Momentum transport in Reversed Field Pinches

Richard Fitzpatrick

Center for Magnetic Reconnection Studies
Institute for Fusion Studies
Department of Physics
University of Texas at Austin

Introduction

- Dominant angular momentum transport mechanism in RFPs is *electromagnetic stress* generated by *magnetic perturbations*.
- What is nature of magnetic perturbations?
- How do these perturbations interact so as to generate EM stresses, and hence to transport angular momentum?

MHD stability theory

- Model RFP equilibrium as periodic cylinder:

$$\mathbf{B} = [0, B_\theta(r), B_z(r)].$$

- Periodicity length in z-direction is $2\,\pi\,R_0$, where R_0 is "major radius".
- Consider magnetic perturbation with m periods in poloidal direction and n periods in "toroidal" direction.
- To very good approximation perturbation dynamics governed by balancing perturbed pressure gradient against perturbed $\mathbf{j}\wedge\mathbf{B}$ force: *i.e.*, plasma inertia, resistivity, viscosity, *etc.* negligibly small.

Newcomb's equation

- This approach yields *Newcomb's equation*:

  $$\frac{d}{dr}\left(f\,\frac{d\psi}{dr}\right) + g\,\psi = 0,$$

  where $\psi = r\,b_r$, and $f = f(r, B_\theta, B_z, m, n)$, $g = g(r, B_\theta, B_z, m, n)$.

- Boundary conditions: ψ well behaved at $r = 0$, and $\psi = 0$ at $r = r_w$, where r_w is radius of conducting wall surrounding plasma.

- Newcomb's equation *singular* (*i.e.*, $f = 0$) at *rational surface*, where

  $$m\,B_\theta - n\,\epsilon\,B_z = 0.$$

  Here, $\epsilon = r/R_0$.

Tearing modes

- Singularity at rational surface resolved by including inertia, resistivity, *etc.* in analysis, since these effects locally important.
- Stability governed by single parameter:

  $$\Delta' = \left[\frac{d\ln\psi}{d\ln r}\right]_{r_{s-}}^{r_{s+}}.$$

  Here, $\psi(r)$ is solution to Newcomb's equation which satisfies boundary conditions, and possesses *gradient discontinuity* at rational surface, $r = r_s$.
- Mode unstable when $\Delta' > 0$. Instability grows on resistive time-scale and saturates at $\sim 1\%$ level. Mode *reconnects* magnetic field in vicinity of rational surface to form chain of *magnetic islands*. This instability known as *tearing mode*.

Tearing modes in RFPs

- Two classes of unstable tearing modes in typical RFP.
- $m = 1$ modes resonant in plasma core. Maybe, five or six unstable modes.
- $m = 0$ modes resonant at *reversal surface* (where $B_z = 0$) close to plasma edge. Maybe, ten to twenty unstable modes.
- Both classes of unstable mode are *global* in nature: *i.e.*, their eigenfunctions extend over whole plasma.

Tearing mode rotation

- RFP plasmas rotate toroidally. Typical rotation velocity $\sim 10\,\mathrm{km/s}$.
- Tearing modes *co-rotate* with plasma at their *rational surfaces*.
- Hence, if toroidal angular velocity profile of plasma is $\Omega_z(r)$, then tearing mode resonant at r_s has toroidal angular frequency

$$V_z = n\,\Omega_z(r_s).$$

- Any effect which slows-down/speeds-up rotation of tearing mode will also slow-down/speed-up plasma rotation at rational surface.

Electromagnetic torques

- Possible to demonstrate that zero net electromagnetic torque associated with tearing mode in any region of plasma governed by Newcomb's equation.
- Follows that electromagnetic torques can only develop in plasma at locations where Newcomb's equation breaks down: *i.e.*, at rational surfaces associated with unstable tearing modes.

Plasma equation of motion

- Plasma toroidal equation of motion takes form:

$$r\,\rho\,\frac{\partial\Omega_z}{\partial t} - \frac{\partial}{\partial r}\left(r\,\mu\frac{\partial\Omega_z}{\partial r}\right) = \sum_n T_n\,\delta(r - r_n),$$

