Quasilinear theory of Hall dynamo driven by two-fluid tearing instability

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Outline

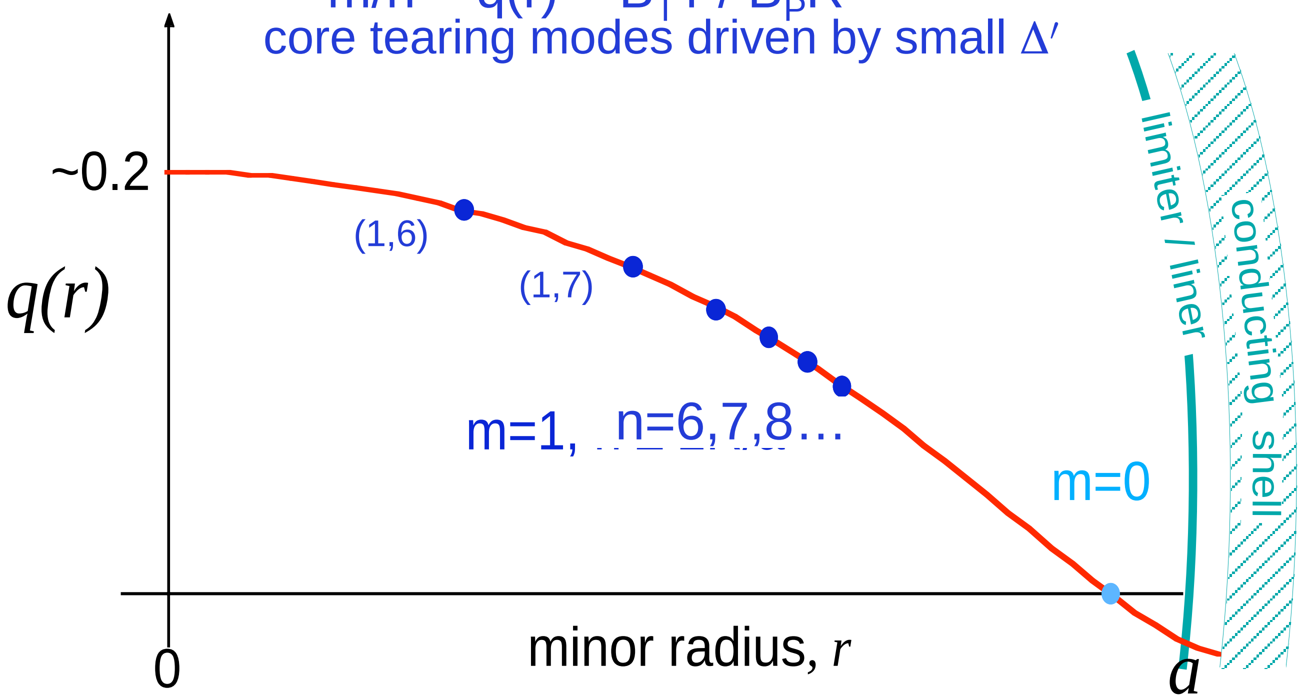
- Motivation
- Two-fluid equations
- Low frequency waves in uniform magnetic field
- Two-fluid tearing mode in cold plasma ($\beta = 0$)
- Hot plasma case
- Quasilinear two-fluid tearing mode dynamo
- Effect of dynamo on mean parallel current and electric field.

RFP equilibrium - resonant unstable modes exist

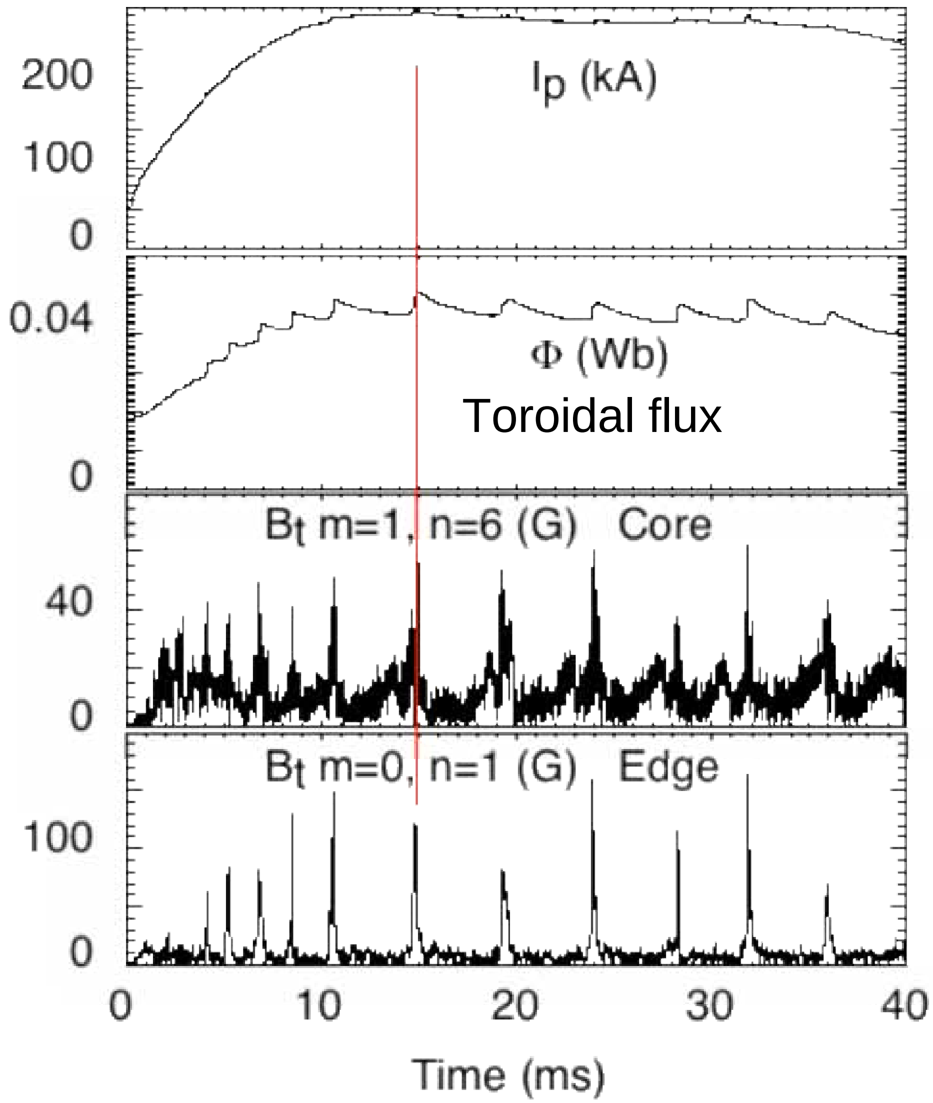
- RFP equilibrium - many resonant modes possible
- Stability depends on $J_{||}/B$ profile
- MHD tearing produces large magnetic fluctuations and flatten the parallel current (and momentum) profile.

MST Reversed Field Pinch

$m/n = q(r) = B_T r / B_P R$



Continuous fluctuations and discrete events drive dynamo



Crashes,
sawteeth,
reconnections,
discrete events...

Sawtooh ensemble - large number (hundreds) of sawteeth ensembled and aligned relative to the maximum rate of magnetic flux change

What balances Ohm's law in the RFP?

