Viscous damping of tearing fluctuations: theory

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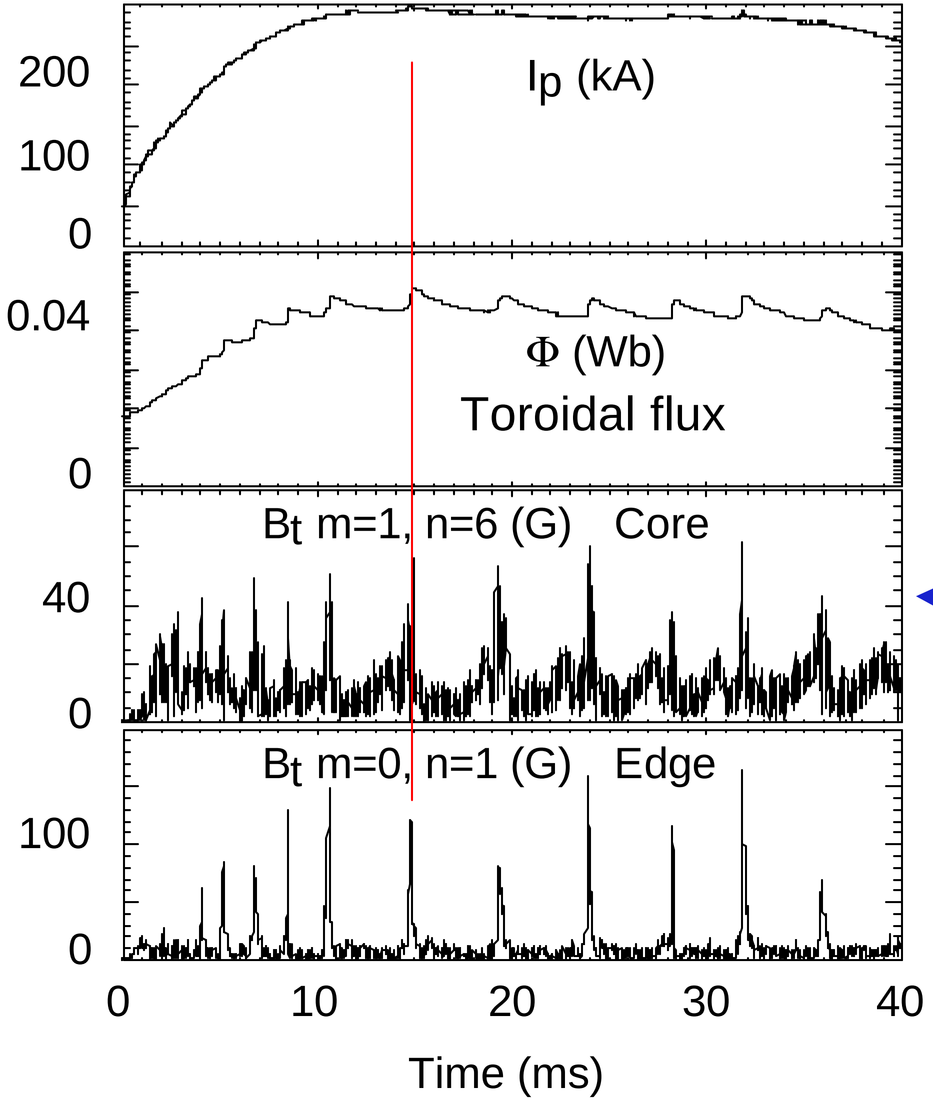
Key physical mechanisms

- Ion heating due to large scale tearing modes:
  - Magnetic field is a source of free energy
  - Generation of cross-field plasma flows
  - Collisional mechanism of energy transfer into heat

- Main problem: ion-ion collisional time is 10 times longer than the heating time
  - Dielectric tensor at weak ion-ion collisions
  - Effect of parallel viscosity on cross-field flows (temporal dynamics)

- Fast parallel flows are accelerated by mean inductive electric field
  - Magnitude and temporal dependence $E_{||}$
  - Damping of parallel flow due to parallel viscosity (spatial dynamics)

Discrete sawtooth events caused by tearing modes (magnetic reconnection)

