Viscous Ion Heating from Lab Reconnection

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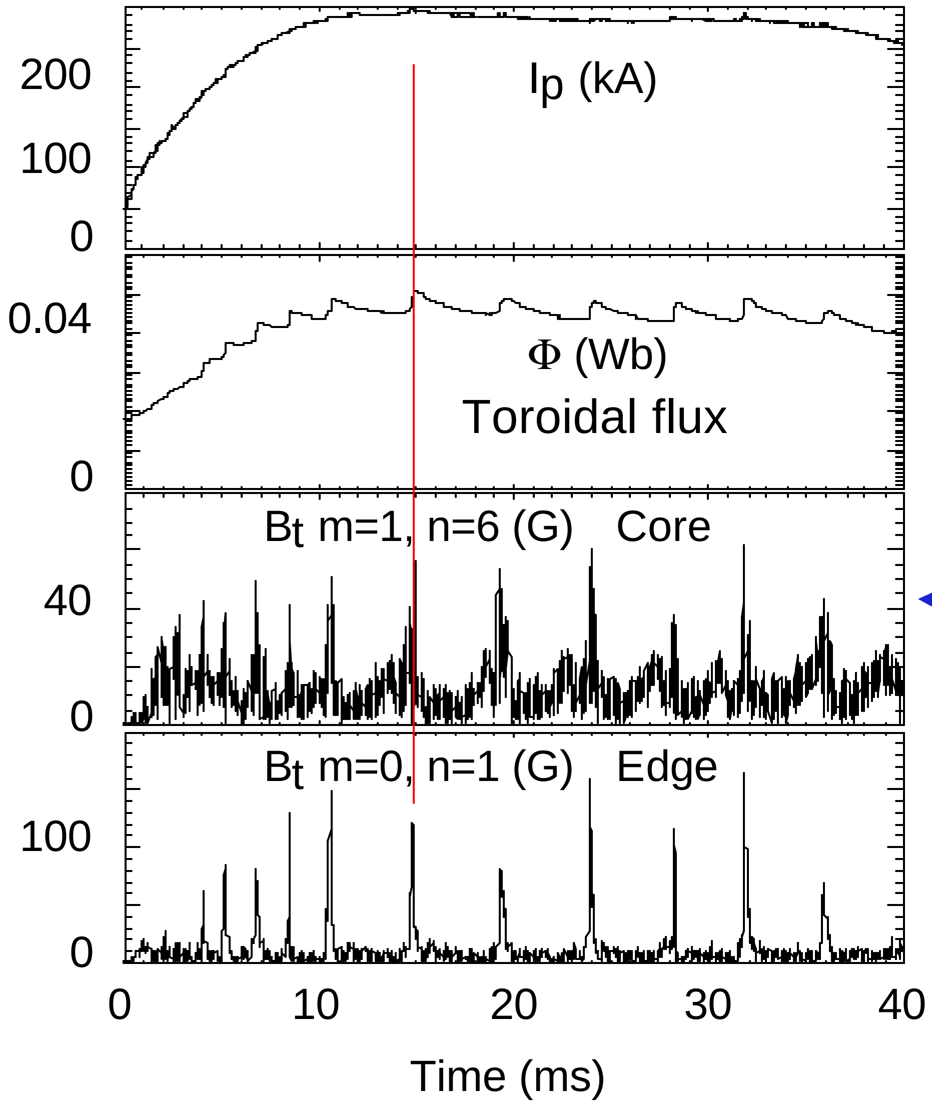
Outline

- Ion heating during reconnection events in MST
- Two views on dissipation: (1) Braginskii viscosity, (2) plasma dielectric tensor
- Dielectric tensor with weak ion-ion collisions
- “Parallel viscosity” at arbitrary ion-ion collision frequency
- “Perpendicular viscosity” in strong and weak collisional regimes
- Numerical treatment of the kinetic equation
- Ion heating by collisional damping of tearing modes

Discrete sawtooth events driven by tearing modes (magnetic reconnection)