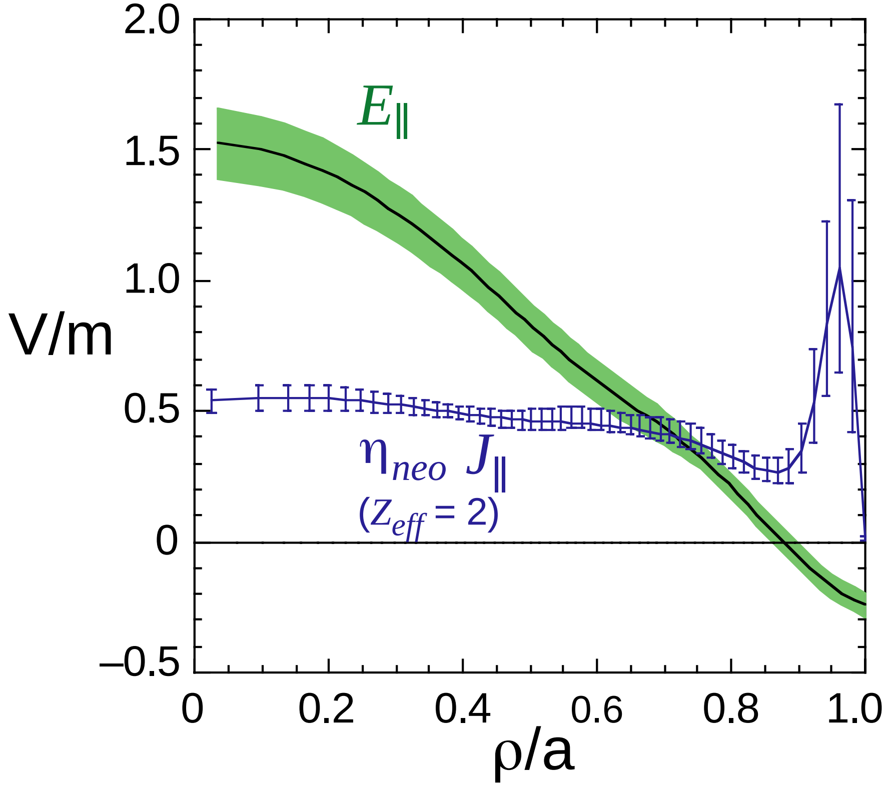
- Generalized Ohm's law:

$$\mathbf{E} - \eta \mathbf{J} =$$

$$-\mathbf{v} \times \mathbf{B} + \frac{1}{en} \mathbf{J} \times \mathbf{B} - \frac{1}{en} \nabla p_e + \frac{m_e}{e^2 n} \frac{\partial \mathbf{J}}{\partial t}$$

- Possible mechanisms:
  - MHD dynamo
  - Hall dynamo
  - Diamagnetic dynamo
  - Kinetic dynamo

Ohm's Law Imbalance in MST



Two-fluid theory is adequate for RFP

Motivations for two-fluid studies:

- High T, low guiding magnetic field $B^{(0)} \Rightarrow$ two fluid effects important
- $B^{(0)}$ of major interest, absent in many Hall-MHD theories
- Effects of β and ρ_s in two-fluid MHD without kinetic closures

Generalized Ohm's law

$$\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B} = \frac{1}{nec}\mathbf{j}\times\mathbf{B} - \frac{\nabla p_e}{ne} - \frac{m_e}{e}\frac{d\mathbf{v}_e}{dt} + \eta\,\mathbf{j}$$

MHD (alpha) dynamo | Hall term | electron inertia | resistivity

Major scales of two-fluid tearing instability:

- External scale L (shear length) $\approx$ 20 cm
- Ion skin depth $d_i = c / \omega_{pi} \approx$ 10 cm
- Ion-sound gyroradius $\rho_s = c_s / \omega_{ci} \approx$ 2 cm
- Combined collisionless and resistive electron skin-depth $\delta^2 = c^2/\omega_{pe}^2 + \eta c^2/4\pi\gamma \approx$ 5 mm

Quasilinear approach to mean field characteristics

Mean field Ohm's law:

$(1/en^{(0)}c) \langle \mathbf{j}^{(1)} \times \mathbf{B}^{(1)} \rangle_{||}$ - $(1/c) \langle \mathbf{v}^{(1)} \times \mathbf{B}^{(1)} \rangle_{||} = \langle E \rangle_{||} - \eta \langle j \rangle_{||}$,

Hall dynamo $\varepsilon_{||}^{(j)}$

MHD (alpha) dynamo $\varepsilon_{||}^{(v)}$

Dynamo can produce $\langle j \rangle_{||}$ and / or $\langle E \rangle_{||}$

$\mathbf{B}^{(1)}$, $\mathbf{v}^{(1)}$ $\mathbf{j}^{(1)}$ are two-fluid linear tearing eigenfunctions

Linear eigenfunctions B(x), V(x) and B_z(x) determine spatial profile of dynamo

- Tearing parity of the eigenfunctions

$$B_x(x,y) = -B(x)\sin ky, \quad B(x) > 0,\ x > 0\ (even),$$
$$v_x(x,y) = V(x)\cos ky, \quad V(x) > 0,\ x > 0\ (odd),$$
$$B_z(x,y) = -B_z(x)\sin ky, \quad B_z > 0,\ x > 0\ (odd)$$

- Parallel components of MHD (alpha) and Hall dynamo terms

$$\mathcal{E}_{||}^{(v)} = -\frac{1}{c}(v_x B_y - v_y B_x)$$

$$\mathcal{E}_{||}^{(j)} = \frac{1}{n^{(0)}ec}(j_x B_y - j_y B_x)$$

- Flux suface average (integration over y)

$$\mathcal{E}_{||}^{(v)}/\mathcal{E}_0 = \frac{d}{dx}(VB), \qquad \mathcal{E}_{||}^{(j)}/\mathcal{E}_0 = kd_i\frac{d}{dx}(B_z B)$$

normalization constant $\mathcal{E}_0 = {B_y^{(\infty)}}^2/(2ck\sqrt{4\pi n^{(0)} m_i})$

MHD waves are modified on short scales by the Hall term

$$F(u) = (u^2 - \cos^2\theta)\left(1 - \frac{\cos^2\theta}{u^2} - \frac{\sin^2\theta}{u^2 - \beta_k}\right) = \frac{k^2 d_i^2 \cos^2\theta}{(1 + k^2 d_e^2)}$$

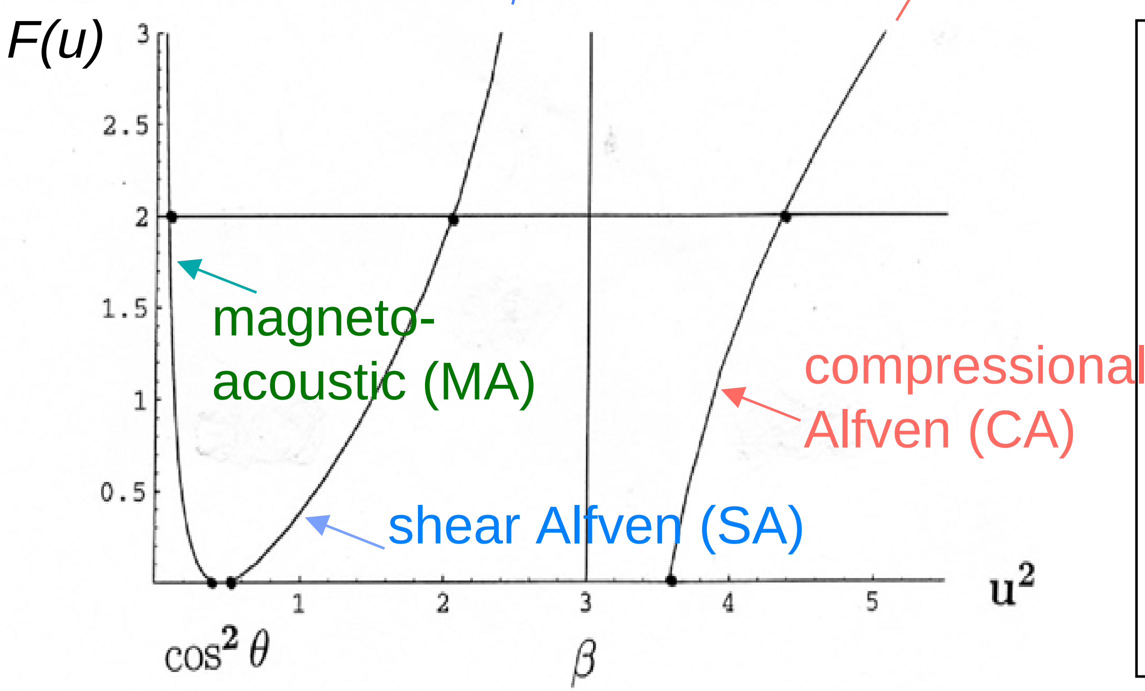
$u = \omega / k v_A$

Hall term

two-fluid "kinetic Alfven" $\omega_{SA}^2 = k_{||}^2 v_A^2 \left(1 + \frac{k^2 d_i^2 \beta_k}{1 + \beta_k}\right)$

