Magnetized spherical Couette flow

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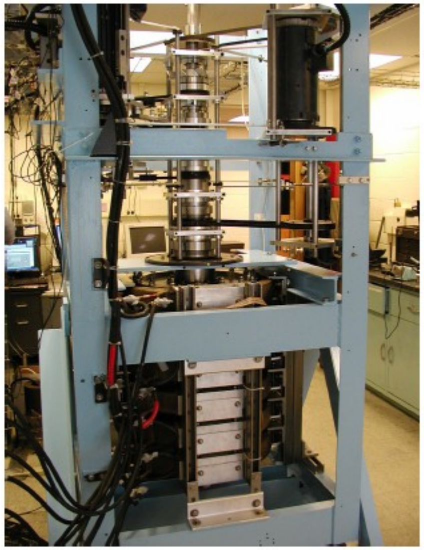
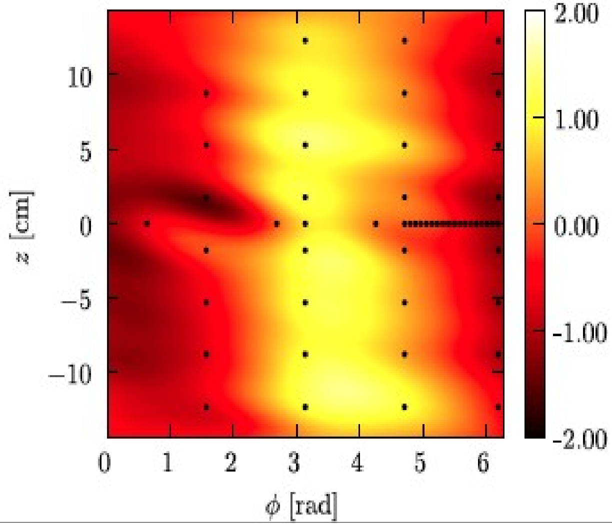
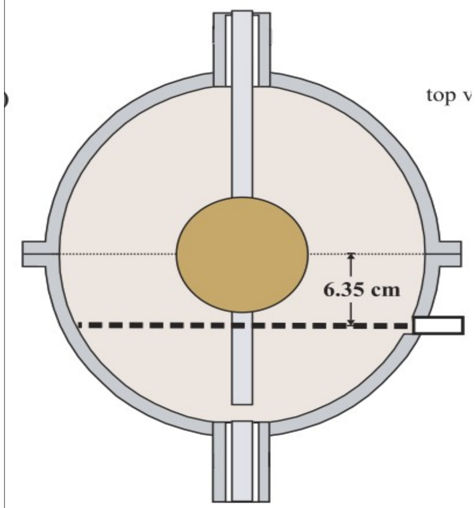
Context

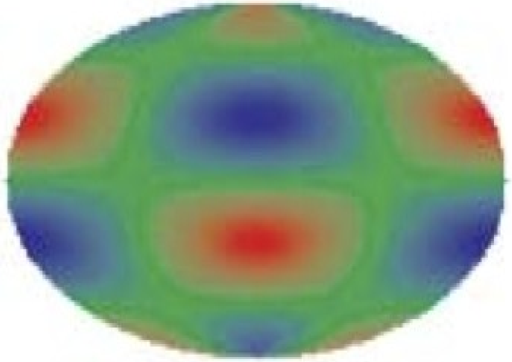
Princeton MRI experiment





Maryland experiment





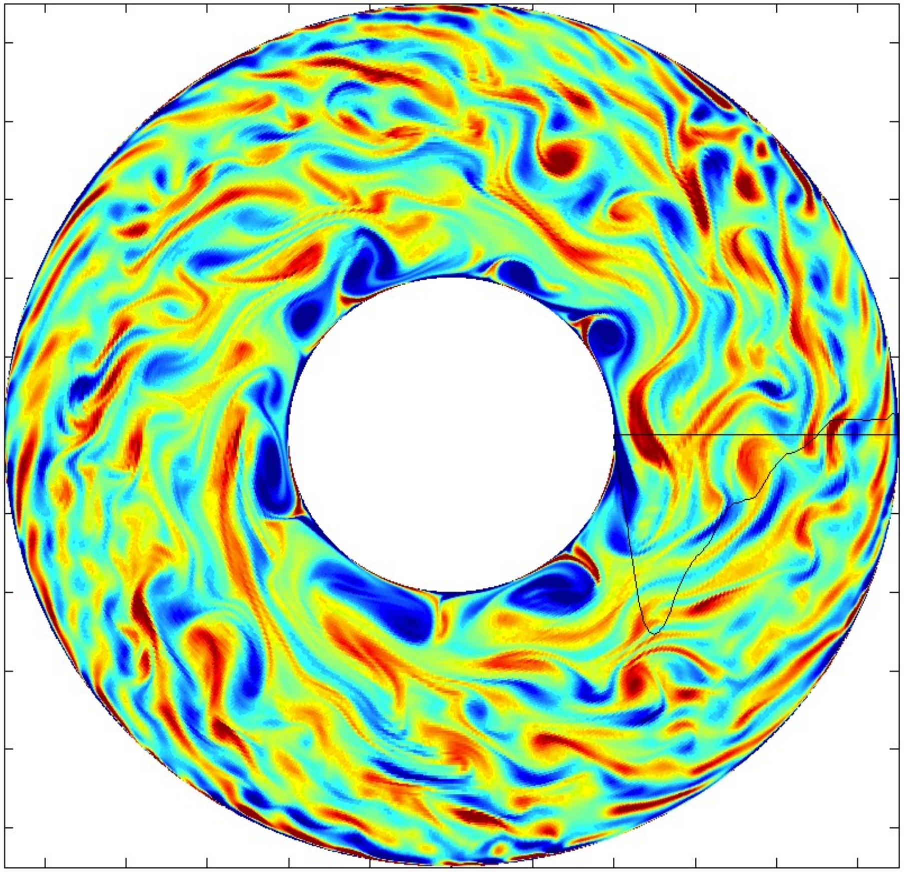
- Non-axisymmetric MHD instabilities
- m=1 or m=2 azimuthal wavenumber
- Symmetric and antisymmetric modes

Nornberg et al., Phys. Rev. Lett. (2010)
Sisan et al., Phys. Earth Planet. Inter. (2003)

Numerical Code

<u>PaRoDy (*Dormy et al*) :</u>

- Spherical geometry
- *Poloidal-Toroidal* decomposition
- Finite differences in radius
- Cranck-Nicholson / Adams Bashford time-stepping .