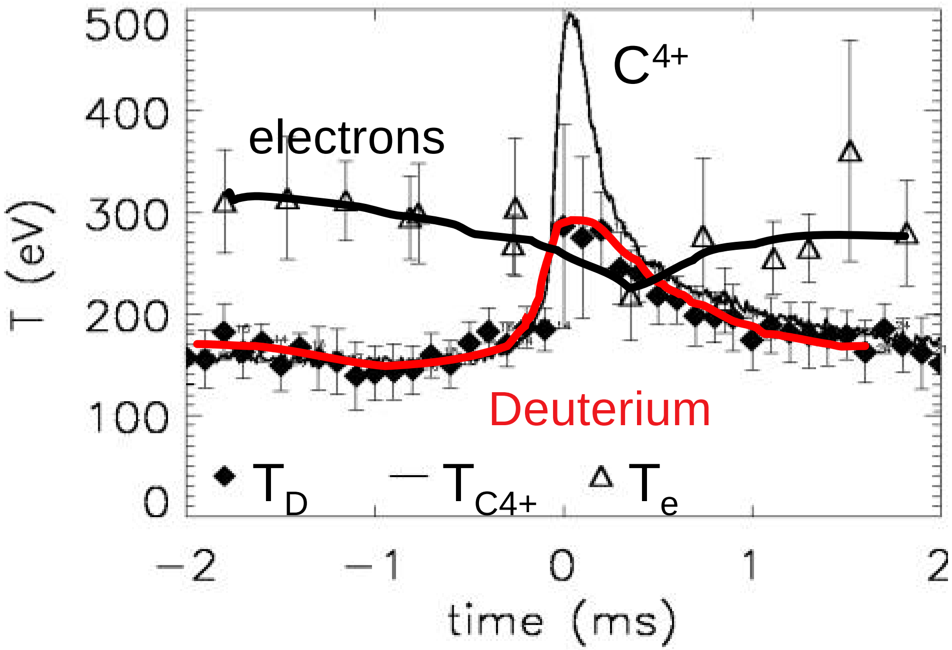


Crashes,
sawteeth,
reconnections,
discrete events...

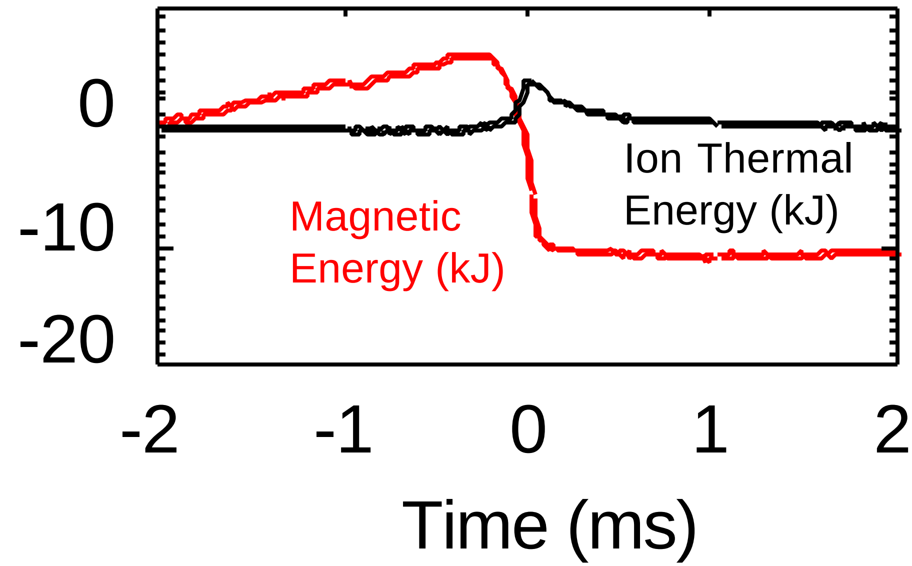
toroidal field fluctuations
$\delta B/B \simeq 2\%$

Strong ion heating in MST during reconnection events

Both impurities and majority components are rapidly heated. Stronger heating of heavy ions.