$\beta_k = (1 + k^2 d_e^2)\beta$

whistlers $\omega_{CA}^2 = k^2 v_A^2 (1 + k_{||}^2 d_i^2)$



Two models for Hall MHD tearing instability:

(1) hot plasma model based on "kinetic Alfven" mode (SA + MA)

(2) cold plasma model based on whistlers (SA + CA)

Quasilinear two-fluid dynamo model

- Force-free equlibrium with uniform plasma density and pressure:
  X ⇒ radial direction, Y ⇒ sheared magnetic field, Z ⇒ guiding field

- Hall and **v ×B** (alpha) dynamo effects in mean field Ohm's law

$$\mathcal{E}_{||}^{(j)} = (1/en^{(0)}c) < \mathbf{j}^{(1)}\times\mathbf{B}^{(1)} >_{||} \propto - d/dx\,(B_z^{(1)} B_x^{(1)}),$$

$$\mathcal{E}_{||}^{(v)} = (1/ c) < \mathbf{v}^{(1)}\times\mathbf{B}^{(1)} >_{||} \propto - d/dx\,(v_x^{(1)} B_x^{(1)})$$

- Two-fluid tearing equations based on plasma compressibility $\nabla\cdot\mathbf{v}$ and the Hall term

Cold plasma case ($c_s << l\,\gamma$)
Compressional (CA) +
Shear Alfven (SA) modes

Hot plasma case ($c_s >> L\,\gamma$)
Shear Alfven (kinetic Alfven) +
magnetoacoustic (MA) modes

- Transition from two-fluid ($\rho_s >> \delta$) to single fluid MHD ($\rho_s << \delta$) as $B^{(0)}$ increases

- In the range $\beta \leq 15\%$ the SA ⇔ MA mode coupling can be ignored

Two-fluid effects are described by out-of-plane magnetic field perturbation $B_{||}^{(1)}$ B_z

Linearization (pure growing tearing mode with **k** = (0, k, 0) and growth rate γ)

$$\mathbf{B}(x,y,t) = \mathbf{B}^{(0)}(x) + \mathbf{B}(x)\ \exp(\gamma t + iky), \quad \mathbf{v}(x,y,t) = \mathbf{v}(x)\exp(\gamma t + iky).$$

B_z is determined by the z-component of the induction equation

$$\gamma(B_z - \delta^2\nabla^2 B_z) = -B_z^{(0)}\ \nabla\cdot\mathbf{v} + \frac{c}{4\pi e n^{(0)}}\left(B_y^{(0)}(x)\ \nabla^2 B_x - \frac{d^2 B_y^{(0)}}{dx^2} B_x\right)$$

Compressibility ∇ **v** is driven by perturbation of magnetic pressure $B_z^{(0)} B_z / 4\pi$

$$\gamma\nabla\cdot\mathbf{v} = -\frac{B_z^{(0)}}{4\pi\rho^{(0)}}\nabla^2 B_z$$

cold plasma limit, $\beta = 0$

$$\frac{c_s^2\rho^{(0)}}{\gamma}\nabla\cdot\mathbf{v} = -p^{(1)} = \frac{B^{(0)}B_z}{4\pi}$$

hot plasma case

In cold plasma case, coupling to the compressional Alfven wave reduces B_z and slows down the instability to resistive MHD time scale

Tearing instability driven by two-fluid SA and CA modes at $\beta = 0$

$\partial B_z/\partial t$ — div **v** + B_z diffusion — Hall

$$B_z - \delta_{eff}^2 \nabla^2 B_z = \frac{d_i}{\gamma \tau_a}\left(B_y^{(0)}(x)\nabla^2 B_x - \frac{d^2 B_y^{(0)}}{dx^2} B_x \right)$$

parallel Ohm's law →

$$\frac{\gamma \tau_a}{k}(B_x - \delta^2 \nabla^2 B_x) = (v + d_i k B_z) B_y^{(0)}(x)$$

vorticity equation →

$$\frac{\gamma \tau_a}{k} \nabla^2 v = \left(\frac{d^2 B_y^{(0)}}{dx^2} B_x - B_y^{(0)}(x) \nabla^2 B_x \right)$$

B_z diffusion — coupling to the CA wave

$$\delta_{eff}^2 = \delta^2 + 1 / \gamma^2 \tau_a^2 \varepsilon^2 \longleftarrow \propto B_0^2 / m_i$$

Electron MHD limit ($m_i \rightarrow \infty$) leads to whistler mediated tearing instability

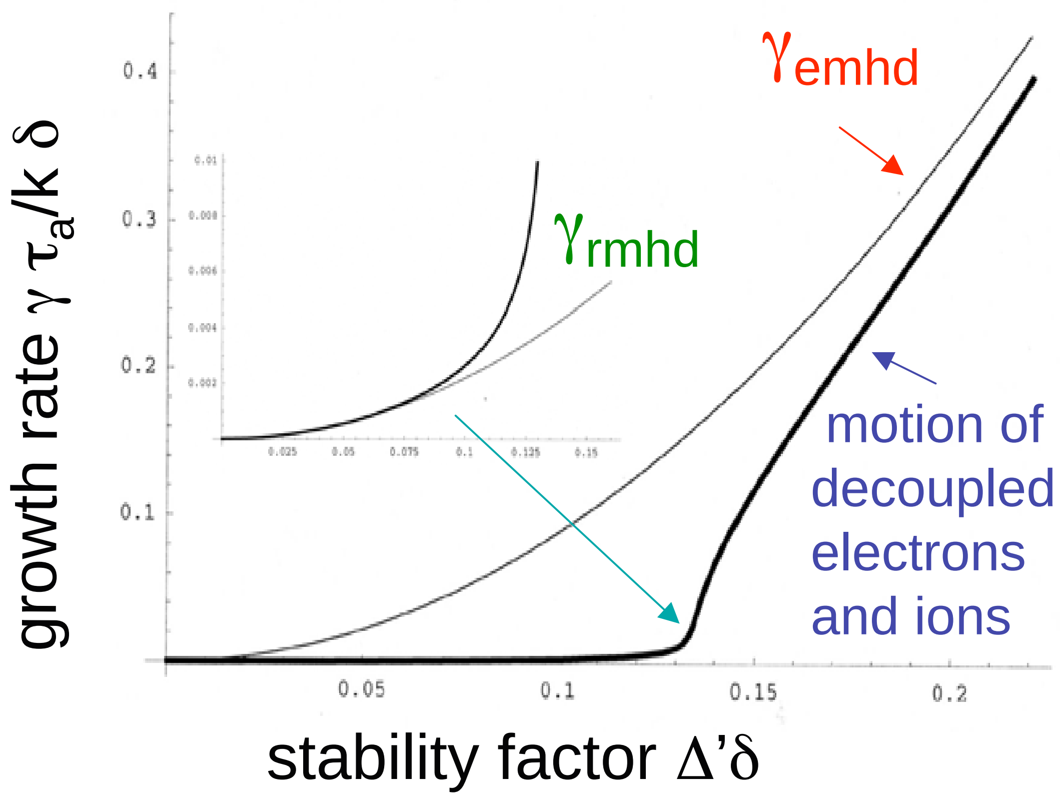
EMHD tearing equations

$$B_z - \delta^2 \frac{d^2 B_z}{dx^2} = \frac{x d_i}{\gamma \tau_a} \frac{d^2 B_x}{dx^2}$$

$$B_x - \delta^2 \frac{d^2 B_x}{dx^2} = \frac{x d_i k^2}{\gamma \tau_a} B_z$$

In the limit $m_i \rightarrow \infty$, the growth rate γ is determined by the electron mass and does not depend on m_i (S.V.Bulanov, F.Pegoraro, A.S.Sakharov, 1992)

