Plasma Physics at High β -- turbulence, field growth, transport etc.

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Opportunity to explore an important and unexplored regime of Plasma Science.

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The Large Prandtl Number Case: Galaxies, Clusters etc.

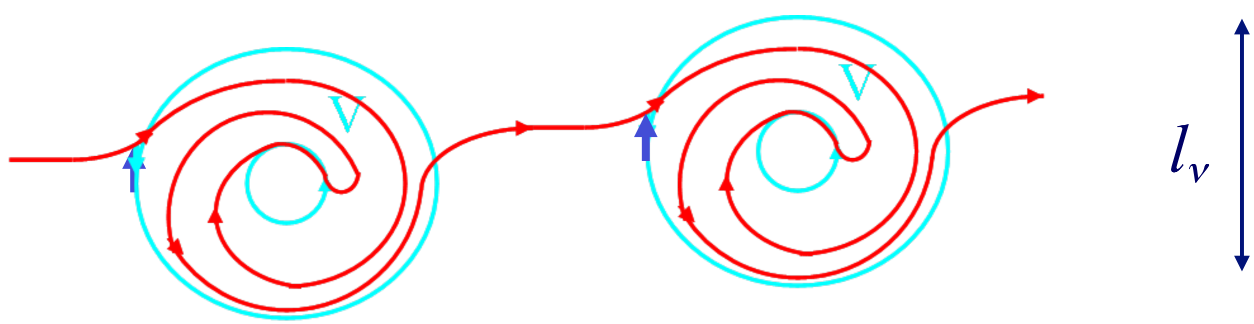
- Turbulent plasmas low collisionality and β
- Magnetic Prandtl number = $Pr = \upsilon/\eta \sim 10^{29}$.
- On the turnover time of the viscous eddies the "seed field" grows. The field develops structure below the viscous scale down to the resistive scale $l_\eta = Pr^{-1/2}\, l_\nu$

$l_\nu = 10$ - 30kpc

Viscous scale

$\tau_\nu = 10^8$ *years*

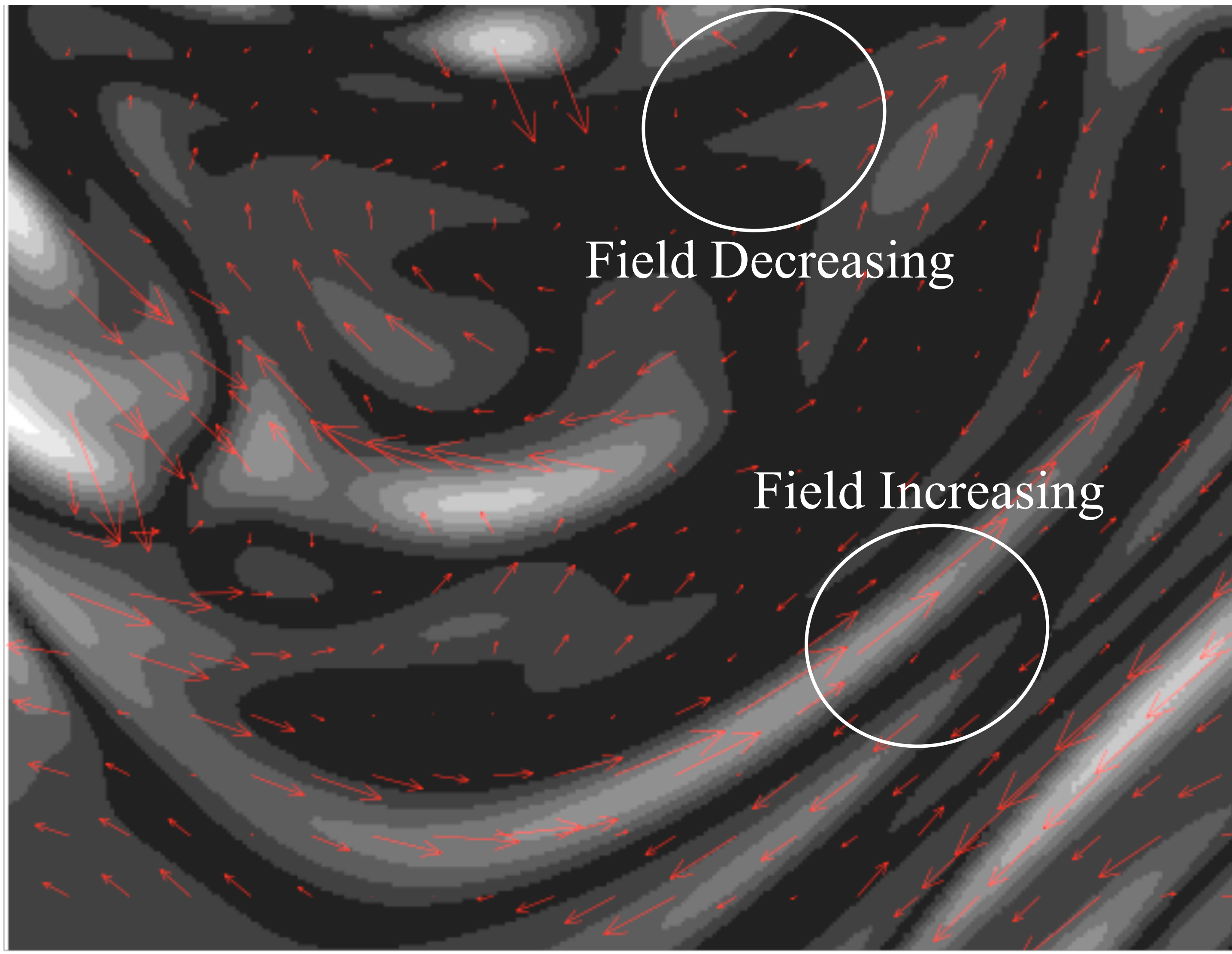
Viscous eddy Turnover time $(Re)^{-1/2}L/V$.