Insulating / conducting / ferromagnetic spheres

Full sphere or Couette problem

No slip / Stress Free BC for velocity

Full MHD problem

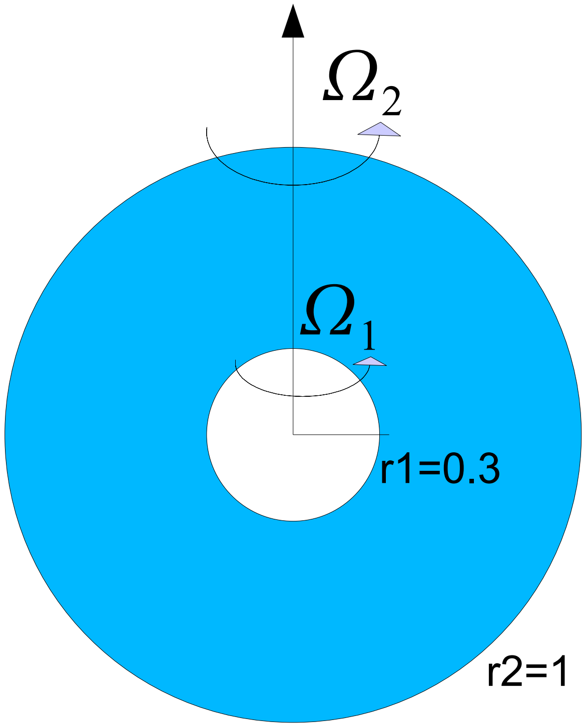
Navier-Stokes equation:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \nabla) \mathbf{u} = -\nabla P + \rho \nu \nabla^2 \mathbf{u} + \mathbf{F} + \mathbf{j} \times \mathbf{B}$$

Induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

- Insulating outer and inner spheres
- Dipolar or vertical magnetic field imposed
- With or without overall rotation :
  - $\Omega_1 = 8\,\Omega_2$
  - $\Omega_2 = 0$



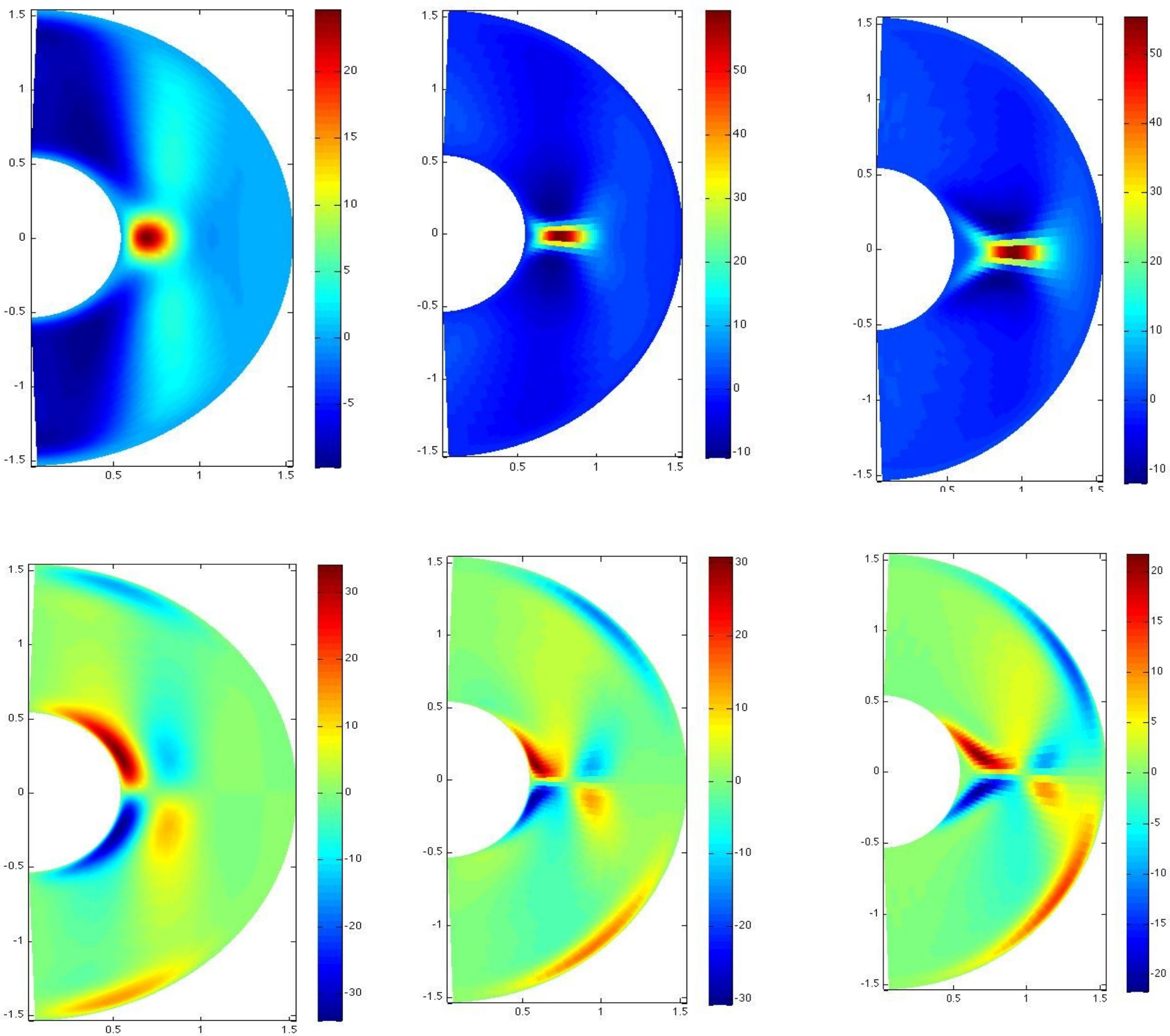
$$\mathrm{Re} = \frac{(\Omega (r_2 - r_1)^2)}{\nu} \qquad \Lambda = \frac{B_0}{\sqrt{(\rho \mu \eta \Omega)}}$$