Transition from single fluid to electron MHD takes place at critical value of Δ’



transition to two-fluid regime requires $J / ne \gtrsim v$

it takes place at

$d_i^2 \gtrsim \delta_{eff}^2 = \delta^2 + (1 / \gamma^2 \tau_a^2 \varepsilon^2)$

if guide field B_0 is small, this condition can be satisfied ($d_i >> \delta$)

in magnetically confined plasmas with strong guide field this branch of instability is suppressed

Δ′ threshold of whistler mediated tearing instability at $\beta = 0$

Whistler mediated instability in cold plasma with guide field requires c/ω_{pi} L

Critical value of Δ' for whistled mediated tearing instability

$$\Delta'_c \delta = \frac{2\pi \Gamma(3/4)}{\Gamma(1/4)} \frac{1}{\sqrt{\epsilon d_i \delta}}$$

Condition for constant - ψ regime ($\Delta'_c\, \delta << \Delta'\, \delta << 1$)

$$\frac{c}{\omega_{pi}} \gg \left(\frac{m_i}{m_e}\right)^{1/4} \frac{2\pi\Gamma(3/4)}{\Gamma(1/4)} \frac{L}{\sqrt{\epsilon}} = 13.9 \frac{L}{\sqrt{\epsilon}}$$

The EMHD whistler regime of tearing instability can exist in cold plasma at sufficiently large c / ω_{pi} L

Tearing layer equations in hot plasma case

- Instability is driven by shear Alfven and magneto-acoustic modes (compressional Alfven wave is decoupled → total magnetic + thermal pressure is equilibrated across the layer, $p^{(1)} + B^{(0)} B_{||}^{(1)}/4\pi = 0$)

$$\Gamma = \frac{\gamma q}{\rho_s k_\theta v_a |q'|}$$ normalized growth rate

$$\left(1 + \frac{\beta x^2}{\rho_s^2 \Gamma^2}\right) B_{||} - \beta\delta^2 \frac{d^2 B_{||}}{dx^2} = -\frac{d_i \beta}{k_\theta}\frac{d^2 v_r}{dx^2},$$ induction equation (parallel component)

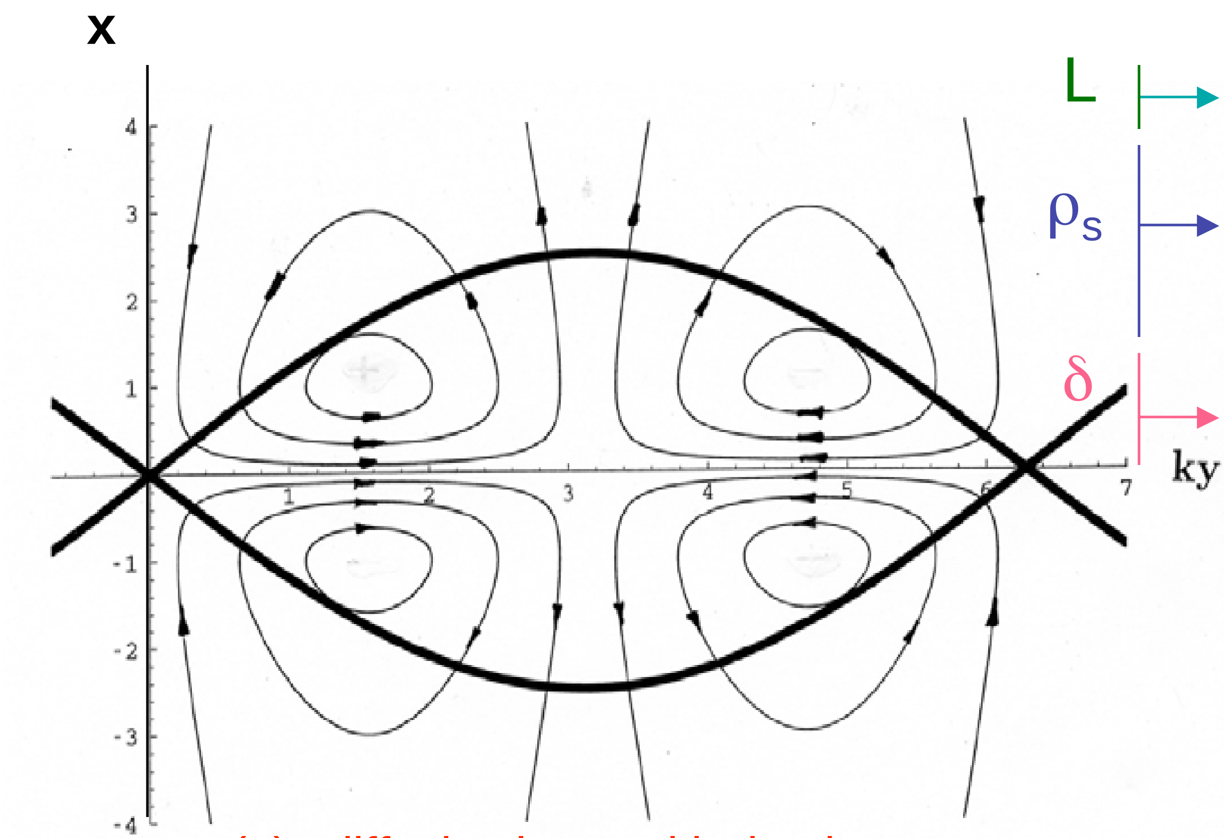
$$\rho_s \Gamma \left(B_r - \delta^2 \frac{d^2 B_r}{dx^2}\right) = x(v_r + d_i k_\theta B_{||}),$$ induction equation (radial component)

$$\rho_s \Gamma \frac{d^2 v_r}{dx^2} = -\left(x \frac{d^2 B_r}{dx^2} - J' B_r\right),$$ vorticity equation

$$J' = \frac{4\pi q}{c B_\theta |q'|} \frac{d j_{||}^{(0)}}{dx}, \quad x = r - r_s.$$

Scales of two-fluid tearing instability in hot plasma case

Three main scales in hot plasma case, $c_s \gg \gamma L$, ("kinetic Alfven" + magnetoacoustic wave)