Isotropic Homogeneous Dynamo Folded Structure at Resistive Scale

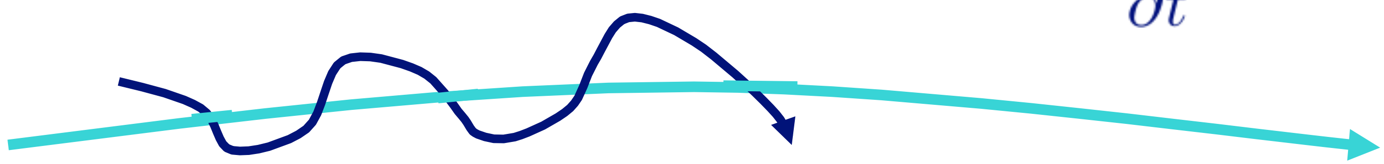
Grayscale is |B|.

Scalar Viscosity



Magnetized Viscosity --Anisotropic Pressure

$$\Omega_i \gg \frac{\partial}{\partial t}$$



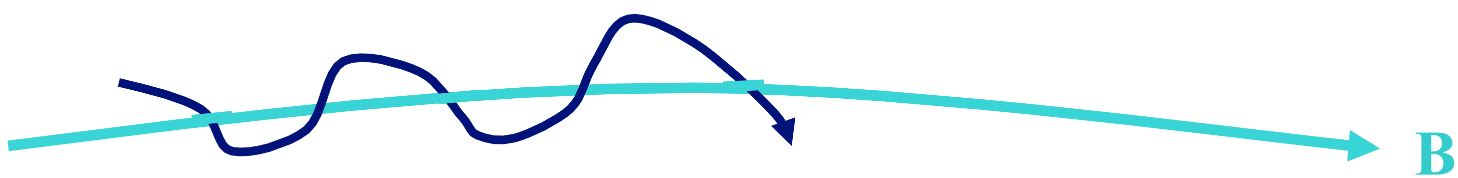
$$\mathbf{P} = \int \mathbf{v}\mathbf{v} f_i(\mathbf{v}, \mathbf{r}, t) d^3\mathbf{v}$$

DEFINITION OF PRESSURE TENSOR.

