac 1999, Galtier 2000, 2002, Lithwick & Goldreich 2003.

Cascading:

$$\epsilon^{\pm} = \frac{(w_\lambda^{\pm})^2 (w_\lambda^{\mp})^2}{\lambda^2} \frac{\Lambda}{v_A}$$

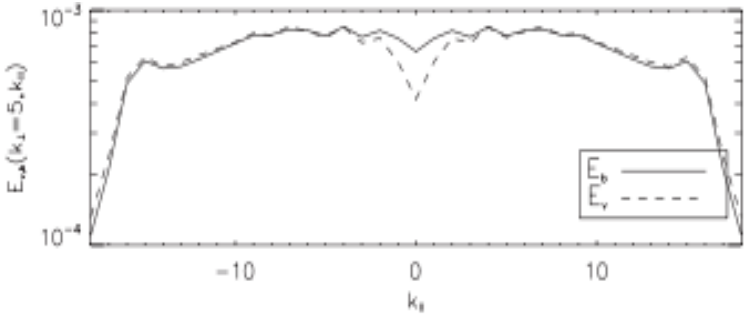
$$\frac{d}{dt} e^{\uparrow}(\boldsymbol{k}, t) = 2 \frac{\Lambda}{v_{\mathrm{A}}} \int d^2\boldsymbol{p} \left[e^{\uparrow}(\boldsymbol{p}, t) - e^{\uparrow}(\boldsymbol{k}, t) \right] |a_{\boldsymbol{k},\boldsymbol{p}}|^2 e^{\downarrow}(\boldsymbol{k} - \boldsymbol{p}, t) \ .$$

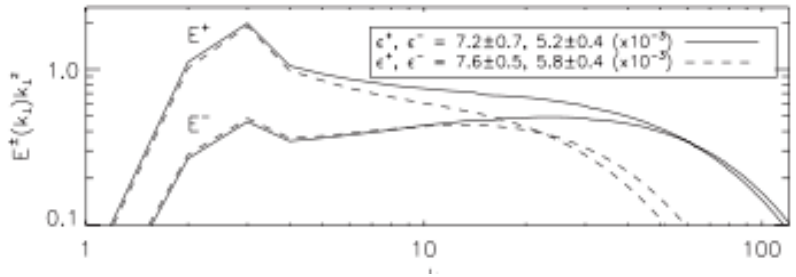
(Lithwick & Goldreich 2003)

Discussion on zero-frequency mode

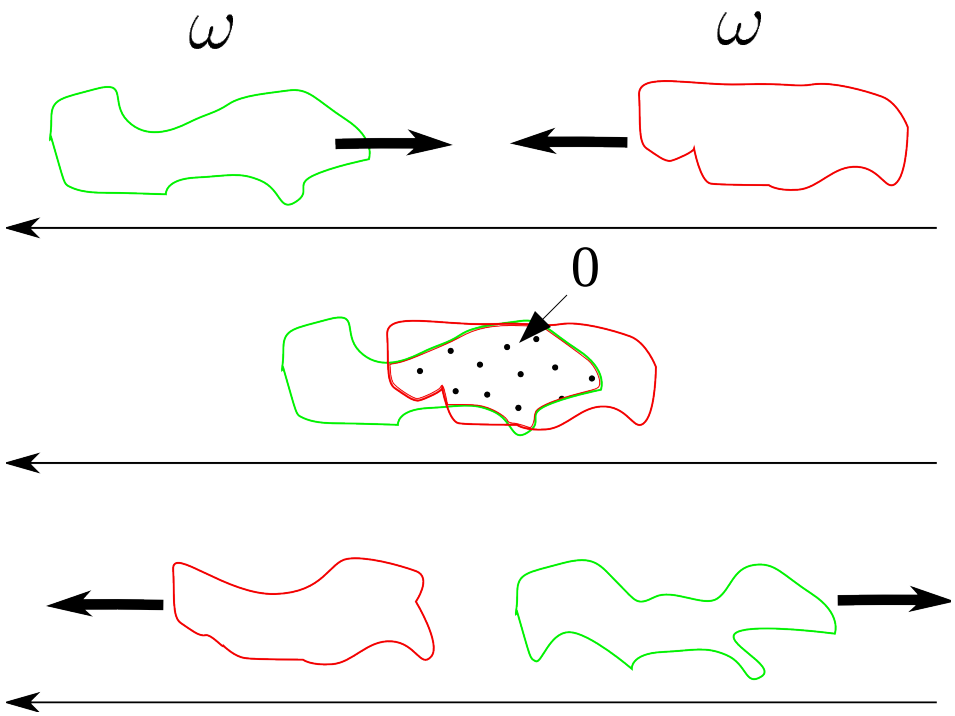
There is a rich history behind zero-frequency mode. Goldreich & Sridhar 1994 were claiming that zero-frequency mode is absent, therefore three-wave interactions are empty. This was wrong and subsequent models were based on three-wave interactions. Galtier et al 2000, 2002 created a correct model, but were saying that there might be a “condensation” of zero frequency mode. Lithwick & Goldreich 2003 disagreed and explained that zero-frequency mode has a transient nature.