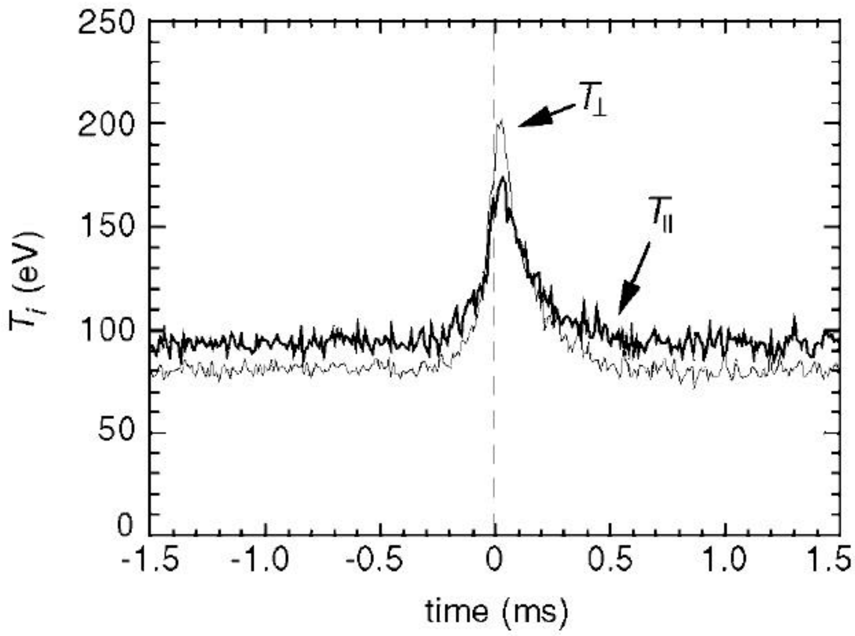
Changes in the thermal energy are large and comparable to the released equilibrium magnetic energy.



Reconnection heating on MST is almost isotropic

Distribution function remains Maxwellian during sawtooth crash

$T_{||} \sim T_{\perp}$



Typical parameters for MST

$n = 10^{13}$ cm^{-3}, $T_i = 200$ eV, $\Delta T_i = 100$ eV

time of reconnection (sawtooth crash) $\leq 100\ \mu$sec $\ll \tau_{ii} = 10^{-3}$ sec

kinematic Braginskii viscosity $\nu_\perp = 0.9 \times 10^3$ cm^2 / sec

viscous relaxation time $\tau_{vis} \simeq L^2/\nu_\perp \simeq 10^{-3}$ sec (at $L \simeq \rho_L \simeq 1$cm)

τ_{vis} is 10 times longer than the time of reconnection

How is the energy delivered and absorbed into the ions?

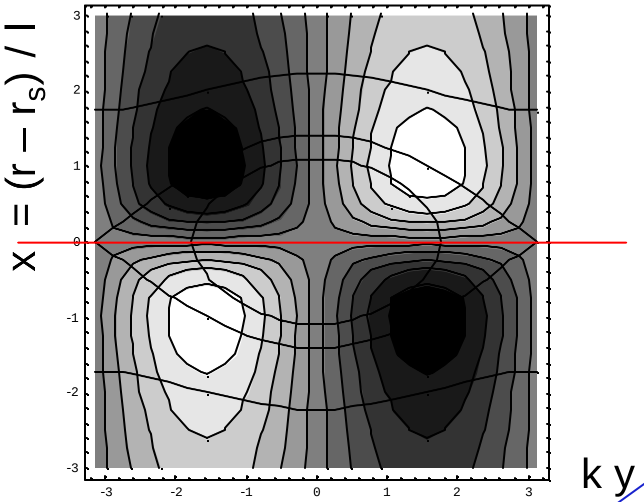
- effect of large parallel viscosity $\nu_\parallel / \nu_\perp \simeq 10^8$?
- small fraction (20%) of magnetic energy is required to explain observed ion heating

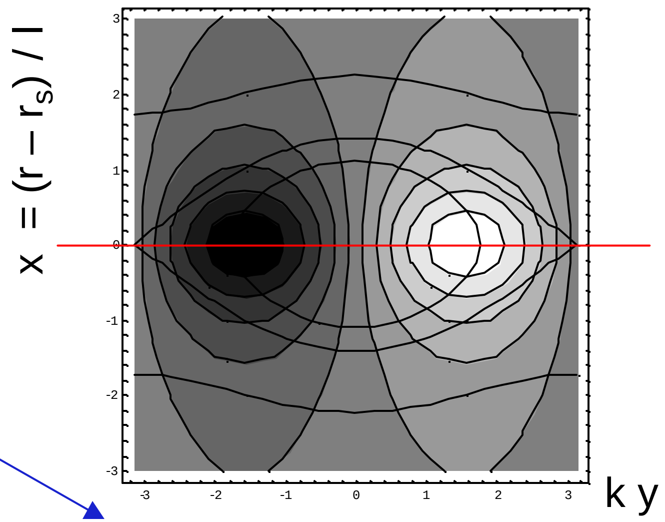
viscous mechanisms should be modified for weakly collisional case