Anisotropic pressure tensor in magnetized plasma. Because of fast motion around the field the tensor must be of the form:

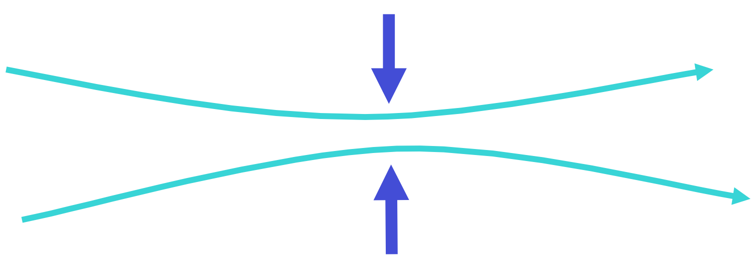
$$\mathbf{P} = P_\perp(\mathbf{I} - \mathbf{b}\mathbf{b}) + P_\parallel \mathbf{b}\mathbf{b} \quad P_\perp = < \frac{1}{2} m_i v_\perp^2 > \quad P_\parallel = < m_i v_\parallel^2 >$$

Magnetized Viscosity.



$$\mu = \frac{v_\perp^2}{B} = \quad constant$$

Collisionless particle motion restricted to being close to field line and conserving μ.



Compressing Field

$$\frac{1}{B}\frac{dB}{dt} \sim \frac{1}{< v_\perp^2 >}\frac{d < v_\perp^2 >}{dt}$$

Collisionless.

$$P_\perp - P_\parallel \sim \frac{1}{\nu B}\frac{dB}{dt}$$

Relaxed by Collisions.

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Firehose Instability.

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p-\nabla\cdot[(P_{\parallel}-P_{\perp})\mathbf{bb}]+\mathbf{J}\times\mathbf{B}$$

Look at instabilities that are smaller scale than the field and growing faster than the stretching rate. We take a constant unperturbed stretching and $\mathbf{B_0}$.

$$\rho\left(\frac{\partial \delta\mathbf{v}}{\partial t}\right)=-\mathbf{k}(\delta p+\delta\mathbf{B}\cdot\mathbf{B})-\mathbf{k}\cdot[(\delta P_{\parallel}-\delta P_{\perp})\mathbf{b_0 b_0}]+[B_0^2-(P_{\parallel}-P_{\perp})]\delta(\mathbf{b}\cdot\nabla\mathbf{b})+(\mathbf{B_0}\cdot\nabla\delta B_{\parallel})\mathbf{b_0}$$

Field still frozen to the plasma. $\delta\mathbf{b}=\mathbf{b_0}\cdot\nabla\xi_{\perp}$

For Alfven wave Polarization. $\xi_{\perp}\propto\mathbf{k}\times\mathbf{b_0}$

$$\gamma^2=-k_{\parallel}^2\left[\frac{B_0^2}{\rho}-\frac{(P_{\parallel}-P_{\perp})}{\rho}\right]$$

Unstable if: $(P_{\parallel}-P_{\perp})>B_0^2$ $\qquad \beta > Re^{-1/2}$

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More Firehose.

Unstable when $B^2 <$ energy in viscous scale eddies (roughly for B<1-5μG). Obviously in early stages of dynamo. Since

