Effect of perpendicular plasma response on ion heating in RFPs

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Outline

- Previous results: collisional dielectric tensor with FLR terms is applied for the estimation of the absorbed power. Field amplitude and spatial scale is found from linear eigenmode.

- Nonlinear 2-D resistive MHD modeling with zero viscosity for validation of electric field (velocity) amplitudes and spatial scales.

- Modeling of absorption based on numerical solution of kinetic equation with Landau collision operator in a driven perpendicular electric field.

- Possible interpretation of localized plasma flow as ion heating for $t < \upsilon_{ii}^{-1}$ if measurement is averaged over several Larmor radii.

- Absorbed power $P_{\text{abs}} = \frac{i\omega}{8\pi} \varepsilon_{ij}^{a} E_i E_j^*$

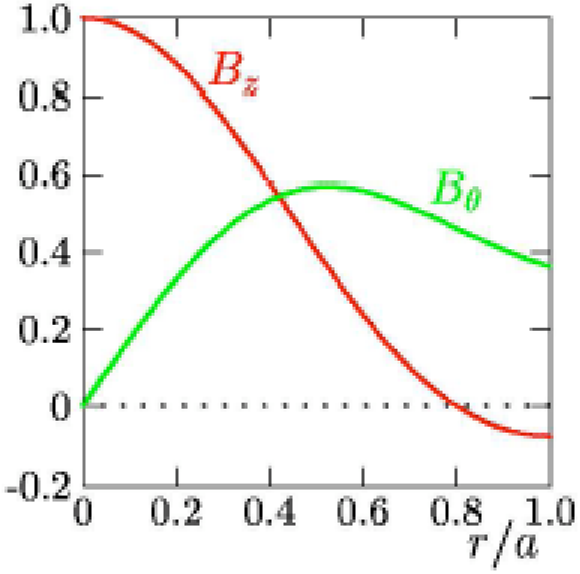
For $k_\perp \rho_L < 1,\ \omega \ll \omega_{ci},\ \omega_{c\alpha}/k_\parallel >> v_{T\alpha}$

$$P_{\text{abs}} = \frac{\omega}{8\pi}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left\{\frac{2}{10}\frac{k_\perp^2 v_{Ti}^2}{\omega_{ci}^2}|E_x|^2 + \frac{4}{5}\frac{\omega_{ci}^2}{\omega^2}\left|\frac{k_\perp v_{Ti}}{\omega_{ci}}E_y + \frac{ik_\parallel v_{Ti}}{\omega}E_z\right|^2 + \frac{4}{5}\frac{\omega_{ci}^2}{\omega^2}\frac{k_\parallel^2 v_{Ti}^2}{\omega^2}|E_z|^2\right\}\frac{\nu_{ii}}{\omega}$$

$$P_{\text{abs}} \propto \eta_\perp \left(\frac{\partial v_y}{\partial x}\right)^2$$

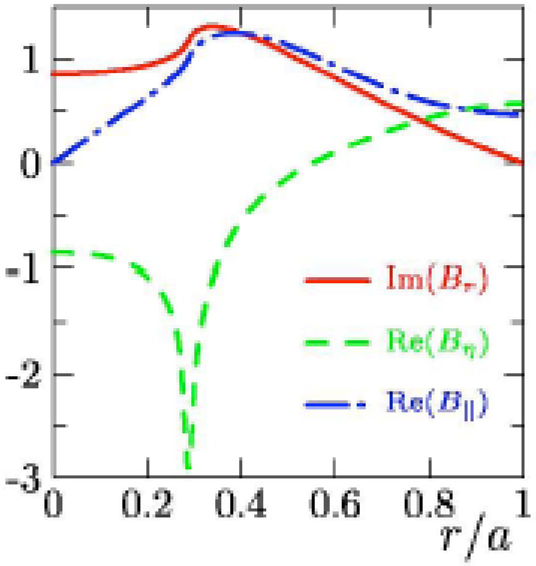
Estimation of E-field from linear eigenmode