Structure of plasma flows in the reconnection layer

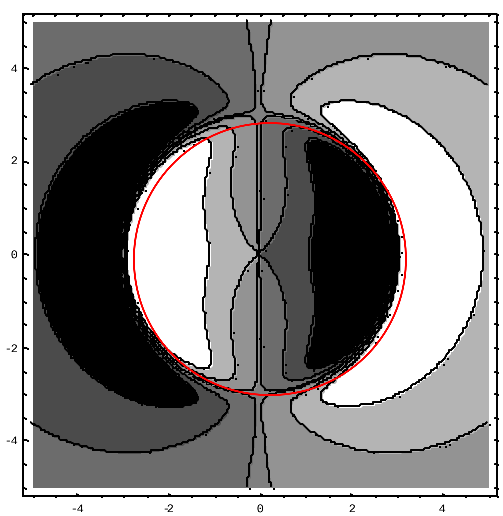
Stream lines for "classical" slab geometry with symmetric current profile ($dJ^{(0)}/dx = 0$ on the resonance surface)

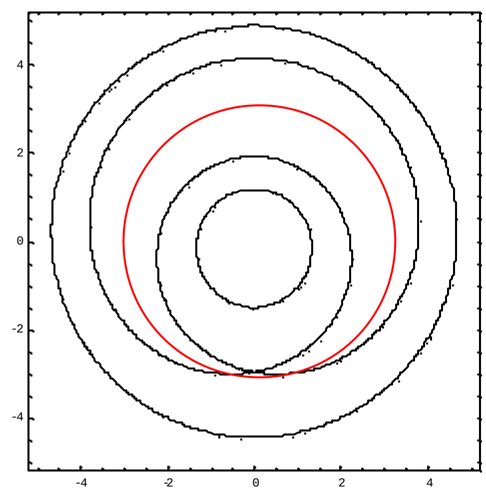
Effect of $dJ^{(0)}/dx \neq 0$ (plasma flows through the rational surface)

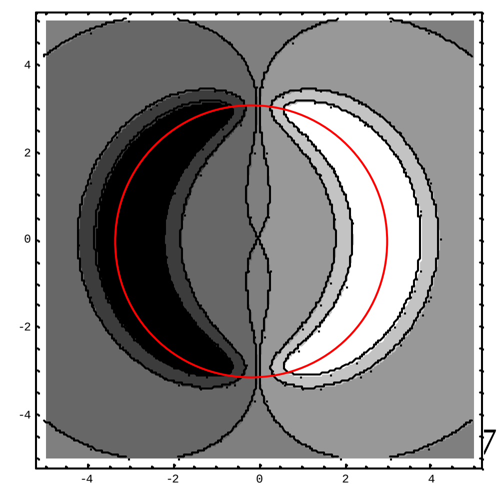




Equivalent flow patterns in cylindrical geometry







Low and intermediate collisionality requires extension of Braginskii approach to weakly collisional case

Braginskii equations for 2-fluid MHD:

$$\frac{3}{2} n_i \frac{dkT_i}{dt} + p_i \nabla \cdot \mathbf{v} = -\nabla \cdot \mathbf{q_i} - \boxed{\Pi : \nabla \mathbf{v}} + \mathbf{Q}$$

reversible adiabatic heating

losses

irreversible viscous heating

e-i collisions

Viscous dissipation rate = $-\pi_{ik} \frac{\partial v_i}{\partial x_k}$ → $\mathbf{v} = c\, \mathbf{E} \times \mathbf{B}^{(0)} / B^{(0)2}$