Suppression of meridional circulation

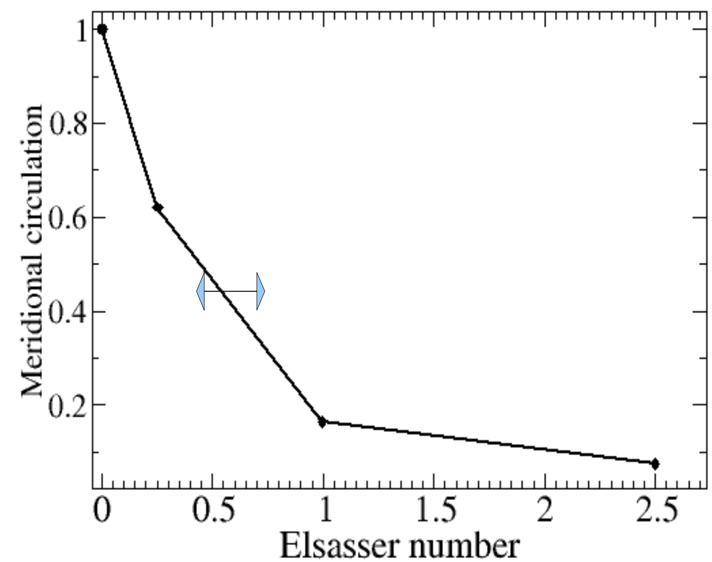
Re=330

Vr

Vt



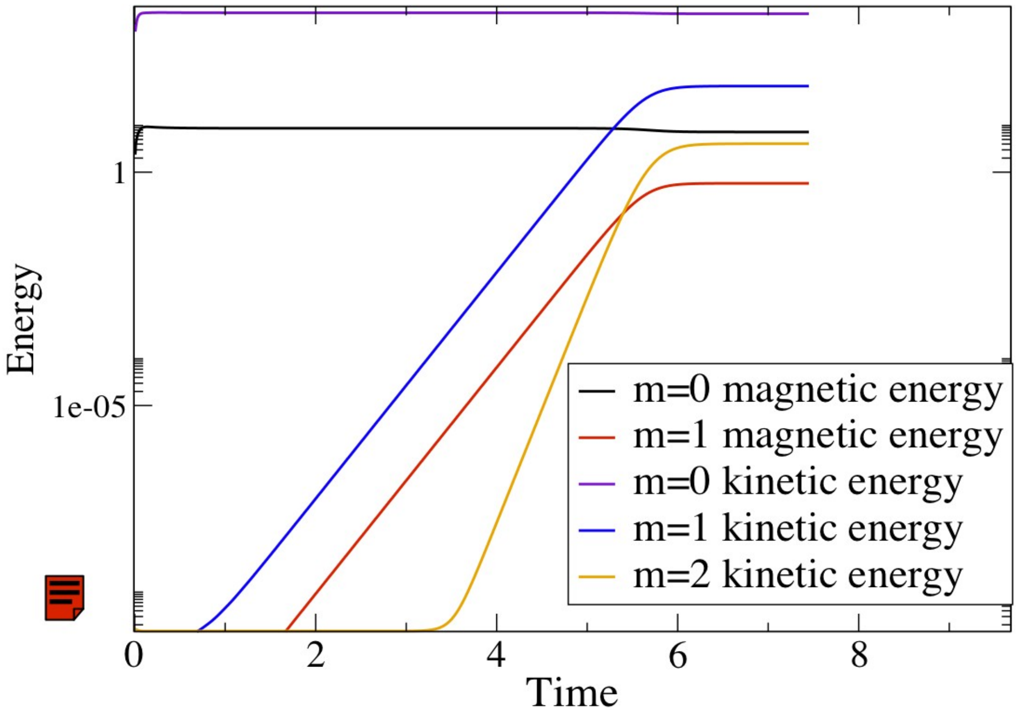
=0 =1 =2.5



Inhibition of the meridional circulation by magnetic field

Non-axisymmetric MHD instability

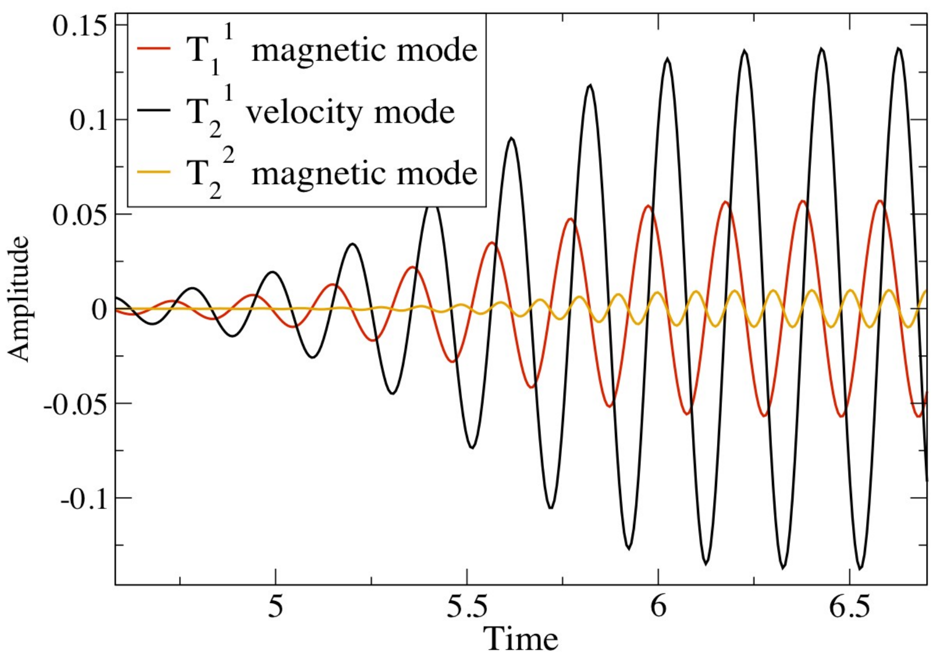
Re=330 =0.5



→ m=1 dominated instability

→ smaller m=2 component

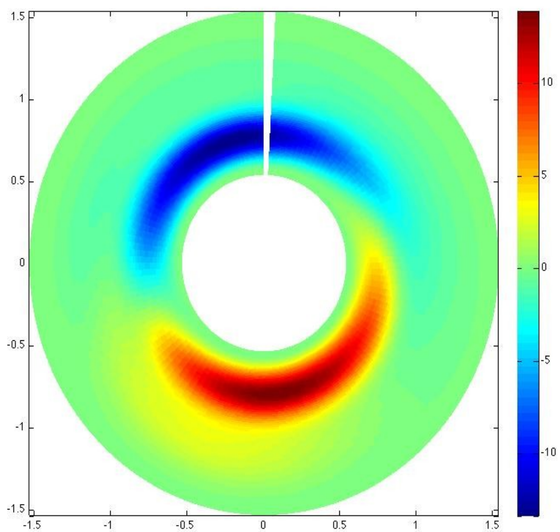
Oscillating structure →



Structure of the instability

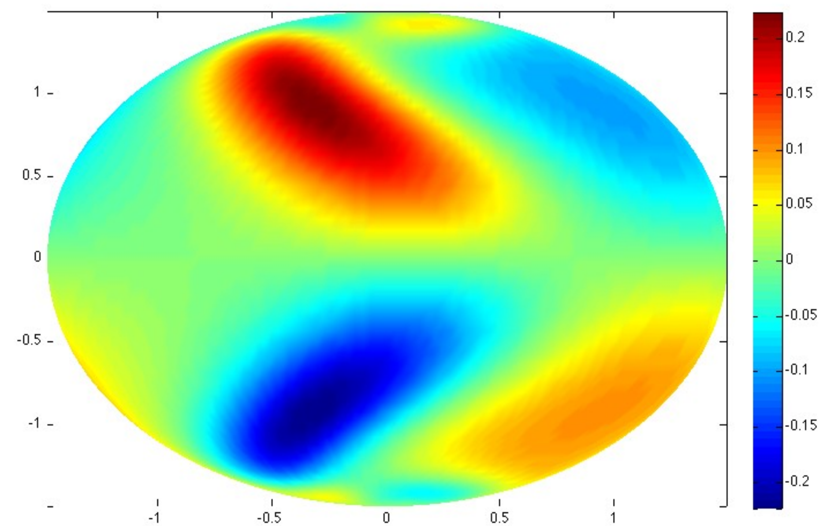