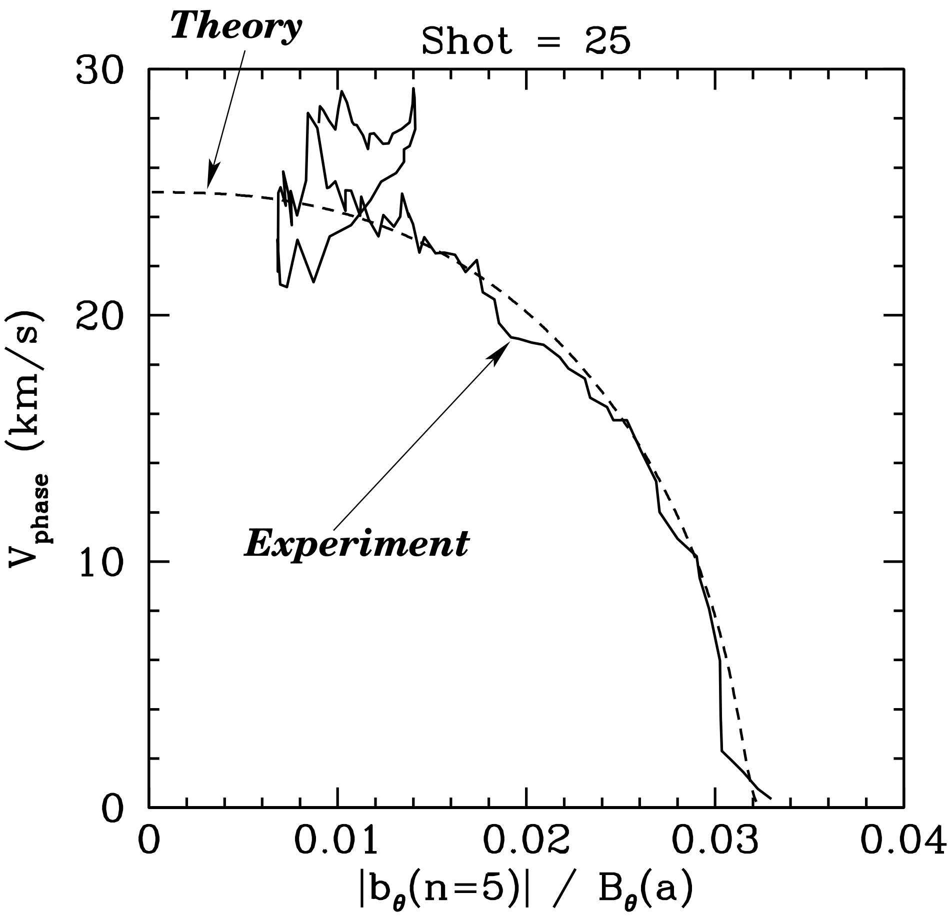
  where $\Omega_z(r)$ is toroidal angular velocity, $\rho(r)$ mass density, $\mu(r)$ viscosity, and the r_n are active rational surfaces.

- Viscosity $\mu(r)$ due to free-streaming along ergodic field-lines in plasma core, and short-wavelength electrostatic turbulence at plasma edge. Viscosity not known very exactly from theory, but can be measured experimentally.

Wall eddy current torques

- RFP plasmas surrounded by close-fitting conducting wall needed to suppress dangerous fast-growing instabilities.
- A rotating tearing mode in plasma induces eddy currents in wall. Eddy current phase *lags* that of tearing perturbation due to finite wall resistivity. Phase lag generates EM torque acting on wall.
- Equal and opposite torque acts on plasma at rational surface. Torque opposes mode rotation. Torque can be calculated from tearing mode eigenfunction and wall characteristics.
- Solve plasma equation of motion with calculated torque. Compare with experimental data.