Absorption rate in terms of anti-hermitian part of dielectric tensor

$$P_{\text{abs}} = \frac{i\omega}{8\pi} \varepsilon^a_{ij} E_i E^*_j$$

Dielectric tensor in weakly collisional magnetized plasma

- Response to electric field $E_{x,y} = E_{x,y}^{(0)} \exp(-i\omega t + ik_x x + ik_z z)$ in uniform $\mathbf{B} = (0,0,B)$
- Linearized kinetic equation with Landau collision operator
- Approximation of weak collisions, $\nu_{ii} \ll \omega$ (opposite to Braginskii)

$$\delta f_\alpha = \delta f_\alpha^{(0)} + \delta f_\alpha^{(1)}, \text{ where}$$

$$-i(\omega - \boldsymbol{k}\boldsymbol{v})\delta f_\alpha^{(0)} - \omega_{c\alpha}\frac{\partial \delta f_\alpha^{(0)}}{\partial \varphi} + e_\alpha \boldsymbol{E}\frac{\partial f_{M\alpha}}{\partial \boldsymbol{p}} = 0,$$

collisionless response

$$-i(\omega - \boldsymbol{k}\boldsymbol{v})\delta f_\alpha^{(1)} - \omega_{c\alpha}\frac{\partial \delta f_\alpha^{(1)}}{\partial \varphi} =$$

$$\frac{\partial}{\partial p_i}\sum_\beta \int d\boldsymbol{p}_\beta I_{ij}^{\alpha\beta}\left(\frac{\partial f_{M\alpha}}{\partial p_{\alpha j}}\delta f_\beta^{(0)} + \frac{\partial \delta f_\alpha^{(0)}}{\partial p_{\alpha j}} f_{M\beta} - \frac{\partial \delta f_\beta^{(0)}}{\partial p_{\beta j}} f_{M\alpha} - \frac{\partial f_{M\beta}}{\partial p_{\beta j}}\delta f_\alpha^{(0)}\right)$$

effect of rare collisions

Strongly and weakly collisional regimes: (a) parallel viscosity

- (a) electric field $\mathbf{E} = E_y \mathbf{e}_y \longrightarrow$ compressible flow $v_x = (c E_y / B) \exp(-i \omega t + i k_x x)$,

  anti-hermitian (resistive) part of j_y is due to parallel viscosity η_0

$$-i\rho\omega v_x = j_y B/c - \frac{\eta_0}{3}\frac{\partial^2 v_x}{\partial x^2}, \qquad \eta_0 = \frac{0.96 n T_i}{\nu_{ii}}$$

$$\delta\varepsilon^a_{yy} = 0.32\frac{i\omega_{pi}^2}{\omega\nu_{ii}}\left(\frac{k_\perp v_{Ti}}{\omega_{ci}}\right)^2, \quad \omega \ll \nu_{ii}$$

from Braginskii equations
(strong collisions)

$$\delta\varepsilon^a_{yy} = i\frac{4}{5}\frac{\omega_{pi}^2}{\omega^2}\frac{\nu_{ii}}{\omega}\left(\frac{k_\perp v_{Ti}}{\omega_{ci}}\right)^2, \quad \omega \gg \nu_{ii}$$

from dielectric tensor
(weak collisions)

- absorption vanishes at large and small collisional frequencies.

- heating due to parallel viscosity can be interpreted as collisional magnetic pumping (gyro- relaxation heating) in a time varying guide magnetic field $B_z = B + (kc / \omega) E_y$

Gyro-relaxation model with constant collision frequency

- time varying guide magnetic field, $B(t) = B\,(1 + a \cos \omega t)$, $a = k_\perp c E_y / \omega B \ll 1$
- perpendicular thermal energy $T_\perp$: conservation of the magnetic moment $T_\perp / B(t)$ and relaxation between $T_\perp$ and $T_{||}$ due to the collisions

$$\frac{dT_\perp}{dt} = \frac{T_\perp}{B}\frac{dB}{dt} - \nu_{ii}\left(\frac{T_\perp}{2} - T_{||}\right)$$

$$\frac{dT_{||}}{dt} = \nu_{ii}\left(\frac{T_\perp}{2} - T_{||}\right)$$

- mean (irreversible) increase of total thermal energy, $T = T_\perp + T_{||}$

$$\frac{dT}{dt} = \frac{(1/6)a^2\omega^2\nu_{ii}T}{\omega^2 + (9/4)\nu_{ii}^2} \quad \longrightarrow \quad \delta\epsilon^a_{yy} = \frac{(\nu_{ii}/2\omega)\ \omega_{pi}^2}{\omega^2 + (9/4)\nu_{ii}^2}\left(\frac{k_\perp v_{Ti}}{\omega_{ci}}\right)^2$$