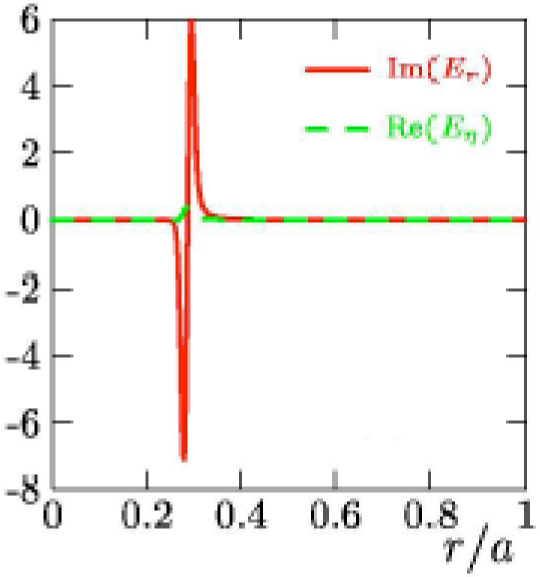
- Force free equilibrium



- Field components of the tearing mode

$S = 10^5$





Ion heating based on dielectric tensor

- $\frac{\Delta T}{T} = \frac{\Delta t \omega}{\beta} \frac{2}{10} \frac{k_\perp^2 v_{Ti}^2}{\omega_{ci}^2} \left|\tilde{E}_x\right|^2 \frac{\nu_{ii}}{\omega}$
- Let $\Delta t = 10^{-4}\,\mathrm{sec},\ T_i = 100\,\mathrm{eV},\ n_i = 10^{13}\mathrm{cm}^{-3},\ k_\perp v_{Ti}/\omega_{ci} = 2,$ $\nu_{ii} = 7 \cdot 10^3 \mathrm{sec}^{-1},\ \delta B_\| / B_0 = 0.02,$ then $\Delta T/T \approx 60$
- This corresponds to $E_r = 2.5 \cdot 10^3 \mathrm{V/cm}$ and $v_\theta = 0.4 v_A$
- $\Delta T/T \sim 1$ is achieved for smaller (more reasonable) field $E_r \approx 3 \bullet 10^2\,\mathrm{V/cm}$ and $v_\theta \approx 0.1 v_\mathrm{A}$
- Heating requires: gyroradius scale flow with amplitude of the order of ion thermal velocity and ion-ion collision time scale.