Re=330 =0.5

Br



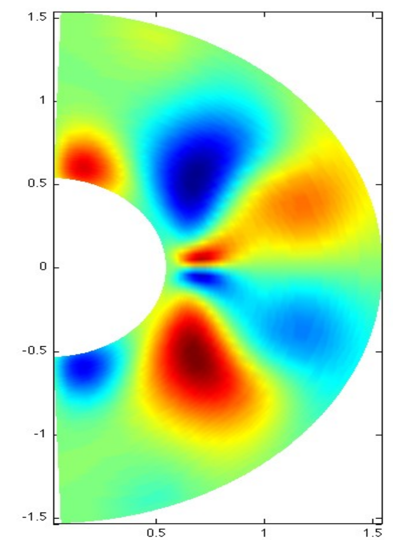
Equatorial plane

Br



Close to the outer sphere

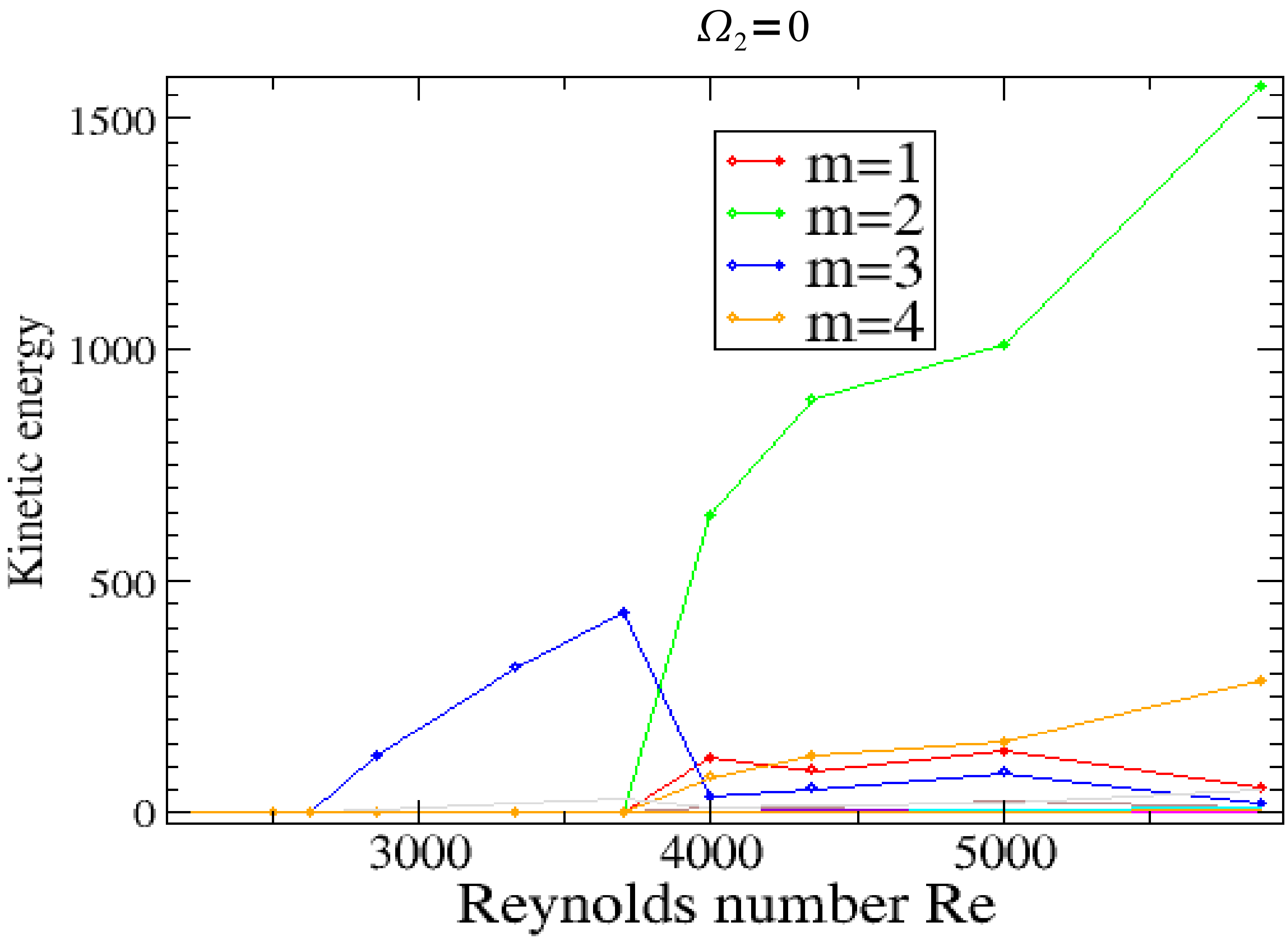
Br



(R,Z) plane

- → Antisymmetric m=1 magnetic field
- → Instability occurs for dipolar and vertical field
- → Instability occurs with or without overall rotation

Hydrodynamical instabilities at Large Re



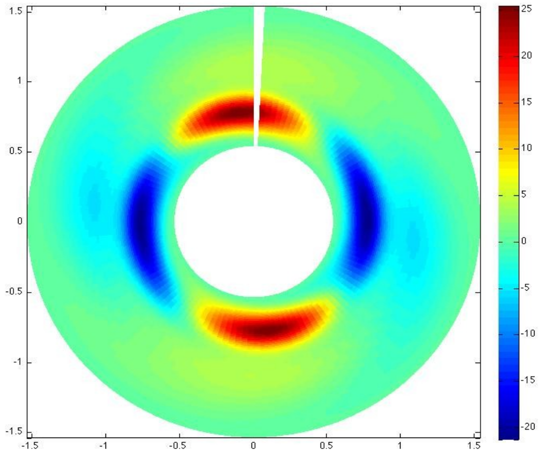
<u>Without rotation:</u> Kelvin-Helmoltz destabilization of the equatorial jet