- maximum of the heating rate, $dT/dt = (\omega/18)\,(\delta B/B)^2\, T$, $(\nu_{ii} = 2\,\omega/3)$
- gyro-relaxation heating is not important in MST

$$\Delta T/T \simeq (1/3)\,(\delta B/B)^2 \simeq 10^{-4}$$

Ion heating caused by parallel viscosity is small in the reconnection layer

- formal comparison of $\delta\varepsilon^a_{yy}$ and $\delta\varepsilon^a_{xx}$ shows that "parallel viscosity" is dominant

$$\frac{\delta\epsilon^a_{yy}}{\delta\epsilon^a_{xx}} \simeq \frac{\omega_{ci}^2}{\omega^2 + (9/4)\nu_{ii}^2} \gg 1$$

$\simeq 10^8$ in MST

- its effect on heating is reduced in the reconnection layer due to $E_y << E_x$

- in potential electric field: (1) $\delta B / B \to 0$, (2) plasma flow becomes incompressible

- ion heating due to parallel Braginskii viscosity vanishes for 2D incompressible flow

$$P_{abs} = \frac{\eta_0}{3}\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} - 2\frac{\partial v_z}{\partial z}\right)^2 \longrightarrow 0$$

Strongly and weakly collisional regimes: (b) perpendicular viscosity

- from Braginskii equations (collisional case) :

  (b) polarization $\mathbf{E} = E_x \mathbf{e}_x$, shear flow $v_y = -(c E_x / B) \exp(-i \omega t + i k_x x)$

$$-i\rho\omega v_y = -j_x B/c - \eta_1 \frac{\partial^2 v_y}{\partial x^2}, \qquad \eta_1 = \frac{3nT_i \nu_{ii}}{10\ \omega_{ci}^2}$$

$\delta\varepsilon^a_{xx}$ is determined by the perpendicular viscosity η_1

$$\delta\varepsilon^a_{xx} = i\frac{3}{10}\frac{\omega_{pi}^2}{\omega_{ci}^2}\frac{\nu_{ii}}{\omega}\left(\frac{k_\perp v_{Ti}}{\omega_{ci}}\right)^2, \omega \ll \nu_{ii}$$

anti-hermitian (resistive) part ε^a_{ik} due to viscous friction of the sheared flows

$$\delta\varepsilon^a_{xx} = i\frac{2}{10}\frac{\omega_{pi}^2}{\omega_{ci}^2}\frac{\nu_{ii}}{\omega}\left(\frac{k_\perp v_{Ti}}{\omega_{ci}}\right)^2 \quad, \omega \gg \nu_{ii}$$

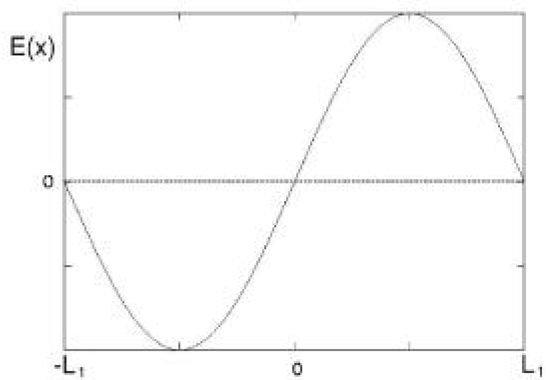
from dielectric tensor (weakly collisional case)

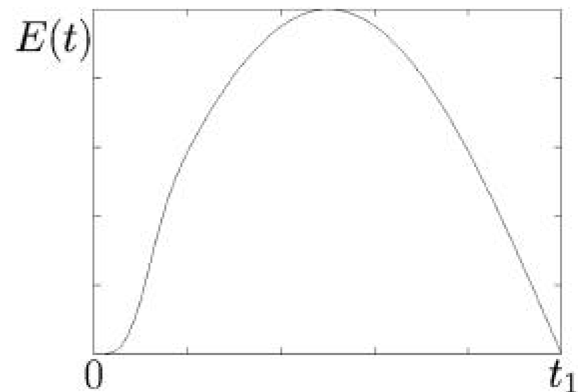
ε^a_{xx} is proportional to ν_{ii} in a wide range of collision frequencies , $\nu_{ii} \ll \omega_{ci}$