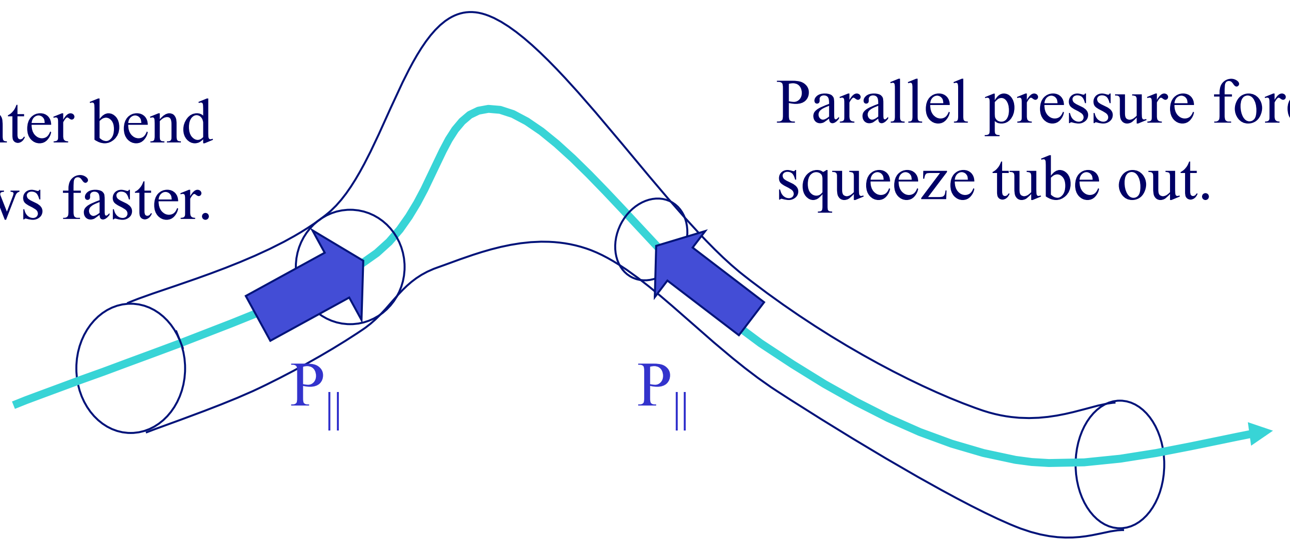
$$\gamma \sim k_{\|} C_{sound} (Re)^{-1/4}$$

Growth at small scales is very fast. Collisionless theory gives same growth formula down to $k_{\|}\rho_i \sim (Re)^{-1/4}$ where:

$$\gamma_{max} \sim \Omega_{ci} (Re)^{-1/4}$$

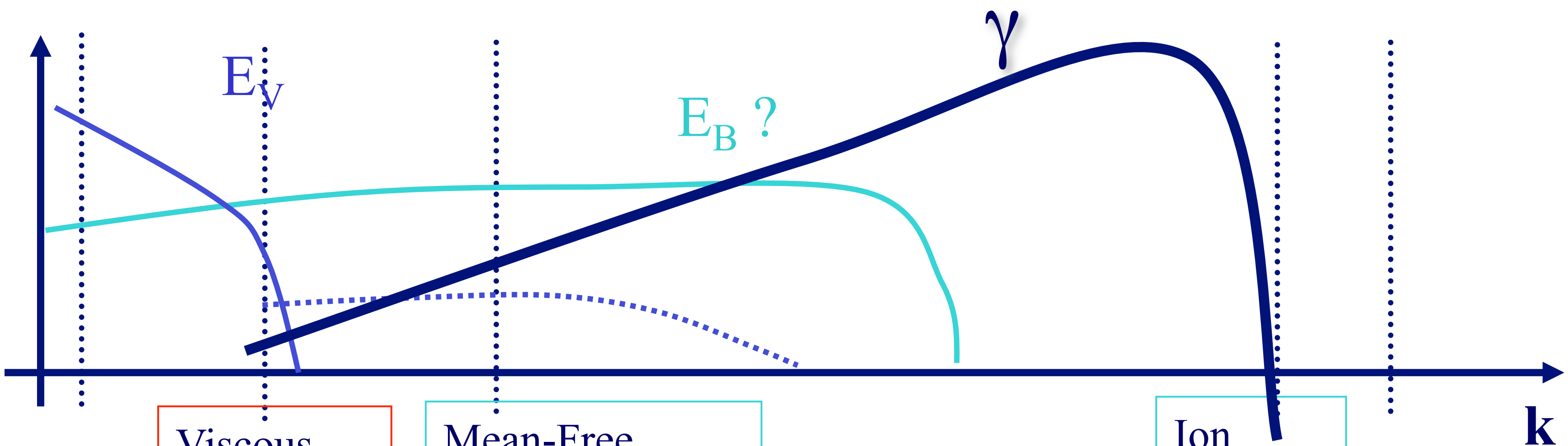
Tighter bend grows faster.

Parallel pressure forces squeeze tube out.