<u>Strong rotation:</u> destabilization of the Stewartson layer

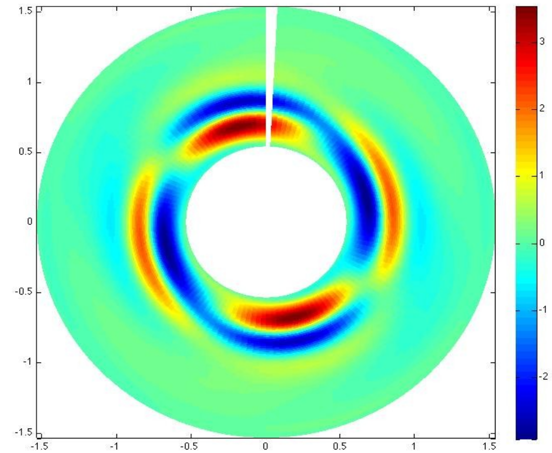
Guervilly and Cardin, GAFD (2010)
Hollerbach et al., Fluids Dyn. Res. (2006)

hydrodynamical instability

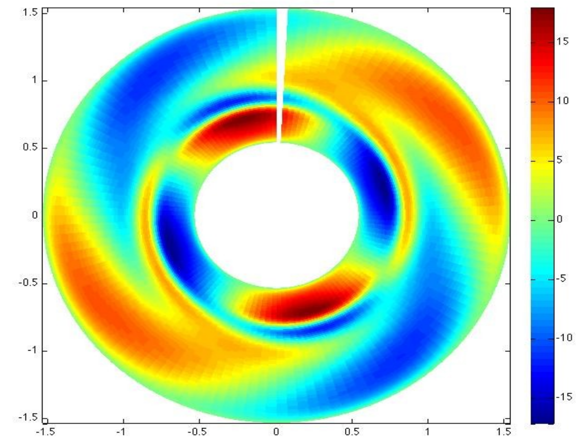
$\Omega_2 =$ ·



Vr

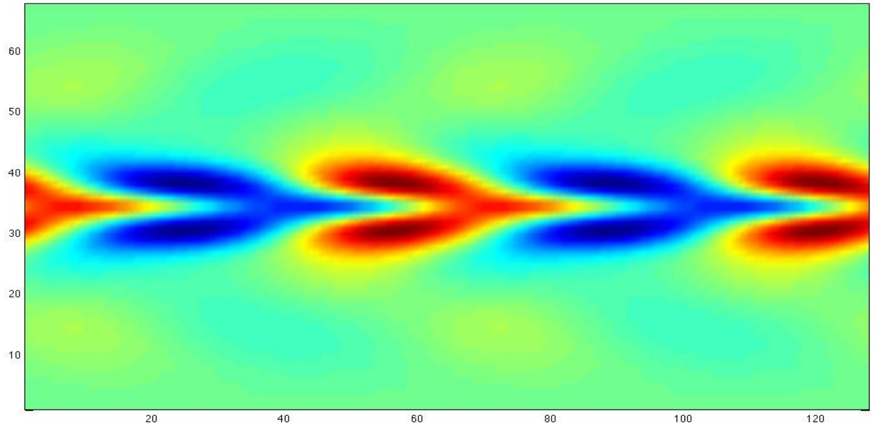


Vt



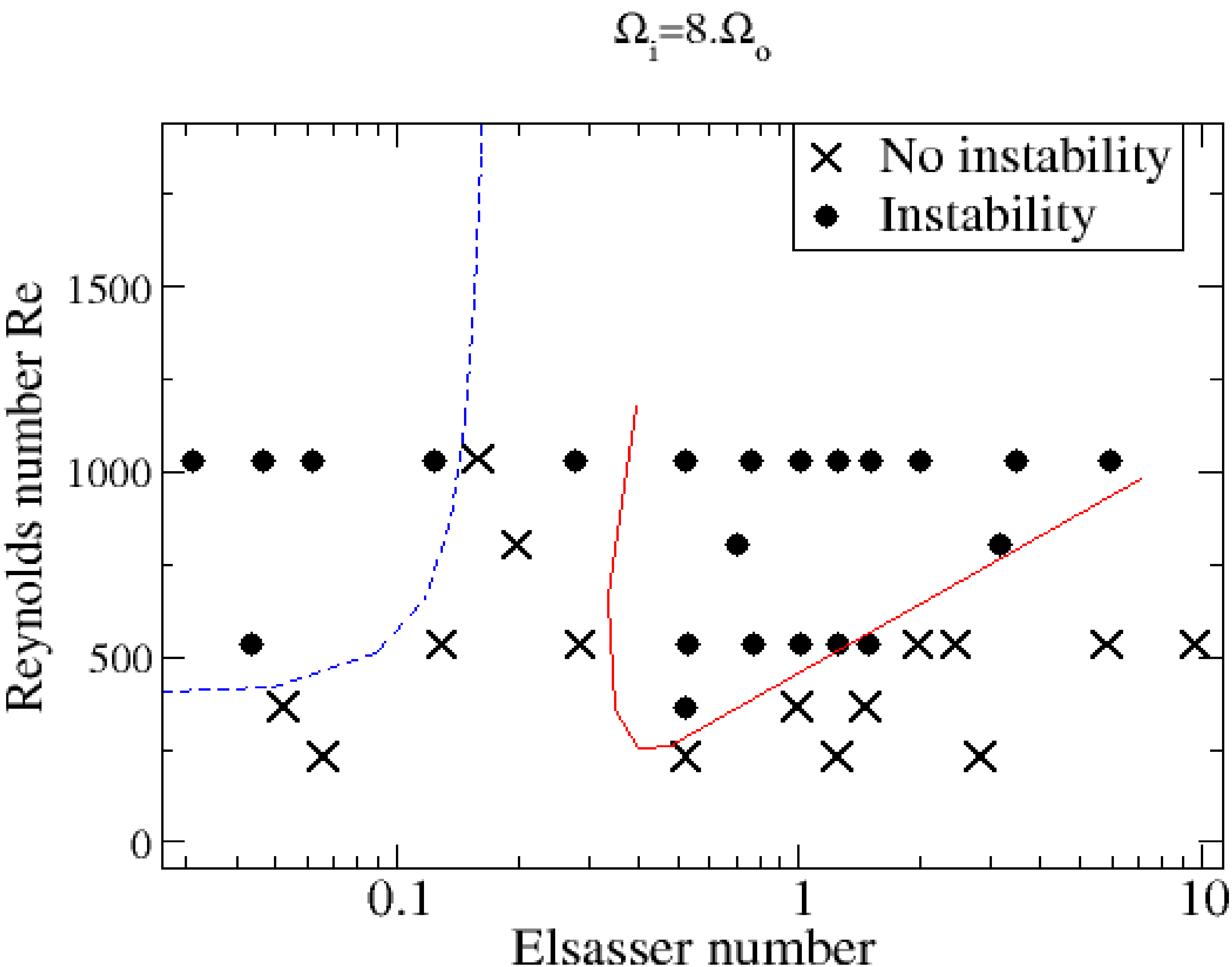
Vp

Vr in (Z,phi) plane :