Numerical modeling of the kinetic equation: (b) perpendicular viscosity

- kinetic equation with Landay ion-ion collision operator in a driven electric field
- uniform magnetic field $\mathbf{B} = B\,\mathbf{e}_z$, $E_x(t, x)$, $-L_1 \leq x \leq L_1$, y, z - uniform
- electric field:

$$E_x(t,x) = E(t)\sin\left(\frac{\pi x}{L_1}\right),\ E(t=0)=0 \qquad E(t) = E_0\left\{1-\exp\left[-(t/\triangle t)^3\right]\right\}\sin(\pi t/t_1)$$



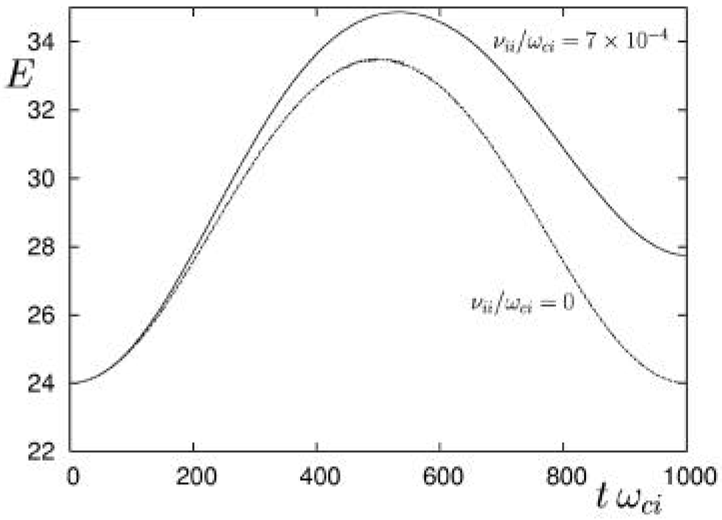


- solution method: Fourier transformation in the box

$$f(t,x,v_x,v_y,v_z) =$$

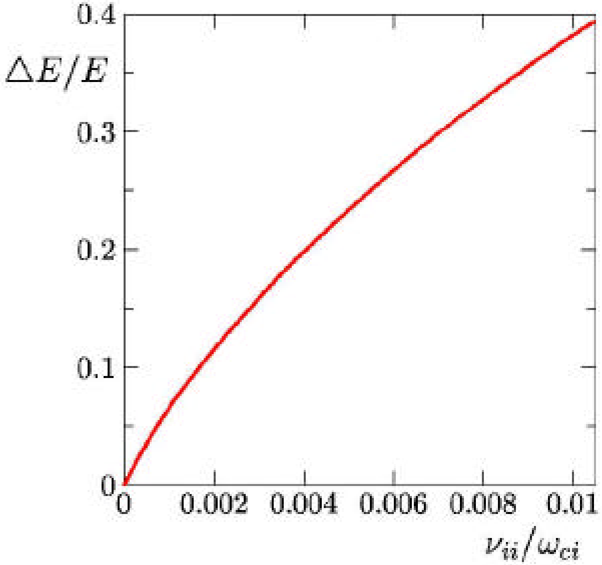
$$\sum_{n=-n_1}^{n_1}\ \sum_{m_x,m_y,m_z=-m_1}^{m_1} f_{n,m_x,m_y,m_z}(t)\exp\left(\frac{in\pi x}{L_1}+\frac{im_x\pi v_x}{v_1}+\frac{im_y\pi v_y}{v_1}+\frac{im_z\pi v_z}{v_1}\right)$$