L — (c) shear length

ρ_s — b) ion-sound gyroradius

$\rho_s = c_s / \omega_{ci}$

δ — (a) electron skin-depth

$\delta^2 = c^2/\omega_{pe}^2 + \eta c^2/4\pi\gamma$

(c) zone: single fluid ideal MHD, $V_e = V_i$

(a) diffusive layer with dominant electron flow, $V_e \gg V_i$

(b) ideal flows of decoupled electrons and ions, $V_e \approx V_i$

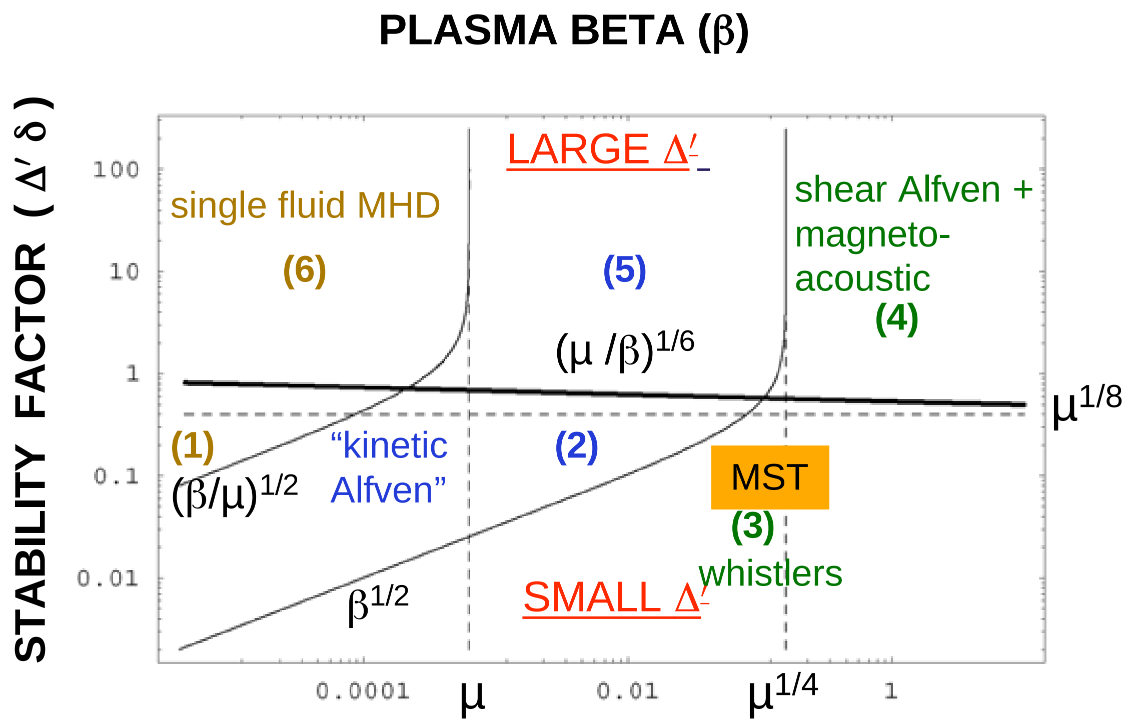
Boundary layer matching yields general dispersion relation

Equation for the growth rate Γ at $\rho_s = d_i\, \beta^{1/2} \gg \delta$

$$\frac{\Gamma^2 \rho_s}{G(\Gamma/\sqrt{\beta})} + \frac{2}{\Delta'} = \frac{2G(\Gamma/\sqrt{\beta})\delta}{\pi \Gamma}$$

Definition of G(a):

Parameter space for two-fluid tearing instability in hot plasma case



Semi-collisional approach:
$m_e / m_i < \mu = (\delta / d_i)^2 << 1$

$\mu = (m_e / m_i) \ (1+ \nu_{ei} / \gamma) = 0.025$
$A = \Gamma (1/4) / (2 \pi \Gamma (3/4)) \simeq 0.5$

Growth rate does not depend on β in small and large beta limits

(A)

(1) $\gamma \tau_a = \Delta'^2 \delta^3 k A^2$

(6) $\gamma \tau_a = k \delta$

FKR, Phys. Fluids, 1963

(B)

(2) $\gamma \tau_a = \Delta' \delta k d_i \beta^{1/2} / \pi$

(5) $\gamma \tau_a = (2 d_i^2 \beta \delta / \pi)^{1/3} k$

F.Porcelli, PRL, 1991
L.Zakharov, B.Rogers, Phys. Fluids, 1992

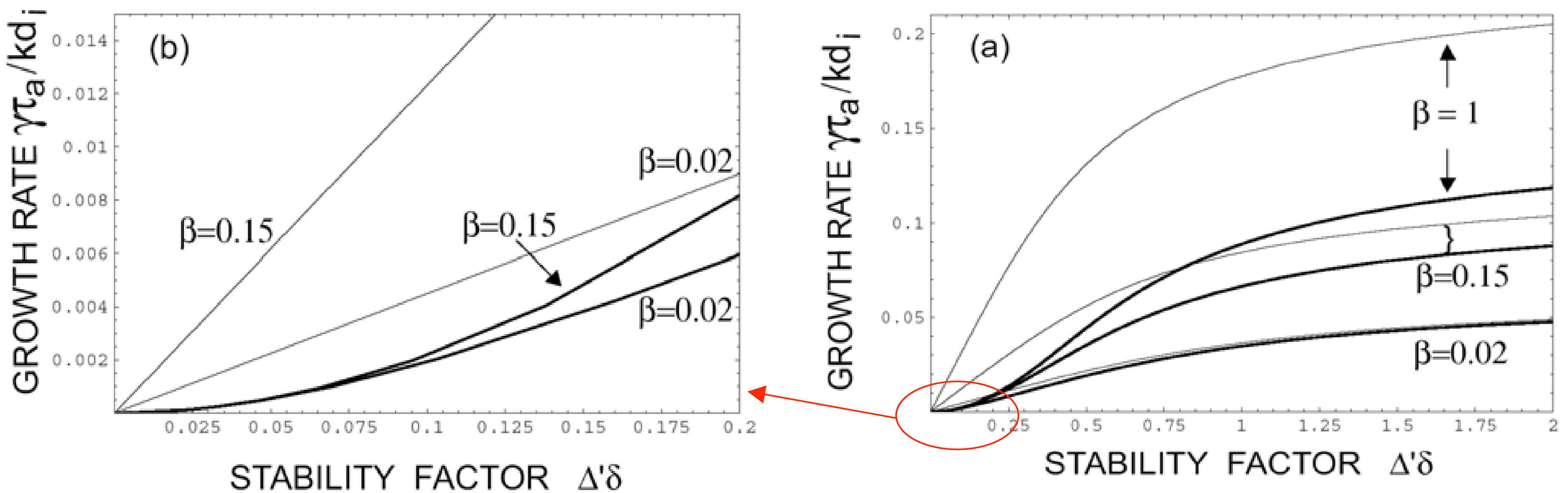
(C)

(3) $\gamma \tau_a = (\Delta' \delta)^2 k d_i A^2$

(4) $\gamma \tau_a = (2 \pi d_i \delta)^{1/2} k A$

V.Mirnov, C.Hegna, S.Prager, PoP, 2004

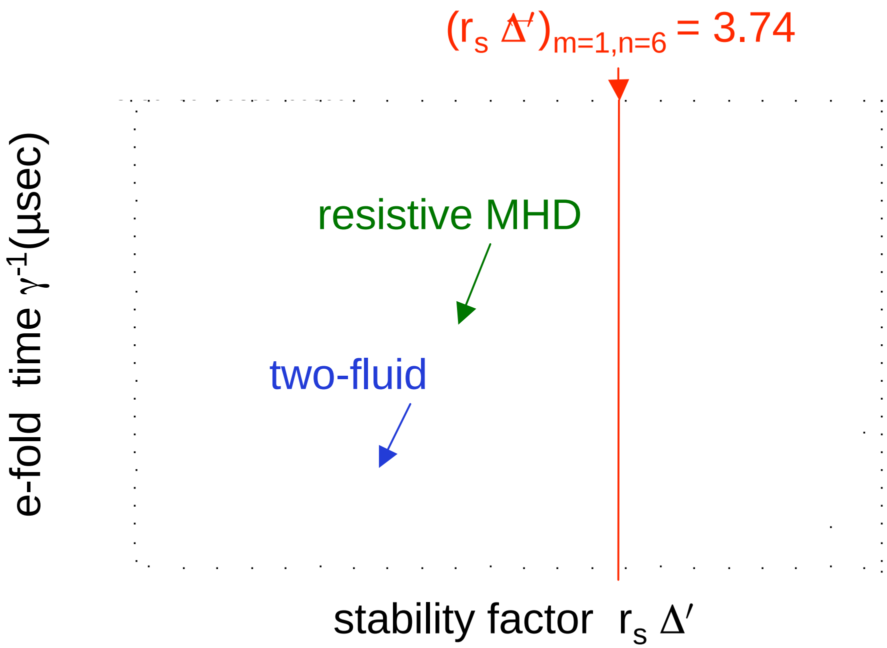
B_z diffusion and coupling with the magneto-acoustic (MA) wave are important at large β and Δ′



Thick curves are exact solutions with $B_{||}$ diffusion and coupling to the MA wave. They describe transition between (B) and (C) regimes.