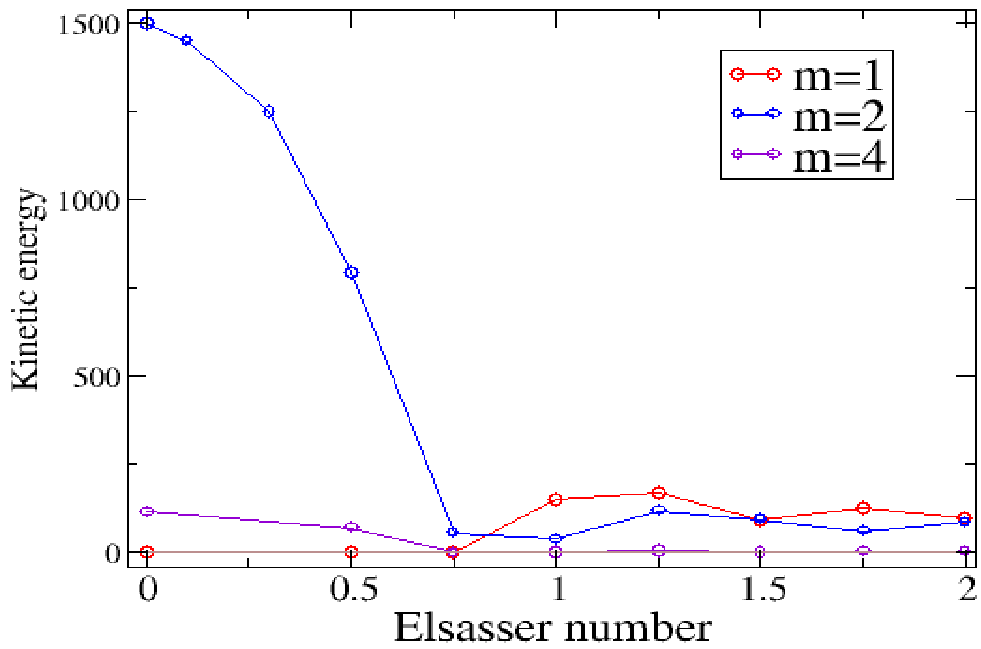
- m=2 non-axisymmetric instability
- Kelvin-Helmoltz instability of equatorial jet
- Equatorially symmetric structure
- Suppressed by magnetic field

Parameter space

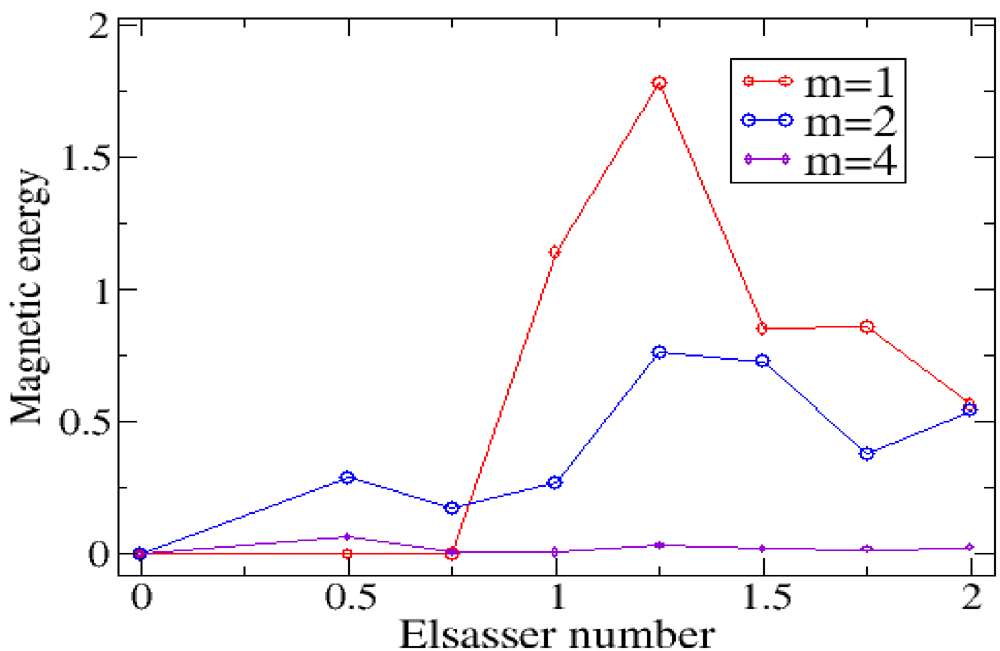


Hollerbach (2010)

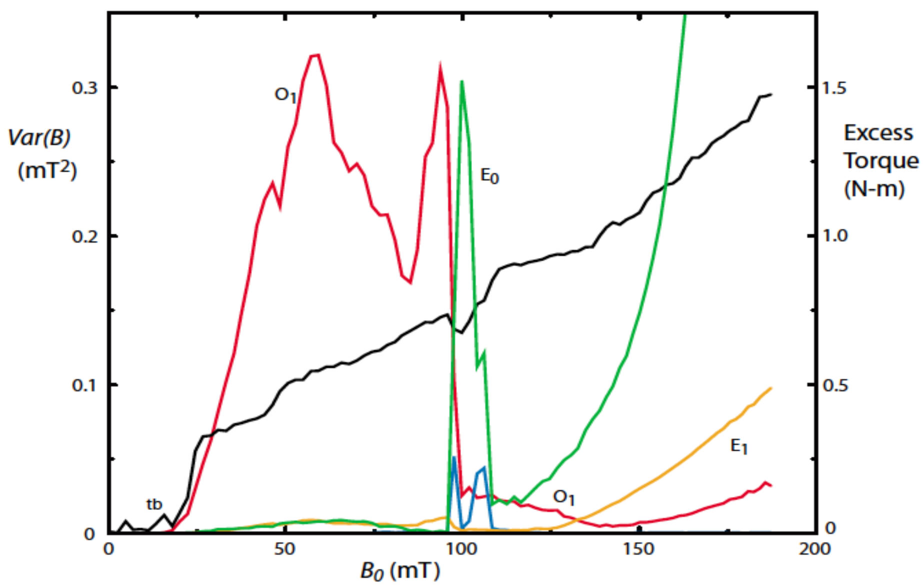
Competition between different modes



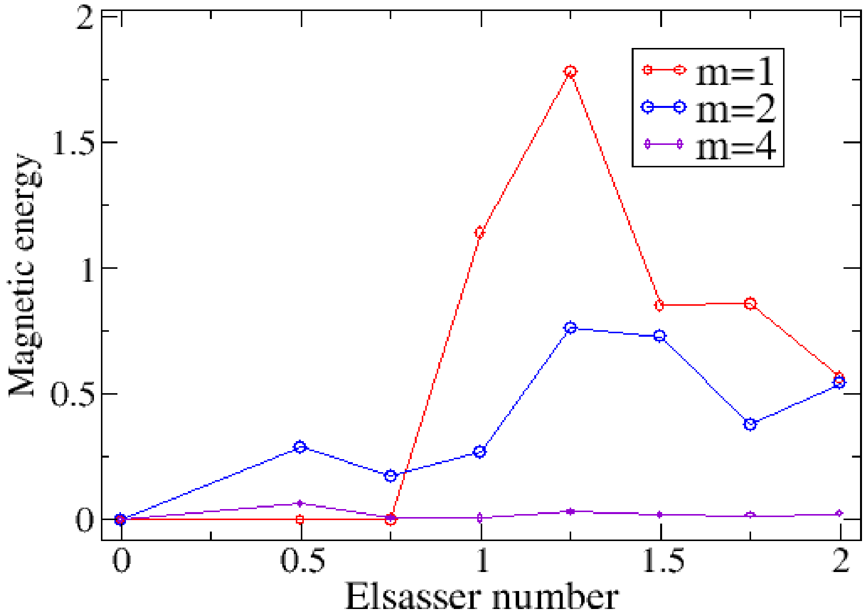
- Transitions between different azimuthal wavenumbers
- Similarities with the Maryland experiment