Kinetic energy of plasma flow is irreversibly transferred to ions

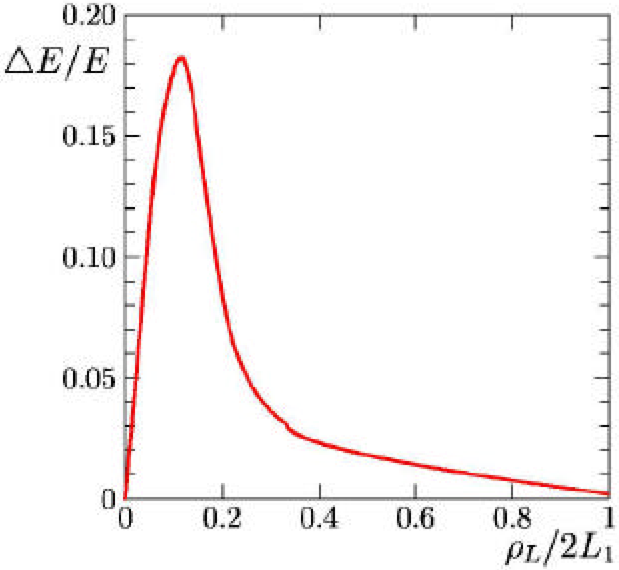


- plasma energy vs. time:
  (a) $\nu_{ii} / \omega_{ci} = 7\times10^{-4}$,
  (b) pure collisionless case $\nu_{ii} / \omega_{ci} = 0$

- plasma energy gain vs. collision rate



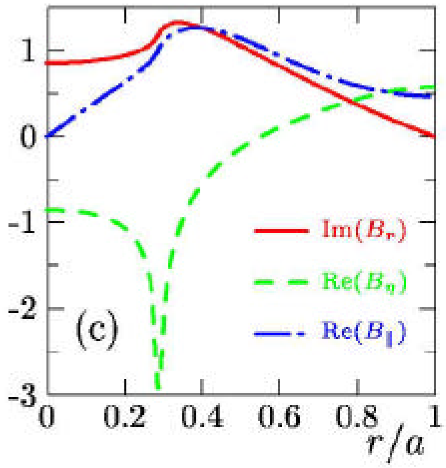
- plasma energy gain vs. k r_L



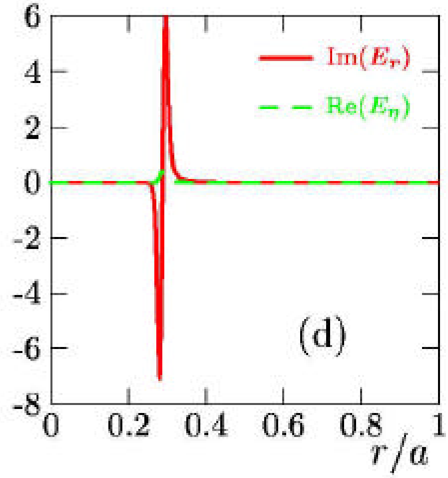
Ion heating by collisional damping of core tearing mode in MST

- field components of the core tearing mode m/n = 1/6

magnetic field



radial electric field



- $S = 10^5$, $\gamma \tau_A = 6.4 \times 10^{-4}$
- relative temperature increase due to "perpendicular viscosity" $\delta\varepsilon^a_{xx}$

$$\frac{\triangle T}{T} = \frac{\triangle t \ \nu_{ii}}{\beta} \frac{2}{10} \frac{k_\perp^2 v_{Ti}^2}{\omega_{ci}^2} \left|\tilde{E}_x\right|^2$$

Perpendicular viscosity is important for ion heating in MST

- $\triangle t = 10^{-4}$ sec, $T_i = 200$ eV, $n_i = 10^{13}$ cm^{-3} , $k_\perp v_{Ti} / \omega_{ci} = 2$, $\nu_{ii} = 3 \cdot 10^3$ sec^{-1} , $\delta B_{||} / B = 0.02$, $\triangle T_i / T_i \approx 60$

- Numerical analysis for linear tearing mode, $E_r \simeq 2.5 \times 10^3$ V / cm, $v_\theta \simeq 0.4\ v_A$

- $\Delta T_i / T_i \simeq 1$ can be achieved at smaller $E_r \simeq 2 \times 10^2$ V / cm and $v_\theta \simeq 0.08\ v_A$

- More accurate estimates of E_r are requires for reliable conclusions

Summary → future plans

Using dielectric tensor we generalized Braginskii's approach to the case of rare ion-ion collisions

Numerical model based ion kinetic equation yields dissipated power in a wide range of temporal and spatial scales of the problem

Ion-ion collisions are adequate for ion heating during magnetic reconnection in the systems with relatively weak guide magnetic field

Effect of compressibility and parallel viscosity in a weakly collisional case

Role of parallel flows generated during reconnection events