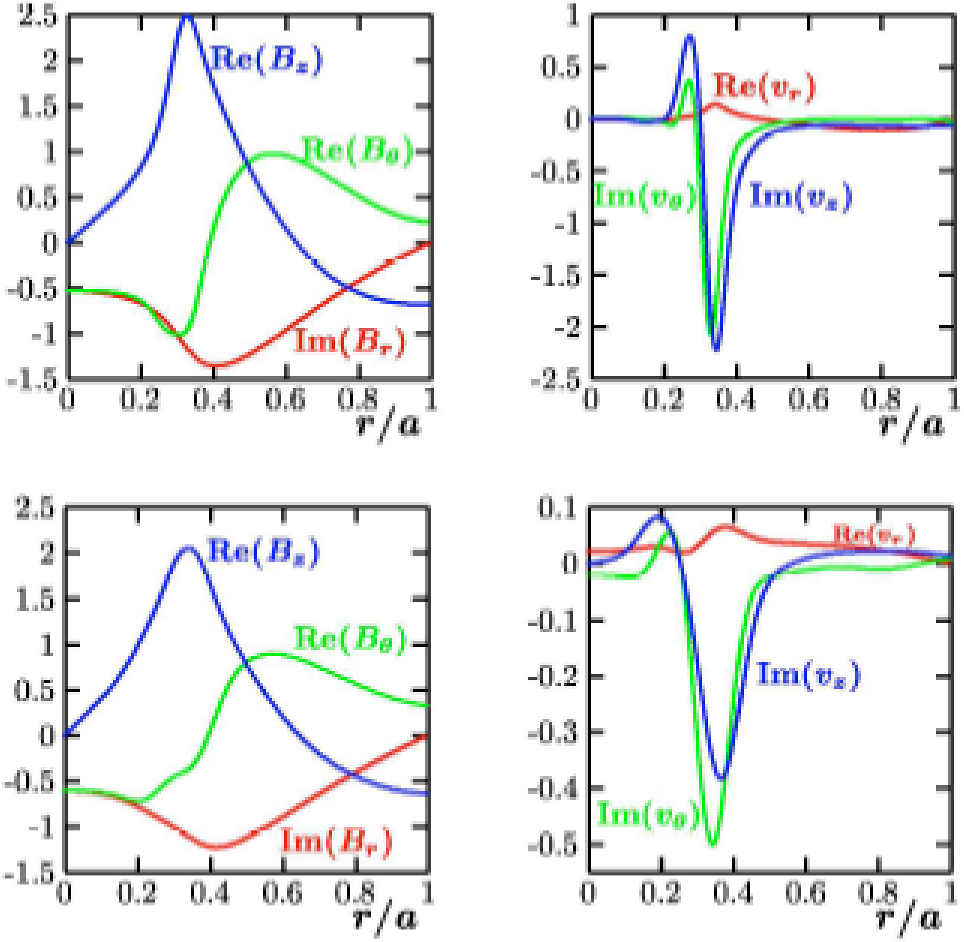
Role of viscosity in numerical modeling

- Effect of viscosity on linear eigenmode

$$\tilde{\eta} = \frac{\eta_0}{\rho_0 a v_A}, \qquad S = 10^4$$

$$\tilde{\eta} = 10^{-6}$$

$$\tilde{\eta} = 10^{-4}$$



- Viscosity widens tearing eigenfunctions and reduces relative amplitudes of velocity components
- Key problem – modeling with realistic perpendicular viscosity

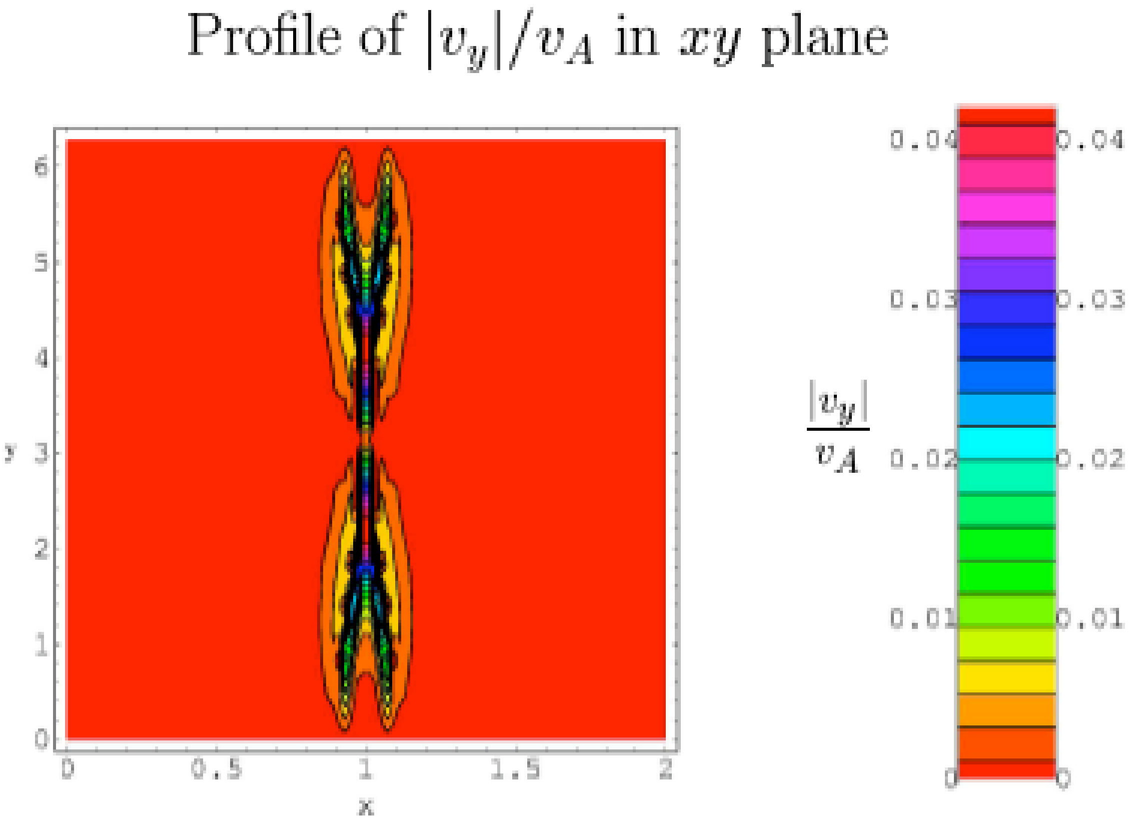
Estimation of E, v from nonlinear simulation