Thin curves are calculated without these two effects and give overestimated values of γ

Two-fluid core tearing mode m=1, n=6 in the MST



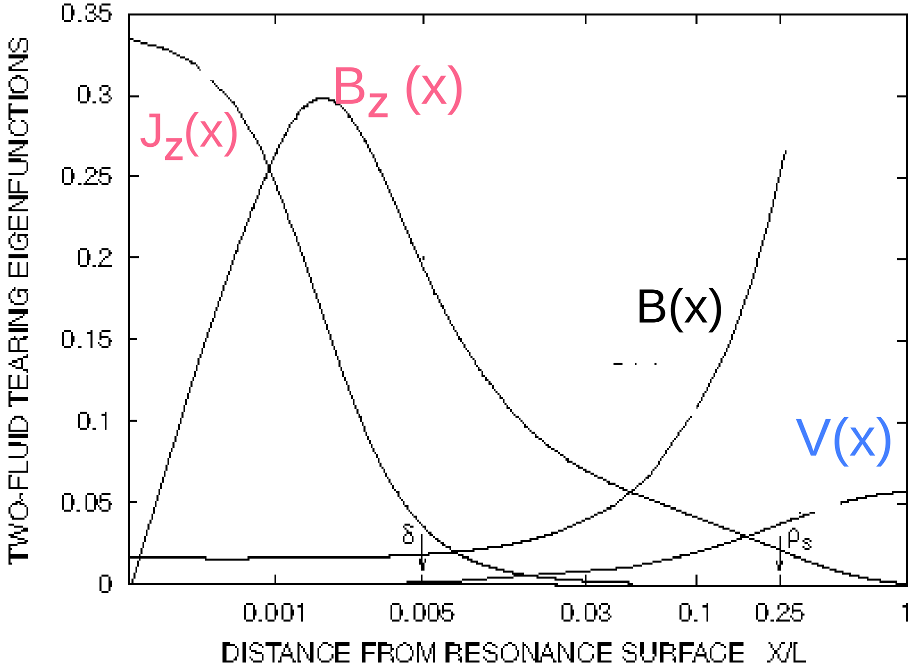
two-fluid tearing instability is fast, $\gamma^{-1} \simeq 50$ μsec

single fluid resistive MHD mode is four times slower, $\gamma^{-1} \simeq 200$ μsec

maximum value of the enhancement, $(m_i/m_e)^{1/2}$, is reduced due to resistive part of electron skin-depth ($\delta_{2F} \simeq 0.4$ cm, $\delta_{MHD} \simeq 0.75$ cm)

- hot plasma regime is adequate to RFP ($c_s/\gamma \simeq 20$ m >> radial width of the mode)
- total (magnetic + thermal) pressure is equilibrated across the layer
  $(p^{(1)} + B^{(0)} B_{||}^{(1)}/4\pi = 0,$ → compressional Alfven mode is decoupled)
- at small $\delta\,\Delta' << 1$ no coupling with the magneto-acoustic mode
- tearing instability is driven by two-fluid "kinetic Alfven" mode
- whistler mediated regime of "kinetic Alfven" mode (MST case): $\beta \simeq 7.5\% > (\delta\,\Delta')^2 \simeq 2.2\%$

Tearing eigenfunctions are characterized by two-scale spatial distributions



Intermediate beta case (B) at large Δ'

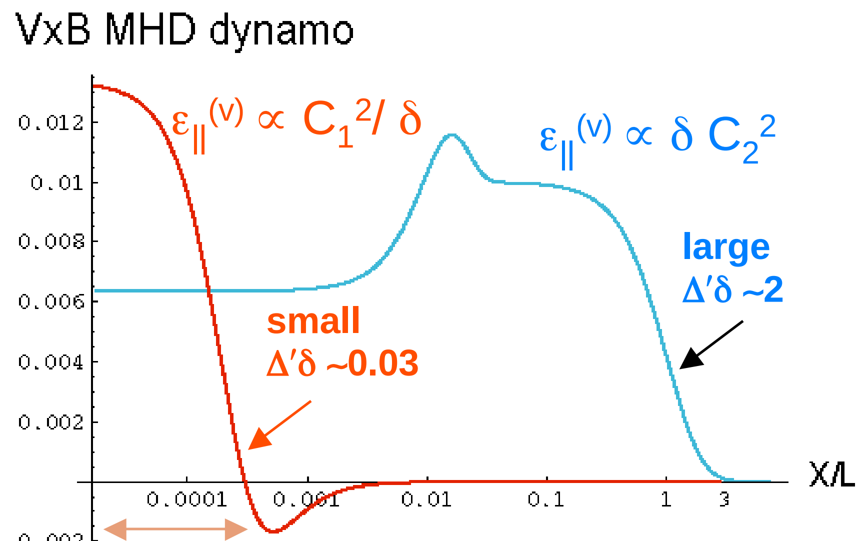
Even B(x), J_z (x) (radial magnetic field and parallel current density) and odd V(x), B_z (x) (radial velocity and parallel magnetic field) eigenfunctions at large Δ'

Growth rate:
$\Gamma = (2\,\delta/\pi\rho_s)^{1/3}$, $\Delta'\delta^{2/3}\rho_s^{1/3} >> 1$

Functions $J_z(x)$, $B_z(x)$ are localized on short scale δ Γ (≤ electron skin depth), V(x) is broaden to a wider scale ρ_s (ion-sound gyroradius)

Intersection of $B_z(x)$ and V(x) at $x \approx \rho_s$ separates area of small x, where electrons and ions are decoupled ($v_e >> v_i$) and large x, where single fluid MHD is applicable ($v_e \approx v_i$)

Single fluid MHD dynamo



<u>Small beta case (A) at small and large Δ'</u>

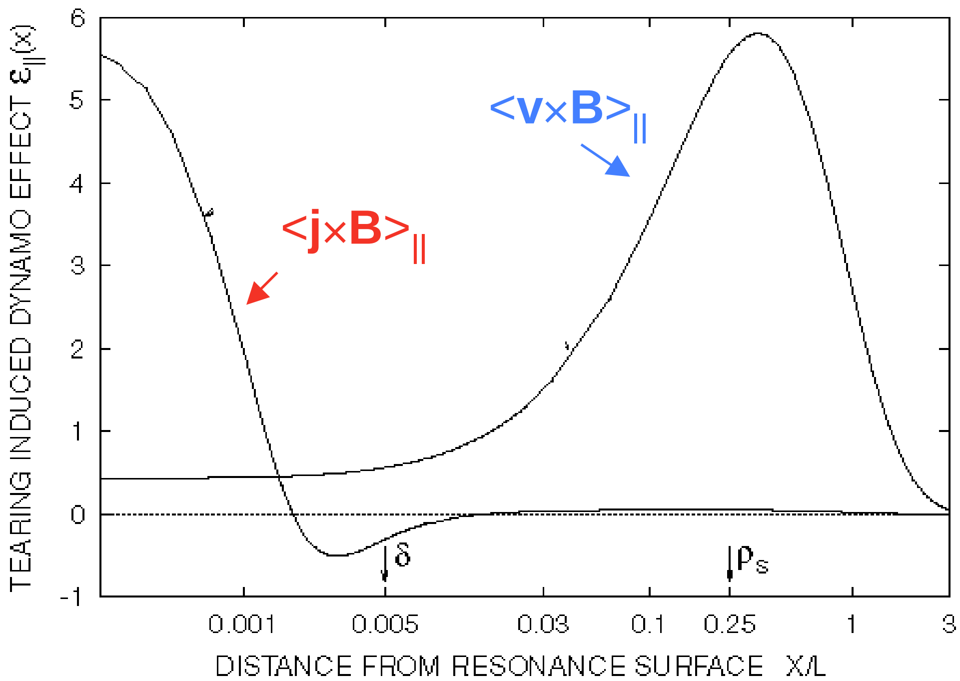
(1) small $\Delta'\delta$ << 1(constant-ψ approximation) dynamo is localized on a small scale $\Delta'\delta^2$