In 2009 Boldyrev & Perez, based on numerics, once again claimed that there is a condensation of residual energy on zero frequency, due to a “mirror symmetry breakdown” in imbalanced turbulence.



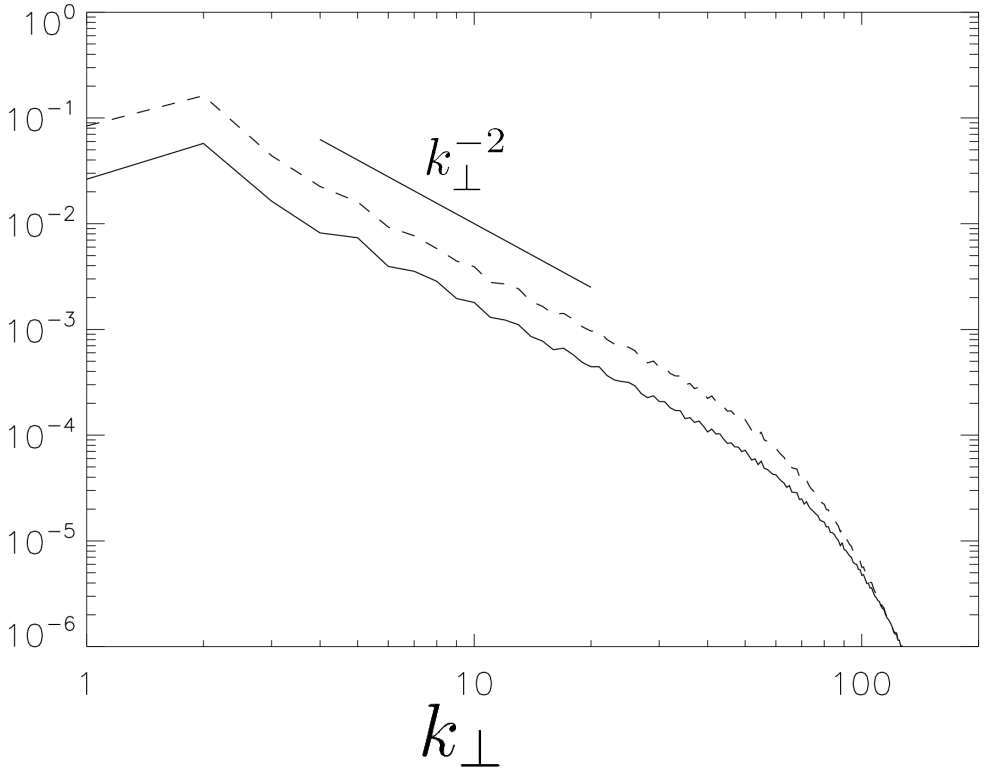


Zero-frequency mode is transient.