- Plane geometry, periodic in y

- Fourier expansion in x,y

$$B(x,y) = \sum_{l,m} B_{l,m} e^{il\frac{\pi}{L_x}x} \cdot e^{im\frac{2\pi}{Ly}y}$$

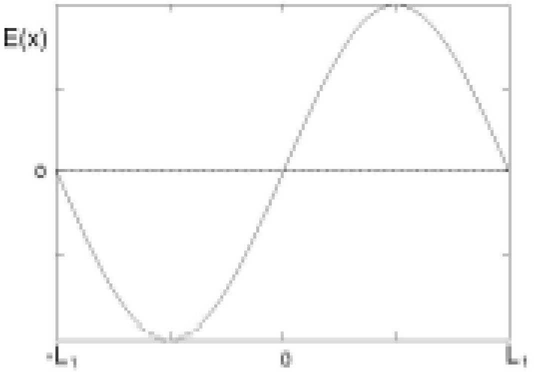
- Zero viscosity does not influence numerical stability

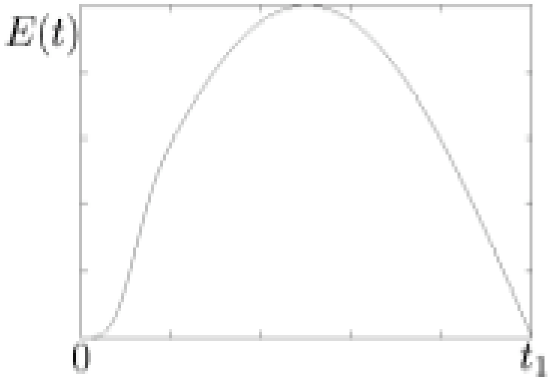


- Magnitude of v_y is comparable to the ion thermal velocity. v_y is strongly localized near the resonance.
- Amplitude of localized ion flow is comparable to one used in the previous analysis.

Solution of kinetic equation in driven electric field

$$\boldsymbol{B} = B_0\boldsymbol{e_z},\ E_x(t,x)$$



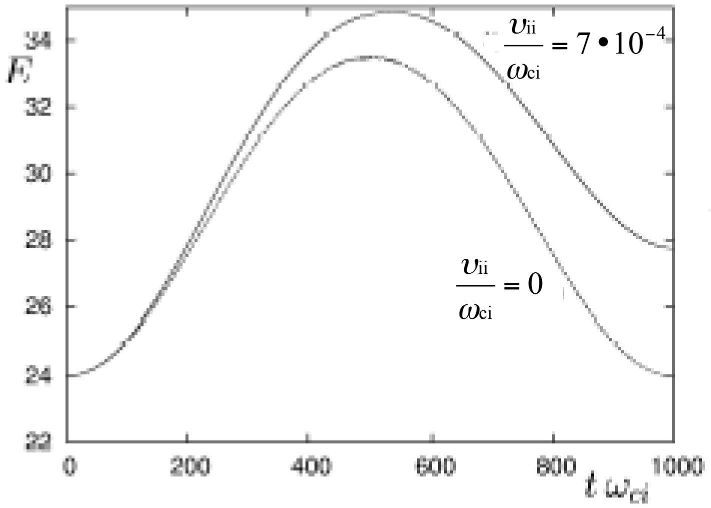


$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + E(t)\sin\left(\frac{\pi x}{L_1}\right)\frac{\partial f}{\partial v_x} + v_y\frac{\partial f}{\partial v_x} - v_x\frac{\partial f}{\partial v_y} = \frac{3\sqrt{\pi}}{2}\frac{\nu_{ii}}{\omega_{ci}}\frac{\partial}{\partial v_i}\int d\boldsymbol{v'}\frac{u^2\delta_{ij}-u_iu_j}{u^3}\left[f(\boldsymbol{v'})\frac{\partial f}{\partial v_j} - f(\boldsymbol{v})\frac{\partial f}{\partial v'_j}\right]$$