Rosenbluth 1956
Southwood and Kivelson 1993

Scales -- Galaxy Clusters



$l_0 \sim$ 1-3Mpc

Viscous Scale $l_\nu \sim l_0 Re^{-3/4} \sim$ 10 - 30 kpc

Mean-Free Path. $\lambda_{mfp} \sim l_0 Re^{-1} \sim$ 1-10kpc

Ion Larmor Radius Scale @ **B** = 1μG ~10^5km.

Resistive Scale ~ $l_0 Rm^{-1/2} \sim 10^4$km

$l_0 / u_0 \sim 10^9$ years

$l_\nu / u_\nu \sim 10^8$ years

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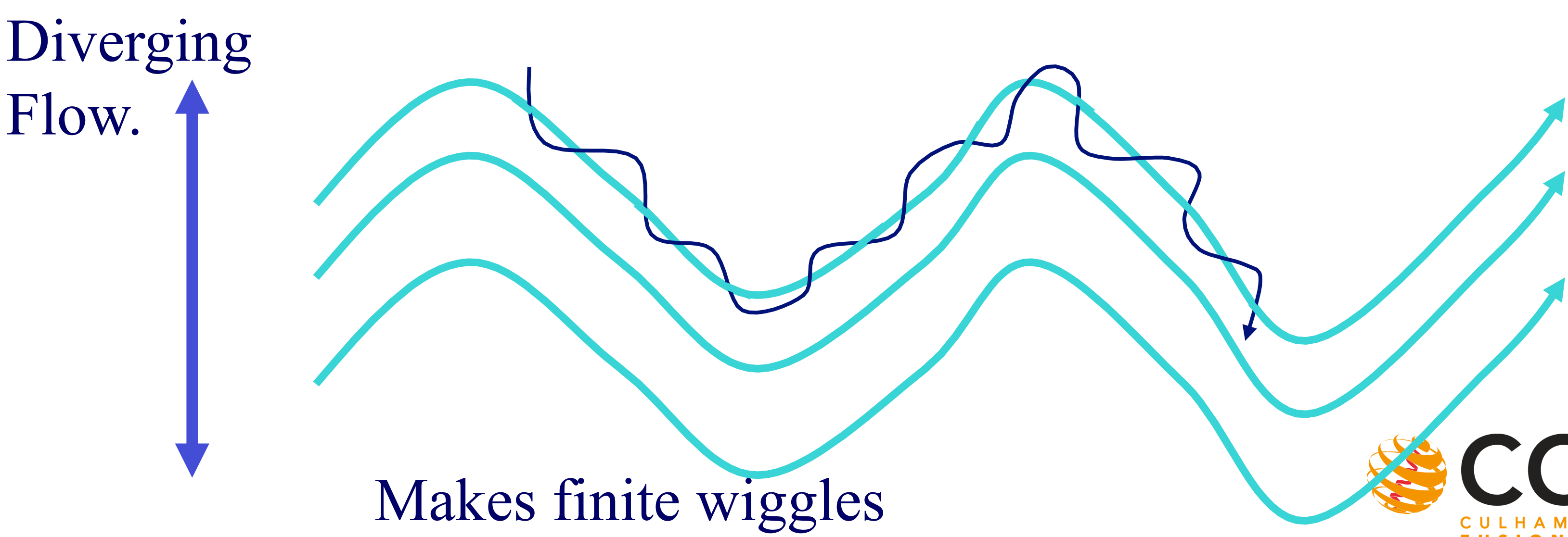
So What!? -- Nonlinear Firehose.

Kulsrud, Schekochihin and SCC

Nonlinear kinetic theory gives:

$$P_{\perp} - P_{\parallel} \simeq < \frac{1}{2\nu B^2} \frac{d(B_0^2 + \delta B^2)}{dt} P >$$

Rate of change of B averaged along B.