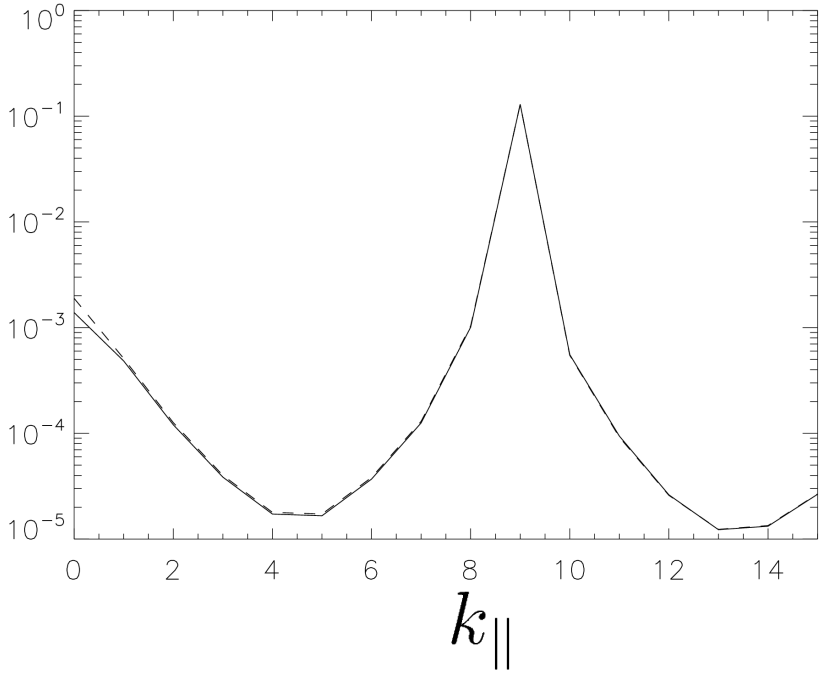


Simulations of weak imbalanced turbulence

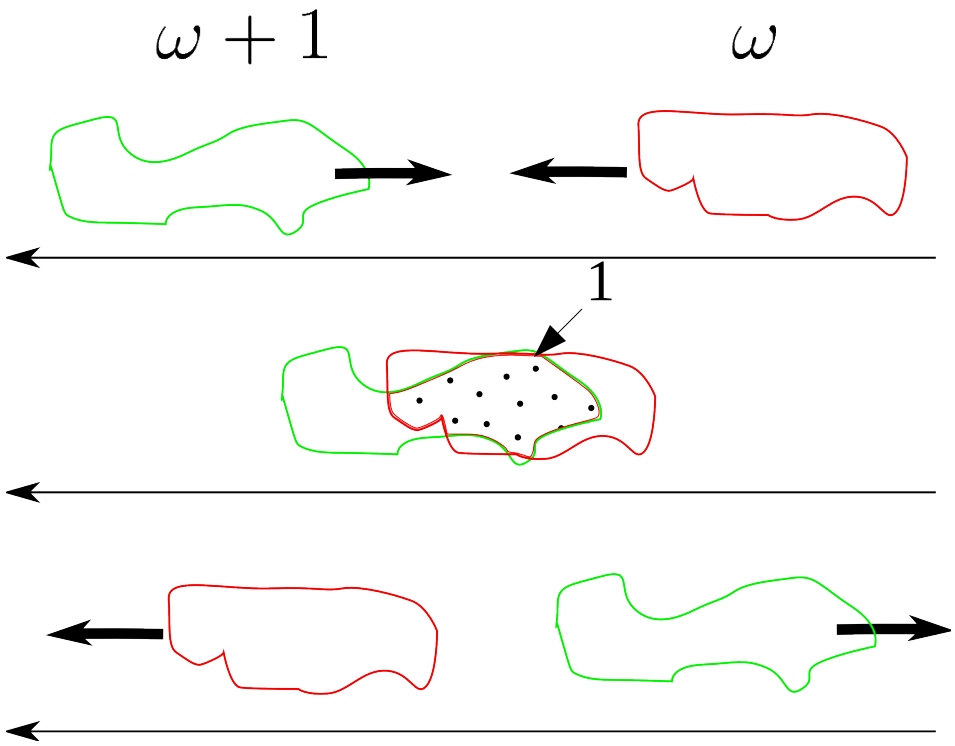
Perpendicular spectrum