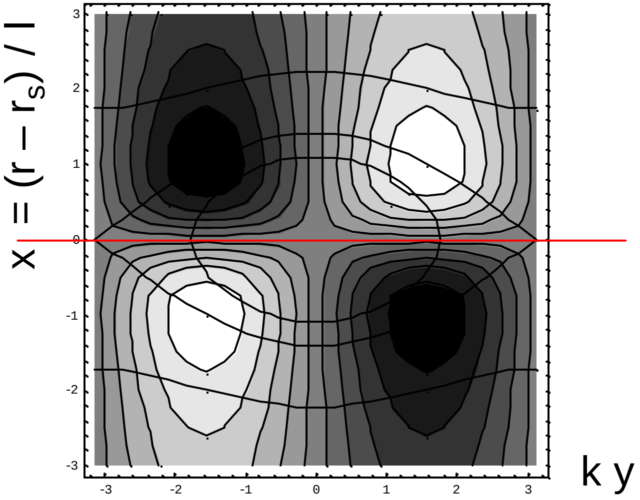


Crashes,
sawteeth,
reconnections,
discrete events...

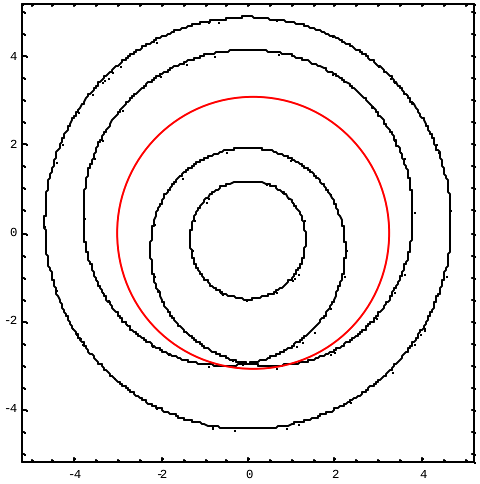
← Magnetic field fluctuations
$\delta B/B \simeq 2\%$ (toroidal component)

Incompressible cross-field flows are driven by potential electric field

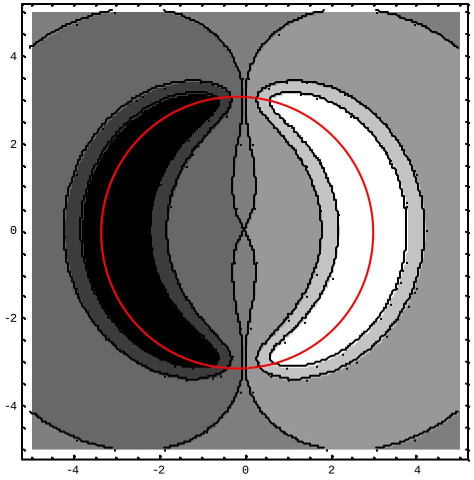
Stream lines for “classical” tearing mode in a slab geometry ($dJ^{(0)}/dx = 0$ on the resonance surface)



Magnetic island in a cylindrical geometry



Cylindrical flow pattern



Viscous damping of cross-field flows

$n = 10^{13}$ cm^{-3}, $T_i = 200$ eV, $\Delta T_i = 200$ eV

reconnection time (sawtooth crash) $\leq 100\ \mu$sec $<< \tau_{ii} = 10^{-3}$ sec

viscous mechanism is modified for weak ion-ion collisions

↓

perpendicular kinematic viscosity $\nu_\perp = 0.9 \times 10^3$ cm^2 / sec

viscous relaxation time is longer than the reconnection time

How is energy delivered
and absorbed by the ions?

→ large parallel viscosity $\nu_\parallel / \nu_\perp \simeq 10^8$?

Effect of large parallel viscosity is not significant for cross-field flows in MST

- electric field $\mathbf{E} = E_y \mathbf{e}_y$, → compressible flow $v_x = (c\,E_y / B) \exp(-i\,\omega\, t + i\, k_x\, x)$
  with anti-hermitian (resistive) current j_y resulting from parallel viscosity

$$-i\rho\omega v_x = j_y B/c - \frac{\eta_0}{3}\frac{\partial^2 v_x}{\partial x^2}, \qquad \eta_0 = \frac{0.96 n T_i}{\nu_{ii}}$$

$$\delta\varepsilon^a_{yy} = 0.32\frac{i\omega_{pi}^2}{\omega\nu_{ii}}\left(\frac{k_\perp v_{Ti}}{\omega_{ci}}\right)^2, \quad \omega \ll \nu_{ii}$$

from Braginskii equations (strong collisions)

$$\delta\varepsilon^a_{yy} = i\frac{4\,\omega_{pi}^2\,\nu_{ii}}{5\,\omega^2\;\omega}\left(\frac{k_\perp v_{Ti}}{\omega_{ci}}\right)^2, \quad \omega \gg \nu_{ii}$$

from dielectric tensor (weak collisions)

absorption vanishes at large and small collisional frequencies

gyro-relaxation heating in time varying guide magnetic field $B_z = B + (kc/\omega)\,E_y$