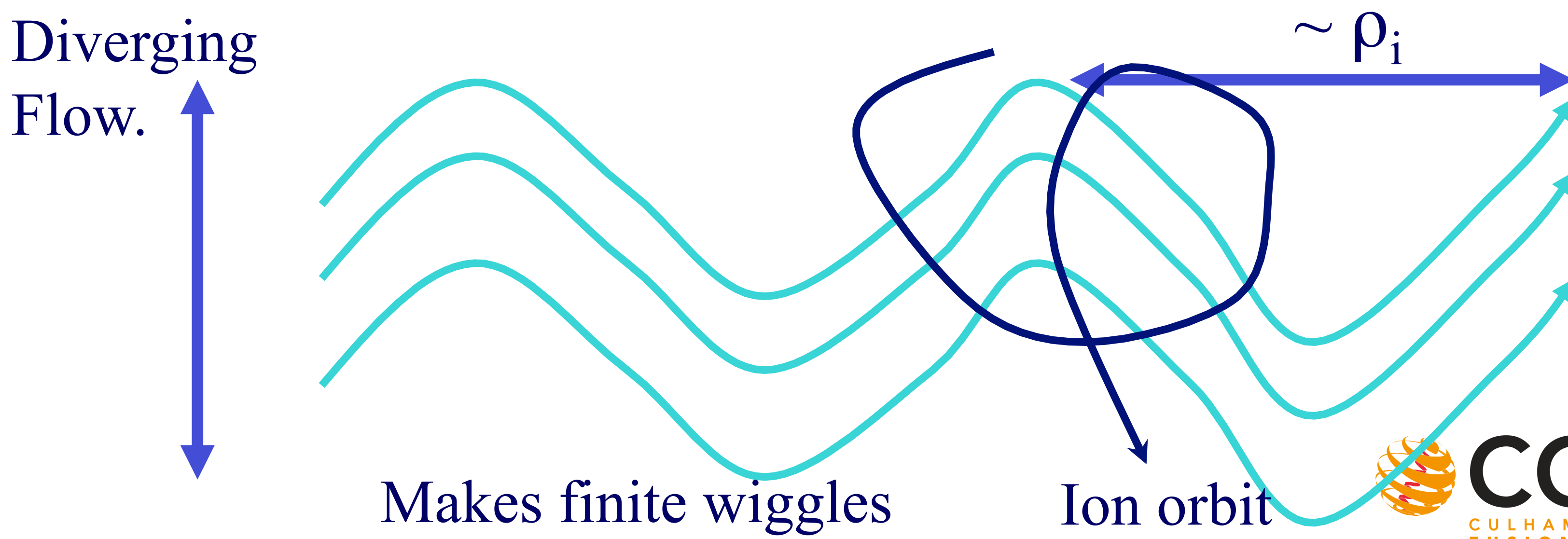


So What!? -- Nonlinear Firehose.

$$\frac{\partial^2 \xi_\perp}{\partial t^2} = -C_s^2 < \frac{1}{\nu B^2}\frac{d}{dt}(B_0^2 + \delta B^2) > \frac{\partial^2 \xi_\perp}{\partial z^2}$$