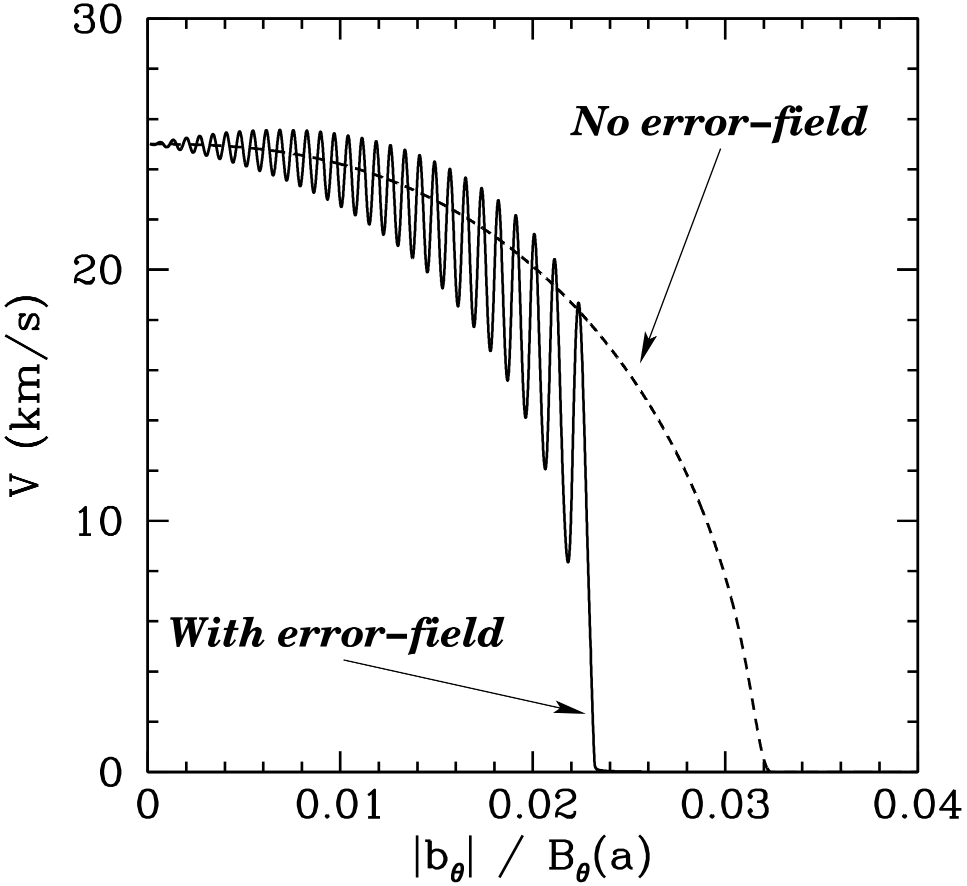
Eddy current rotation braking



Error-field torques

- Conducting wall possesses toroidal and poloidal *gaps* needed to allow magnetic flux from external field-coils to reach plasma.
- Gaps generate small magnetic perturbations, some of which have same helicity as active tearing modes in plasma. This type of perturbation known as *error-field*.
- An error-field resonant with given tearing mode exerts torque on rational surface proportional to $\sin(\varphi - \varphi_0)$, where φ is helical phase of tearing mode, and φ_0 is (fixed) error-field phase.
- Can calculate error-field torque from tearing eigenfunction and amplitude/phase of error-field. Can then solve plasma equation of motion.

Error-field rotation braking



Nonlinear coupling torques

- Tearing modes in RFP plasma of sufficiently high amplitude that substantial EM torques arise from nonlinear coupling.
- Two core tearing modes: the $1, n_1$ and $1, n_2$ modes, say: can couple with an edge tearing mode: the $0, n_3$ mode, say: provided that

$$n_1 - n_2 = n_3.$$

- EM torques generated at $1, n_1$, $1, n_2$, and $0, n_3$ rational surfaces take form $-n_1\,T$, $n_2\,T$, and $n_3\,T$, respectively. Constant T can be calculated from tearing mode eigenfunctions and equilibrium.
- Can write equation of motion of plasma under influence of nonlinear torques.

Nonlinear coupling torques

- Many tearing modes present in typical RFP plasma. Many mode triplets generating nonlinear torques. Equation of motion of plasma very complicated. Seek some simplifying concept.
- Consider model dissipative system:

$$m\,\frac{d^2x}{dt^2} + \nu\left(\frac{dx}{dt} - v_0\right) + F(x) = 0.$$

If conseravtive force, $F = -d\phi/dx$, large, and inertia, m, dissipation, ν, and drag v_0, small, then expect system to eventually settle down close to stable equilibrium point $[F(x) = 0$, $F'(x) > 0]$. If we are only interested in final state then we can just search for such points, rather than solving equation of motion.

Torque minimization principle

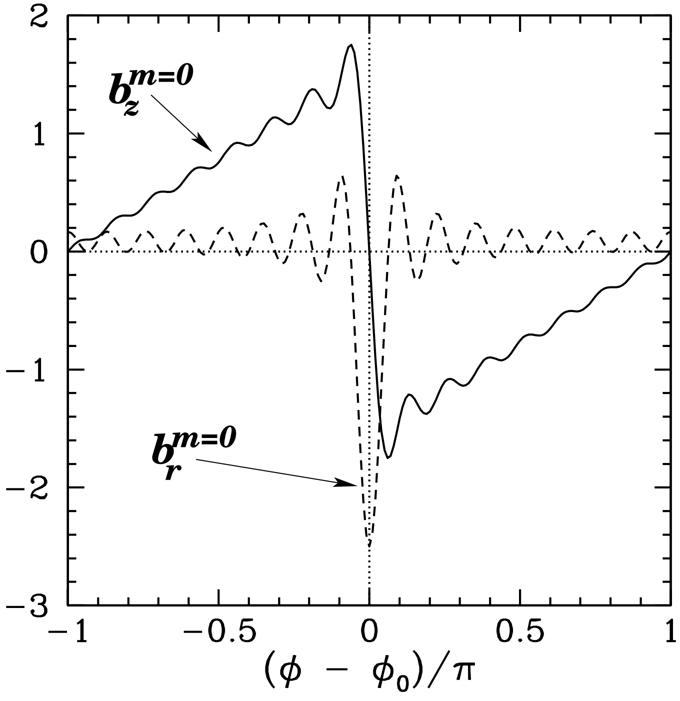
- RFP plasma is dissipative system in which nonlinear EM coupling torques significantly greater in magnitude than inertial and viscous terms in equation of motion.
- In accordance with previous discussion, expect plasma to find final state in which EM coupling torques *minimized*. This is achieved by adjusting *relative phases* of coupled modes. This, in turn, requires redistribution of angular momentum within plasma, since mode angular velocities related to angular velocity of plasma at rational surfaces.