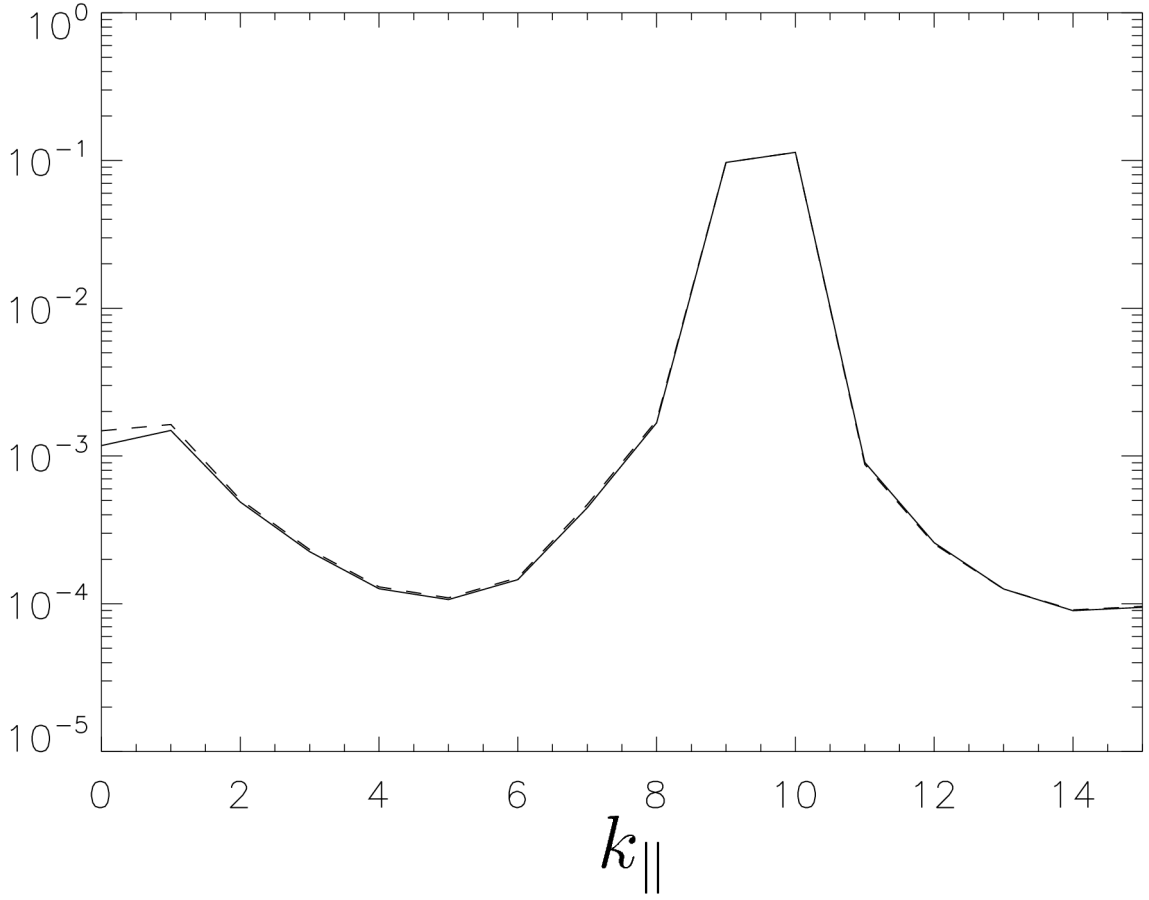


Parallel spectrum



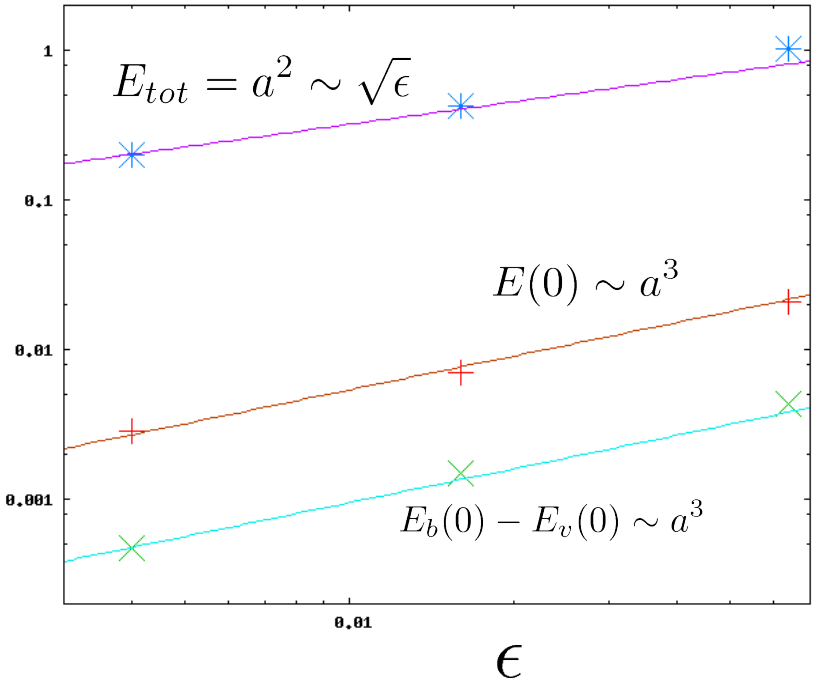
In fact, there other transient modes that don't necessarily provide any cascading:



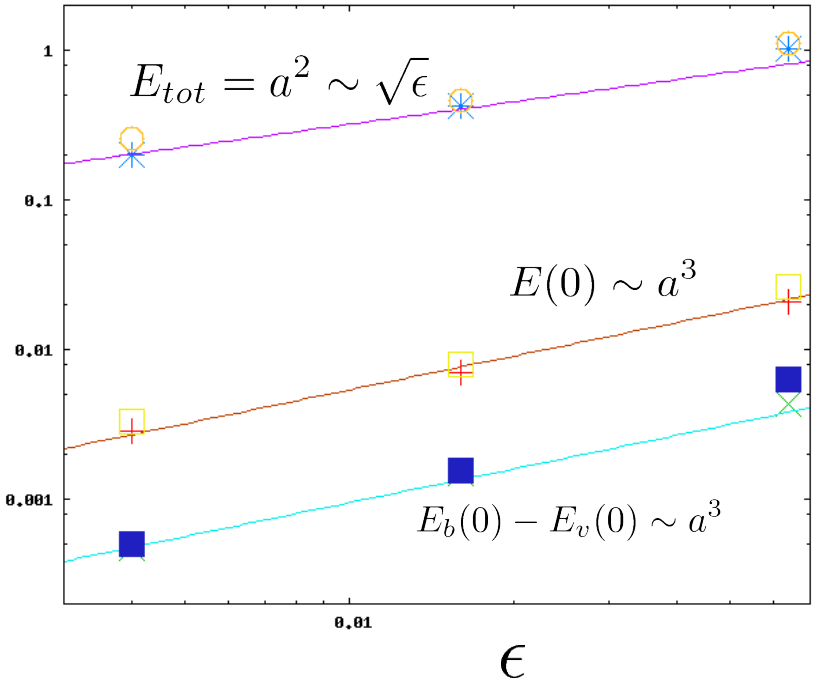

10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
2
4
6
8
10
12
14
$k_{||}$

Stationary state energies:

balanced



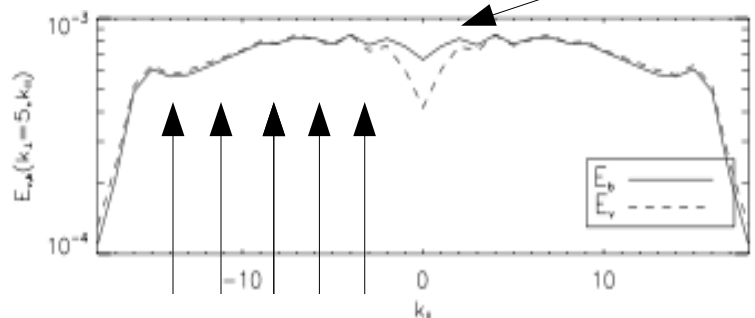
balanced+imbalanced



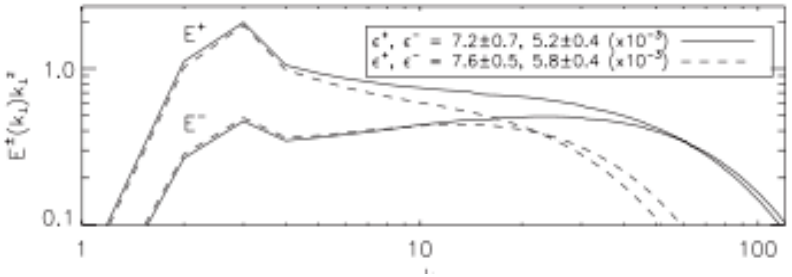
Zero-frequency energy and residual energy become negligible in the limit of weak turbulence.

Boldyrev & Perez 2009

Possibly strong cascading



many frequencies are driven



Many overlapping independent cascades

$$\epsilon^{\pm} = \frac{(w_\lambda^{\pm})^2 (w_\lambda^{\mp})^2}{\lambda^2} \frac{\Lambda}{v_A}$$

Summary

- Zero-frequency mode is a transient phenomenon and has negligible energy in the proper limit of weak turbulence, *balanced or imbalanced*.
- A partucular simulation setup of Boldyrev & Perez 2009, namely a decision to drive many frequencies and a decision to drive frequency k_parallel=1 (which could lead to strong cascade) most likely explains their observation of very high residual energy at zero frequency.