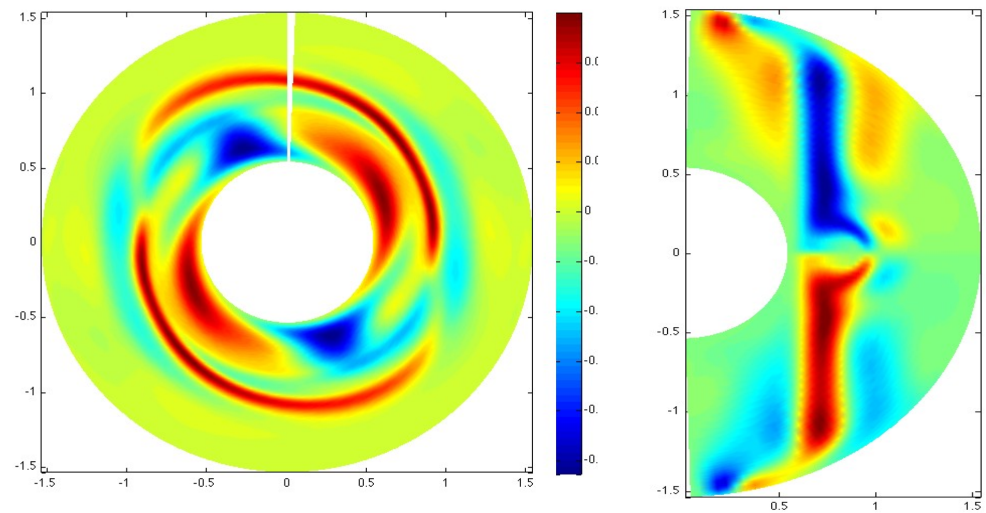
Maryland experiment :



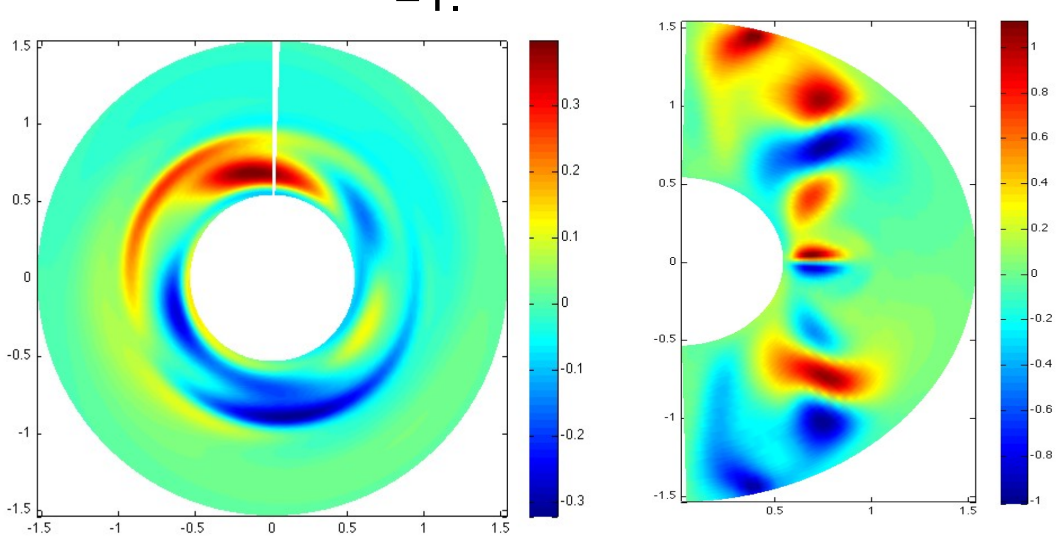
Non-axisymmetric instability



=0.5



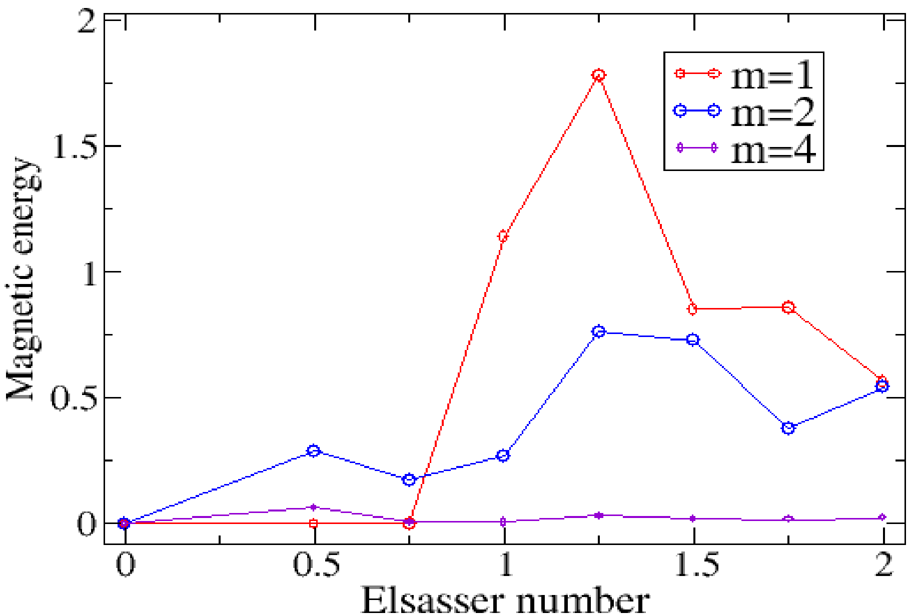
=1.