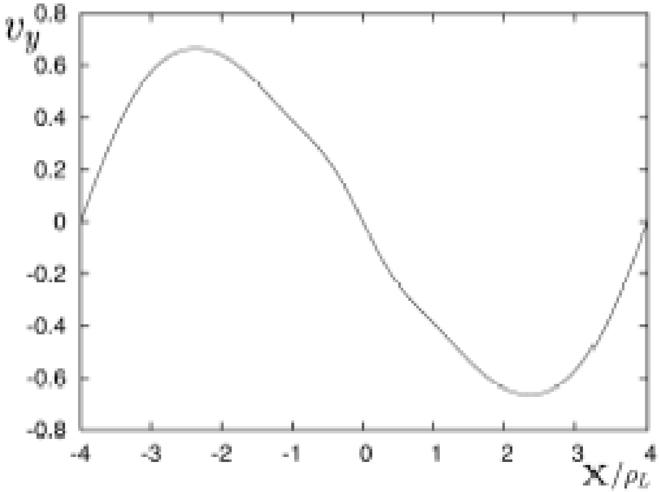
Plasma energy vs. time

$$E_0 = 4\times 10^2\mathrm{V/cm}$$
$$L_1/\rho_L = 4$$

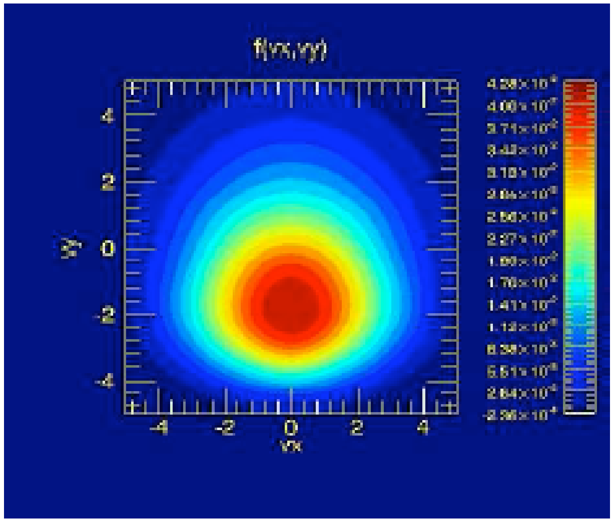
- Heating is reduced



- For intermediate t v_y is a function of x

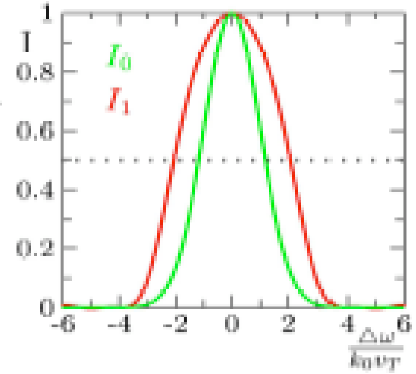


Bulk velocity component v_y vs. x at $t\omega_{ci} = 500$



$x/\rho_L = 2,\, v_z = 0$

- Such localized plasma flow could be interpreted as ion heating when measurement is averaged over several Larmor radii



$$E_0 = 4 \times 10^2 \mathrm{V/cm}$$

$$\frac{T_1}{T_0} = 3.1$$

Conclusions

- On ion-ion collision time scale perpendicular viscosity is important only for strong localized flows.
- Due to lack of measurements of such flows in RFPs the role of viscosity in ion heating is not clarified.
- On shorter time scales (which are also studied in astrophysical applications) ion-ion collisions are not important.