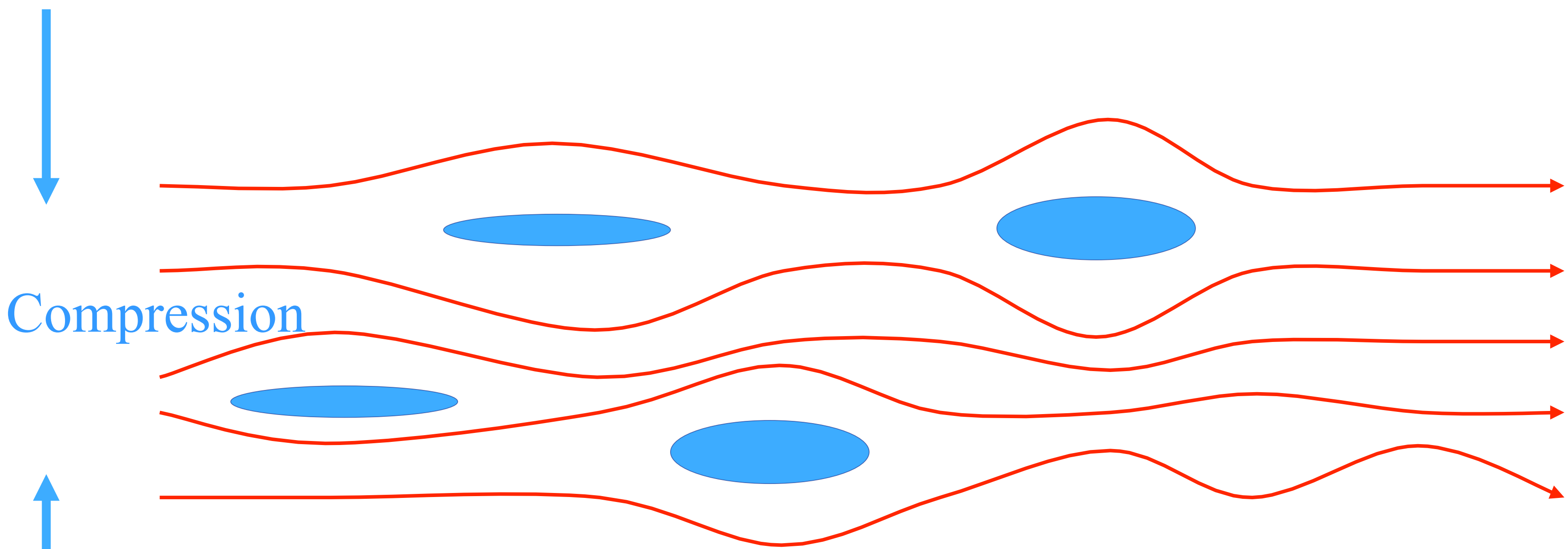
$$\delta B = \mathbf{B_0} \cdot \nabla \xi_\perp$$

Instability tries to keep average B constant by bending the field.



Mirror-Mode

Slow mode type polarization.



Particles collect in the magnetic mirrors trying to weight the region where **B** field is not changing.

What does small scale field do? Macroscopic consequences.

Schekochihin, Kulsrud and SCC

- Enhanced particle scattering?
- Effective collisions increase -- $\nu \rightarrow \Omega_i$?
- Effective Mean free path $\rightarrow \sim \rho_i$?

Transport properties changed.

- Heat conduction suppressed.
- Viscosity suppressed.
- Electron scales -- enhanced resistivity?

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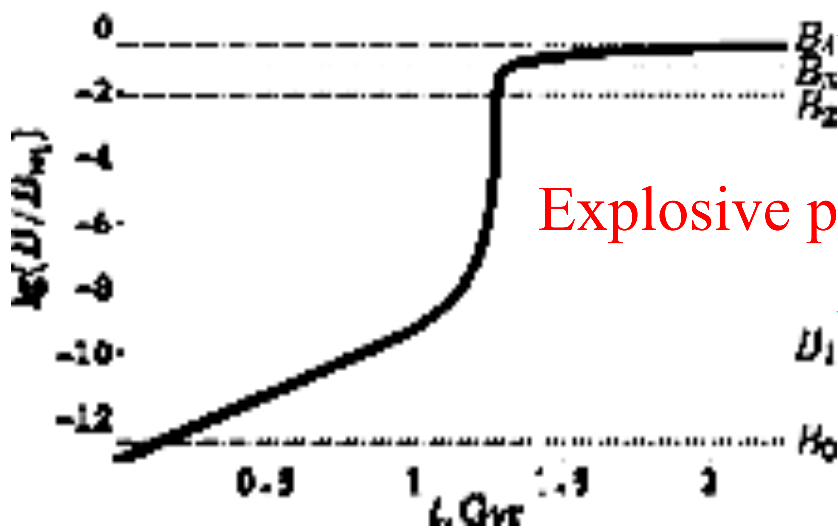
Explosive dynamo?

Schekochihin, Kulsrud and SCC

If viscosity decreases -- **Re** increases
turbulence has faster motions at a smaller viscous scale.

➔**Dynamo Growth Time:** $\tau_\nu \sim \tau_0(\rho_i/L)^{(1/2)} \sim$ 1000 years!

MAGNETIC FIELD CAN GROW ON TRIVIAL TIMESCALES.



Explosive phase

20μG

10^{-16} G

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Thoughts

We will not be able to proceed in several areas of astrophysics without a better understanding of turbulent partially collisionless plasmas -- especially the macroscopic consequences of the Microturbulence.

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