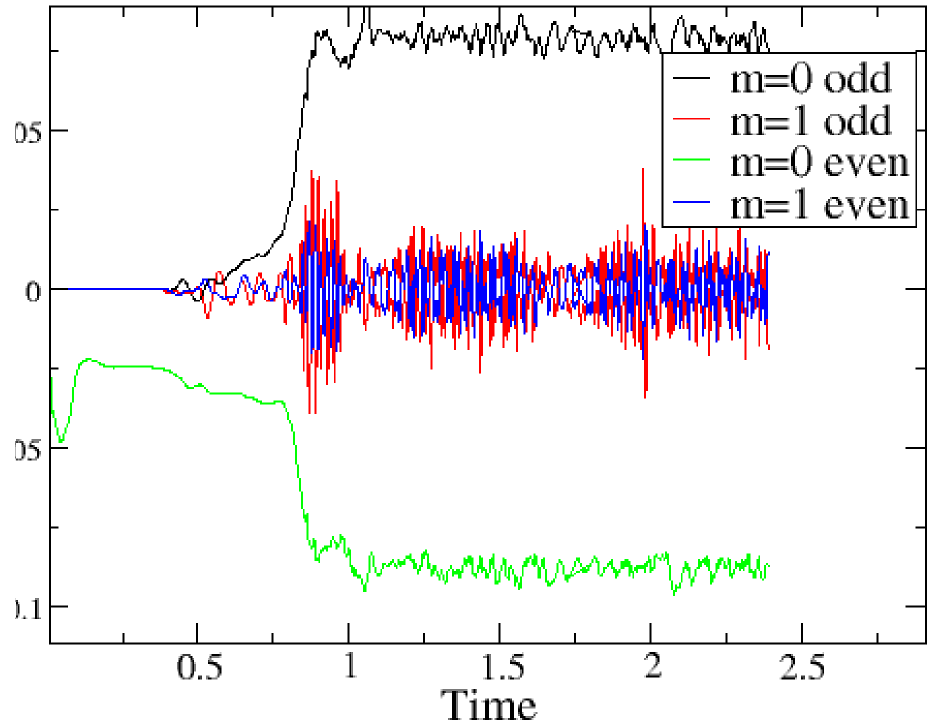


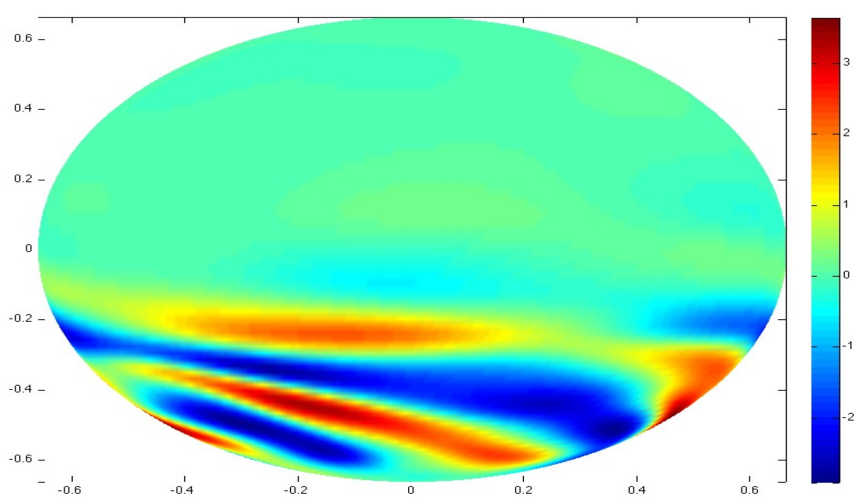
- Instability localized close to the tangent cylinder
- Large vertical wavenumber
- Interaction symmetric/antisymmetric modes
- Related to Magnetostrophic MRI (see *Petitdemange 2009*)

Non-linear dynamics

=2





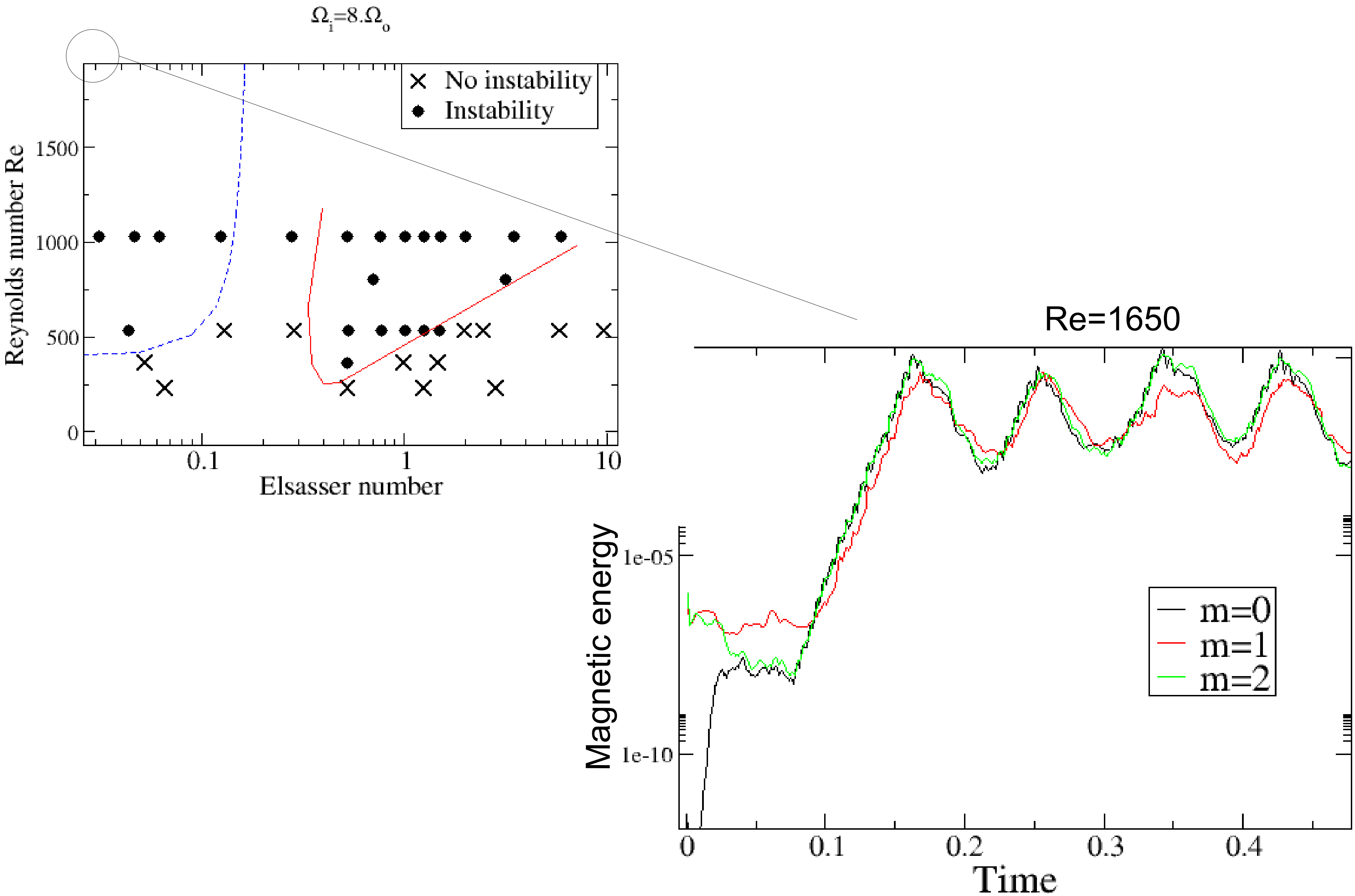


→ Hemispherical magnetic field

→ Phase locking of symmetric and antisymmetric modes

Gallet and Petrelis, PRE (2009)

Interaction with dynamos



Guervilly and Cardin, GAFD (2010)

Conclusion

- Magnetized spherical Couette flow generates non-axisymmetric instabilities
- Non-linear competition between modes
- Similarities with Maryland 'MRI' experiment.

Perspectives

High Reynolds number simulations and effect of Pm

Investigate non-linear dynamics

Comparison with cylindrical simulations

Interaction between MRI and dynamo action