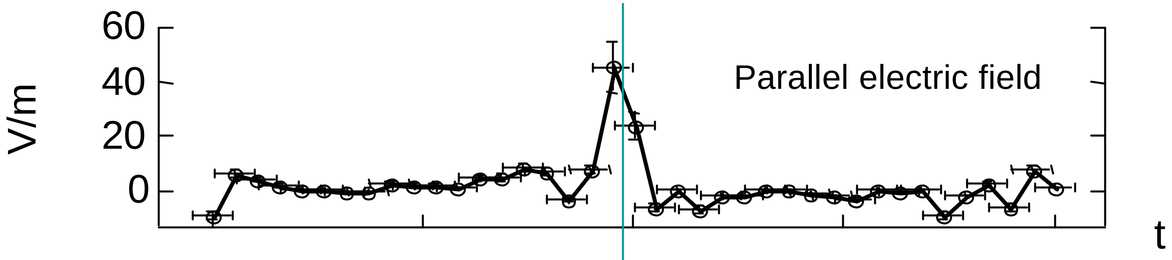
maximum of the heating rate $dT/dt = (\omega/18)\,(\delta B/B)^2\,T$ at $\nu_{ii} = 2\,\omega/3$

not effective in MST: $\Delta T/T \simeq (1/3)\,(\delta B/B)^2 \simeq 10^{-4}$

Temporal profile of mean inductive electric field during sawtooth events

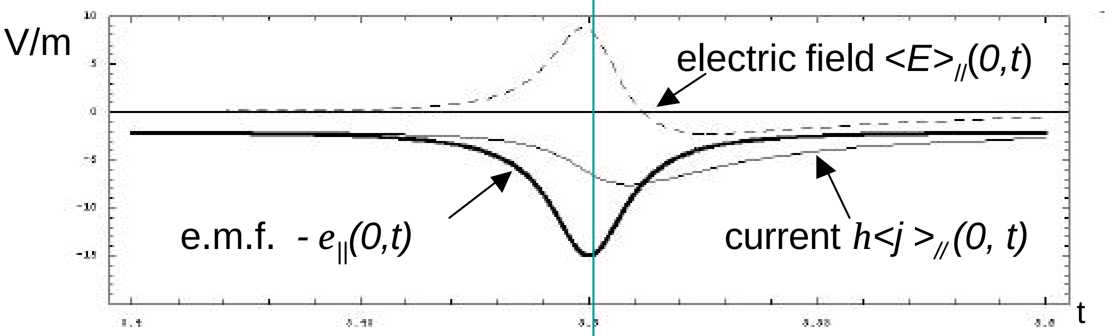
Experimental results from MST:

W. Ding et al.
PRL, **93**, 2004



sawtooth
ensemble

Numerical simulations:



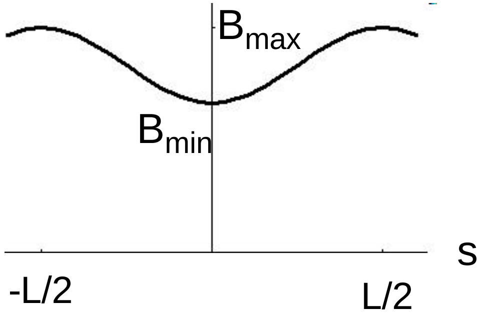
Mechanism of damping of parallel flows

- Neoclassical viscosity slows down parallel flows in spatially varying static magnetic field.

Model

- Parallel inductive electric field accelerates ions up to velocity u.
- Variations of **|B|** along field lines due to tearing perturbations or toroidal geometry of equilibrium magnetic field.