$m = 0$ **mode locking rule**

- Principle yields following mode locking rule for $m = 0$ phases:

$$\varphi^{0,n} = n\,\phi_0 \pm \frac{\pi}{2}.$$

- Rule generates characteristic pattern in $m = 0$ field perturbation.



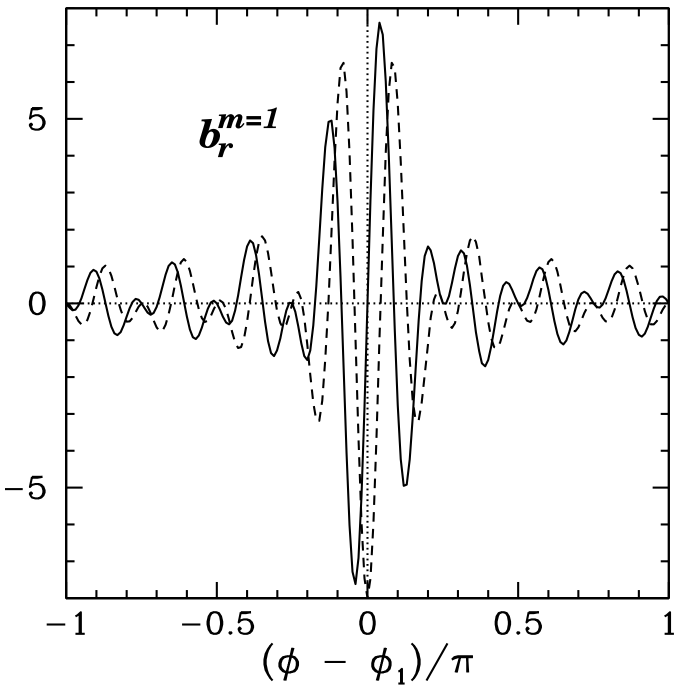
$m = 1$ mode locking rule

- Torque minimization principle yields following mode locking rule for $m = 1$ phases:

$$\varphi^{1,n} = n\,\phi_1 - \Delta_1,$$

  where ϕ_1 very close to ϕ_0, and Δ_1 arbitrary.

- Rule generates characteristic pattern in $m = 1$ field perturbation.



Slinky pattern

- Nonlinear coupling between $m=0$ and $m=1$ modes in RFP plasma generates characteristic *toroidally localized* pattern in perturbed radial field known as *slinky pattern*. Edge heat flux funneled through peak of slinky pattern, giving rise to "blow-torch" effect on surrounding wall. Effect far more dangerous if pattern is *non-rotating*.

- According to mode locking rules, slinky pattern rotates at angular rotation frequency of plasma at reversal surface.

- $m=1$ components of pattern generally rotate at much higher angular frequencies which are all different: *i.e.*, slinky pattern is merely *interference pattern* in perturbed magnetic field.

- In general, formation of slinky pattern does not require collapse in central plasma rotation.

Effect of error-fields on slinky pattern

- Suppose one of constituent $m = 1$ components of slinky pattern: the $1, n_l$ mode, say: is locked to an error-field: *i.e.*, its angular frequency is zero.
- Phase locking rules now yield

  $$\dot{\varphi}^{1,n} = (n - n_l)\,\dot{\varphi}_0,$$

  where $\dot{\varphi}_0$ is angular frequency of pattern. $m = 1$ modes with $n > n_l$ rotate in *same* direction as pattern, modes with $n < n_l$ rotate in *opposite* direction. All angular friquencies integer multiples of pattern frequency. Formation of pattern requires collapse in central rotation.
- If $0, 1$ mode also locked to error-field then plasma rotation at reversal surface halted. Slinky mode and *all* constituent modes stationary. Plasma rotation pinned to zero at all rational surfaces.

Conclusions

- Tearing mode theory able to account pretty well for electromagnetic momentum transport seen in RFP plasmas.
- Theory explains slowing down of mode/plasma rotation at very high mode amplitudes.
- Theory also explains characteristic patterns seen in $m = 0/m = 1$ fields due to nonlinear coupling.