(2) large $\Delta'\delta \simeq 1$; dynamo is broadened to the external scale L;

(3) at $B_x \propto \sin ky$, the MHD polarization currents are, $j_x \propto v_y \propto \sin ky$, $j_y \propto -v_x \propto \cos ky$

→ flux surface average Hall dynamo equals zero ($< j_x B_y - j_y B_x> \propto <\sin ky \cos ky> = 0$)

Hall dynamo is large in two-fluid MHD



Intermediate beta regime (B) at large Δ′

Total dynamo effect $\varepsilon_{||}(x) = \varepsilon_{||}^{(v)} + \varepsilon_{||}^{(j)}$ and contribution $\varepsilon_{||}^{(v)}(x)$ from **v** × **B** term (10^2 fold) (all functions are even on x)

Conclusion

(1) in two-fluid regime ($\rho_s \gg \delta$) the Hall dynamo is larger than the **v** × **B** dynamo by a factor $(\rho_s/\delta)^{4/3}$ and localized on a short scale $\delta^{4/3}/\rho_s^{1/3}$.

(2) **v** × **B** dynamo is broadened to a spatial scale of order ρ_s

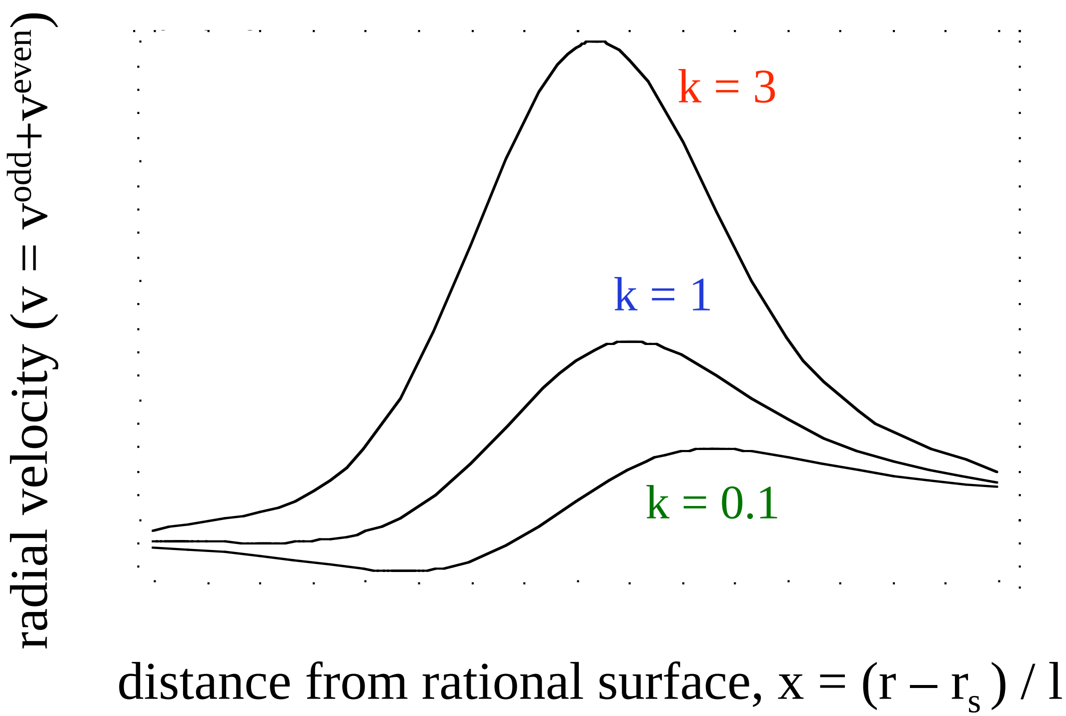
Effect of equilibrium $dJ^{(0)}/dr$ on plasma flow profiles

- Plasma velocity $v(x) = i\, v_r$ in constant -ψ approximation ($v \rightarrow v / v_0$, $x \rightarrow x / l$,

  $B_r \rightarrow B_r / B_\theta^{(0)}$, $v_0 = l\, B_r / \delta$, $l^{\,2} = (\gamma\, \tau_a / k_\theta)\,(q\, \delta / |q'|\, r_s)$, dispersion relation $l = \Delta'\, \delta^2 A$)

$$\frac{d^2 v^{odd}}{dx^2} = -x + x^2 v^{odd}$$

$$\frac{d^2 v^{even}}{dx^2} = x^2 v^{even} - k, \qquad k = \frac{|J'|}{\Delta'^2 \delta^4 A^2}$$

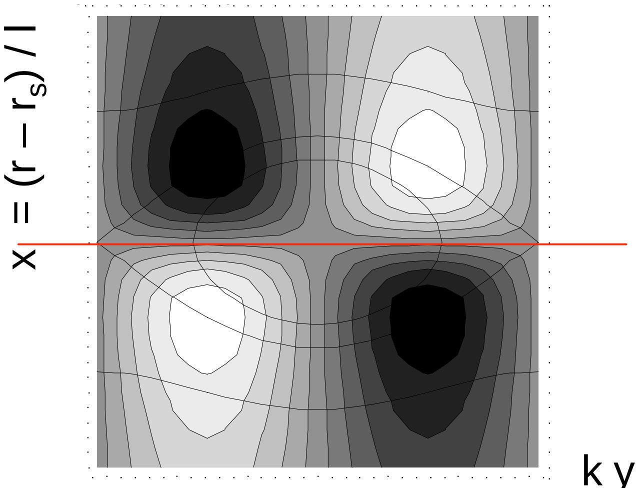


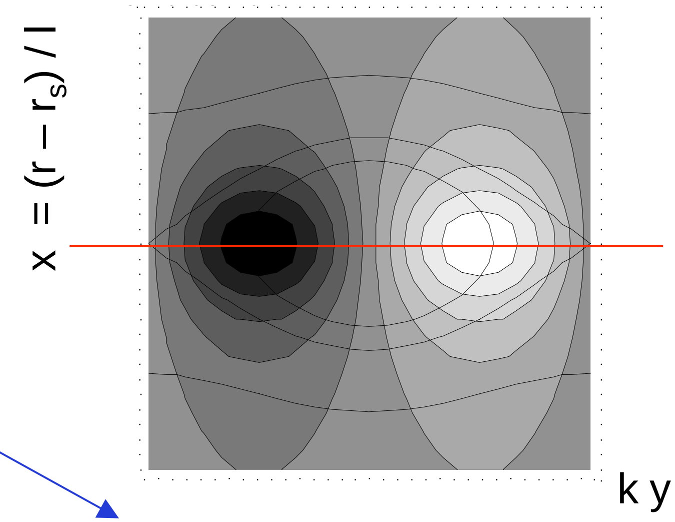
- term proportional to $dJ^{(0)}/dr$ mixes odd and even parities in radial velocity profile←
- at small k “classical” odd profile
- at large k plasma velocity is even in $x = r - r_s$
- strong shear of poloidal flow v_θ across the resonant surface (sign reversed)
- dynamo e.m.f.$<\mathbf{v} \quad \mathbf{B}>_{||} \propto$ dv_r/dr flattens equilibrium current profile

Effect of equilibrium $dJ^{(0)}/dr$ on plasma flow profiles (cont.)