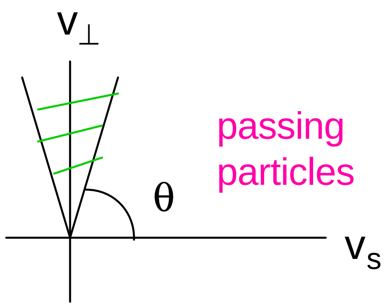
$$B(s) = B_{min} + (B_{max} - B_{min}) \sin^2 \frac{\pi s}{L}$$



- Energy transfers to heat due to friction between passing and trapped ions

$$\left|\frac{\pi}{2} - \theta\right| \le \left(\frac{\Delta B}{B}\right)^{1/2} \simeq \frac{\Delta n}{n}$$

density of
trapped ions



$$\Delta B = B_{max} - B_{min}$$

Damping rate strongly depends on ion-ion mean free path

- Friction force (per unite volume)

$$F_{fr} = - n \nu_{eff} m_i u ,$$

ν_{eff} is effective frequency of momentum transfer between passing ions and magnetic field

Characteristic time of damping is ν_{eff}^{-1}. It depends on ion m.f.p λ, spatial scale L and amplitude $\Delta B/B << 1$ of |**B**| variations.

1) in strongly collisional case , ν_{eff} is determined by parallel Braginskii viscosity

$$\nu_{eff} \simeq \frac{v_{Ti}\lambda}{L^2}\left(\frac{\Delta B}{B}\right)^2 \qquad \lambda << L$$

2) at intermediate λ, when the bounce time of trapped ions is longer than the time of entrapping, friction force does not depend on λ

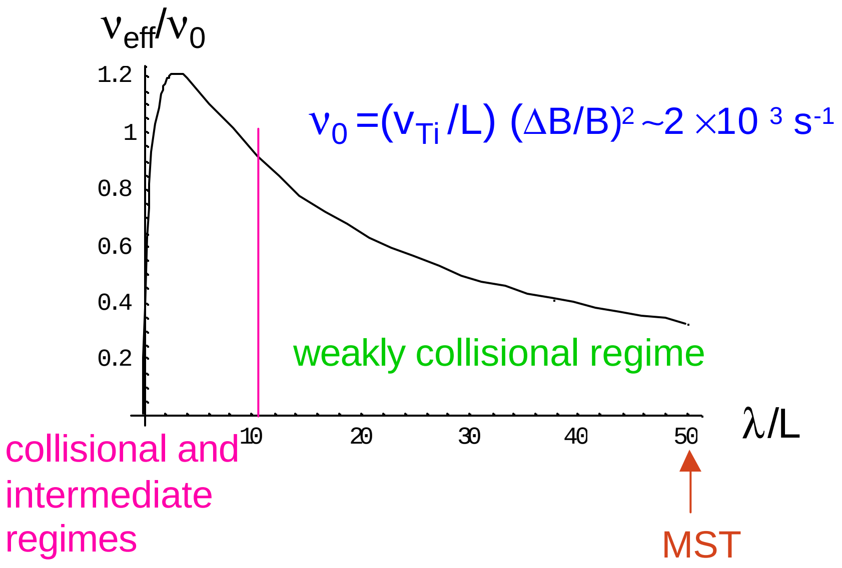
$$\nu_{eff} \simeq \frac{v_{Ti}}{L}\left(\frac{\Delta B}{B}\right)^2 \qquad L << \lambda << L(\Delta B/B)^{-3/2}$$

3) in weakly collisional case, the bounce time is much shorter than the lifetime of trapped ions,

$$\nu_{eff} \simeq \frac{v_{Ti}}{\lambda}\left(\frac{\Delta B}{B}\right)^{1/2} \qquad L(\Delta B/B)^{-3/2} << \lambda$$

Summary

- Inductive electric field accelerates ions up to the energy ≈ 0.5 KeV
- Friction force has a maximum at intermediate λ



$\nu_0 = (v_{Ti}/L)\,(\Delta B/B)^2 \sim 2\times10^{3}\ \mathrm{s}^{-1}$

- Taking L = 3 m, $(T_i/m)^{1/2} = 1.5\times10^{5}$ m/s, λ = 150 m, and ΔB/B ~20% shows that MST is in a weakly collisional regime with characteristic flow damping time ≈ 1 - 2 msec.
- Similar effects of particle trapping in multiple-mirror magnetic field are widely discussed in astrophysics (R.Kulsrud, L.Malishkin, B.Chandran). Verification of this concept in lab experiments could be very important.