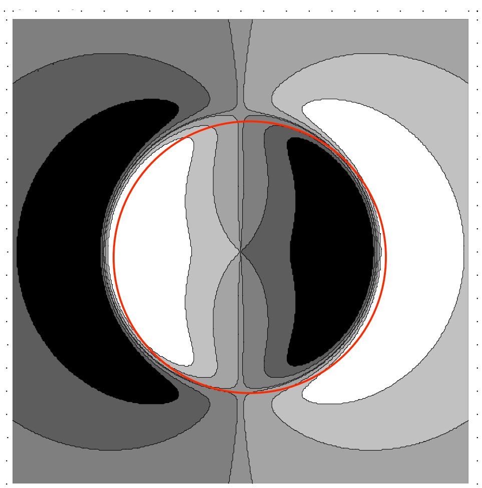
Stream lines for "classical" slab geometry with symmetric current profile (the case, when $dJ^{(0)}/dr = 0$ on the resonance surface)

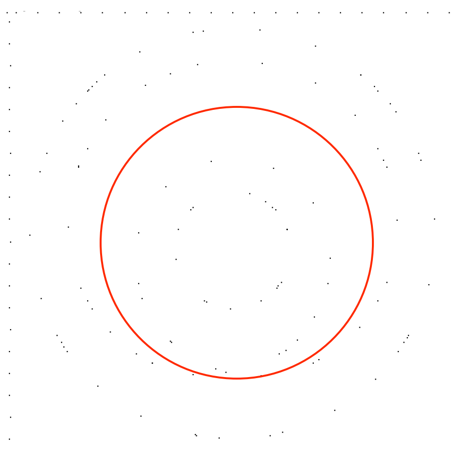
Effect of $dJ^{(0)}/dr \neq 0$ on plasma vortices (plasma flows through the rational surface)

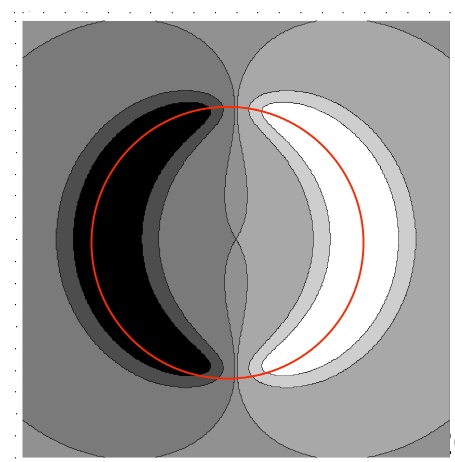




Equivalent flow patterns in cylindrical geometry







Effect of dynamo on mean electric field and current density

- Growing tearing mode generates parallel (with respect to mean magnetic field) electromotive force, $\varepsilon_{||}(r, t)$, localized at the resonant magnetic surface.

λ Both the MHD ($\langle \mathbf{v}\times\mathbf{B}\rangle$) and the Hall dynamo ($\langle \mathbf{j}\times\mathbf{B} \rangle$) contribute to $\varepsilon_{||}$, $\langle\ \rangle$ denotes mean (flux surface average) value.

λ Deviation of mean electric field and current density from their equilibrium values

$$\langle E\rangle_{||} = E_{EQ} + \langle\delta E\rangle_{||}, \qquad \langle j\rangle_{||} = j_{EQ} + \langle \delta j \rangle_{||}$$

- Generalized Ohm's law for $\langle\delta j\rangle_{||}$ and $\langle\delta E\rangle_{||}$

$$\varepsilon_{||}(r, t) = \langle \delta E \rangle_{||} - \eta \langle \delta j \rangle_{||}$$

λ This equation does not determine $\langle\delta E\rangle_{||}$ and $\langle\delta j\rangle_{||}$ separately. We combine two-fluid tearing theory with a 1D temporal model based on Faraday's law for two cases:

1) $\varepsilon_{||}$ is driven by exponentially growing tearing mode, $\varepsilon_{||}(r, t) = \varepsilon_{||}(r) \exp(2 \gamma t)$
2) $\varepsilon_{||}$ is a periodic function of time with experimentally observed sawtooth temporal profile $\varepsilon_{||}(r, t) = \varepsilon_{||}(r)\, g(t)$

Exponentially growing tearing mode

- Ampere's law and the induction equation → 1D radial model for flux surface average (in poloidal and axial directions) $\langle\delta E\rangle_{||}(r, t)$
- Radial profile of $\langle\delta E\rangle_{||}(r, t) = f(r) \exp(2\gamma t)$ in the core $(\langle\delta E\rangle_{||} \simeq \langle\delta E\rangle_z)$

$$f - \frac{\delta'^2}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right) = \varepsilon_{||}(r)$$

- Radial profile of $\langle\delta E\rangle_{||}(r, t)$ near the reversal surface $(\langle\delta E\rangle_{||} \simeq \langle\delta E\rangle_\theta)$

$$f - \delta'^2\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rf) = \varepsilon_{||}(r)$$

- Factor $\delta'^{2} = c^2/\omega_{pe}^2 + c^2\eta/8\pi\gamma$ is approximately equal to $\delta^{2} = c^2/\omega_{pe}^2 + c^2\eta / 4\pi\gamma$

Exponentially growing tearing mode (cont.)

- Plane geometry

$$f - \delta^2 \frac{d^2 f}{dx^2} = \epsilon_{||}(x), \qquad x = r - r_s$$

- General solution decaying at x $\rightarrow \pm \infty$

$$f(x) = \frac{1}{2\delta}\int_{-\infty}^{+\infty} \exp\left(-\frac{|x-x'|}{\delta}\right)\ \epsilon_{||}(x')dx'$$

- Approximate solutions for narrow (l << δ) and broad (l >>δ) functions $\varepsilon_{||}(x)$

$$f(x) \simeq \exp\left(-\frac{|x|}{\delta}\right)\frac{1}{2\delta}\int_{-\infty}^{+\infty}\epsilon_{||}(x)dx \simeq \epsilon(0)\frac{l}{\delta}\exp\left(-\frac{|x|}{\delta}\right) \rightarrow 0, \qquad l \ll \delta$$

$$f(x) \simeq \epsilon_{||}(x), \qquad l \gg \delta$$

Exponentially growing tearing mode (conclusions)

- Hall dynamo contribution to $\varepsilon_{||}(x)$ is localized on short electron scales: $l \simeq \Delta' \delta^2$ (at small Δ'), and $l \simeq \delta^{4/3}/\rho_s^{1/3}$ (at large Δ')

- As $l \ll \delta$, the Hall dynamo contributes mainly to the mean current while its effect on the electric field is small

- The **<v** x **B>** part of $\varepsilon_{||}(x)$ is broadened to ρ_s scale or even wider up to the external scale L

- Since $l \gg \delta$, the **<v** x **B>** dynamo contributes mainly to the mean electric field while its effect on the mean current is small

1D temporal model for electric field and current density

- Generalized Ohm's law with electron inertia

$$E - \eta j - \frac{4\pi}{\omega_{pe}^2}\frac{\partial j}{\partial t} = \epsilon_{||}(x,t)$$

- Faraday's and Ampere's laws → equation for E

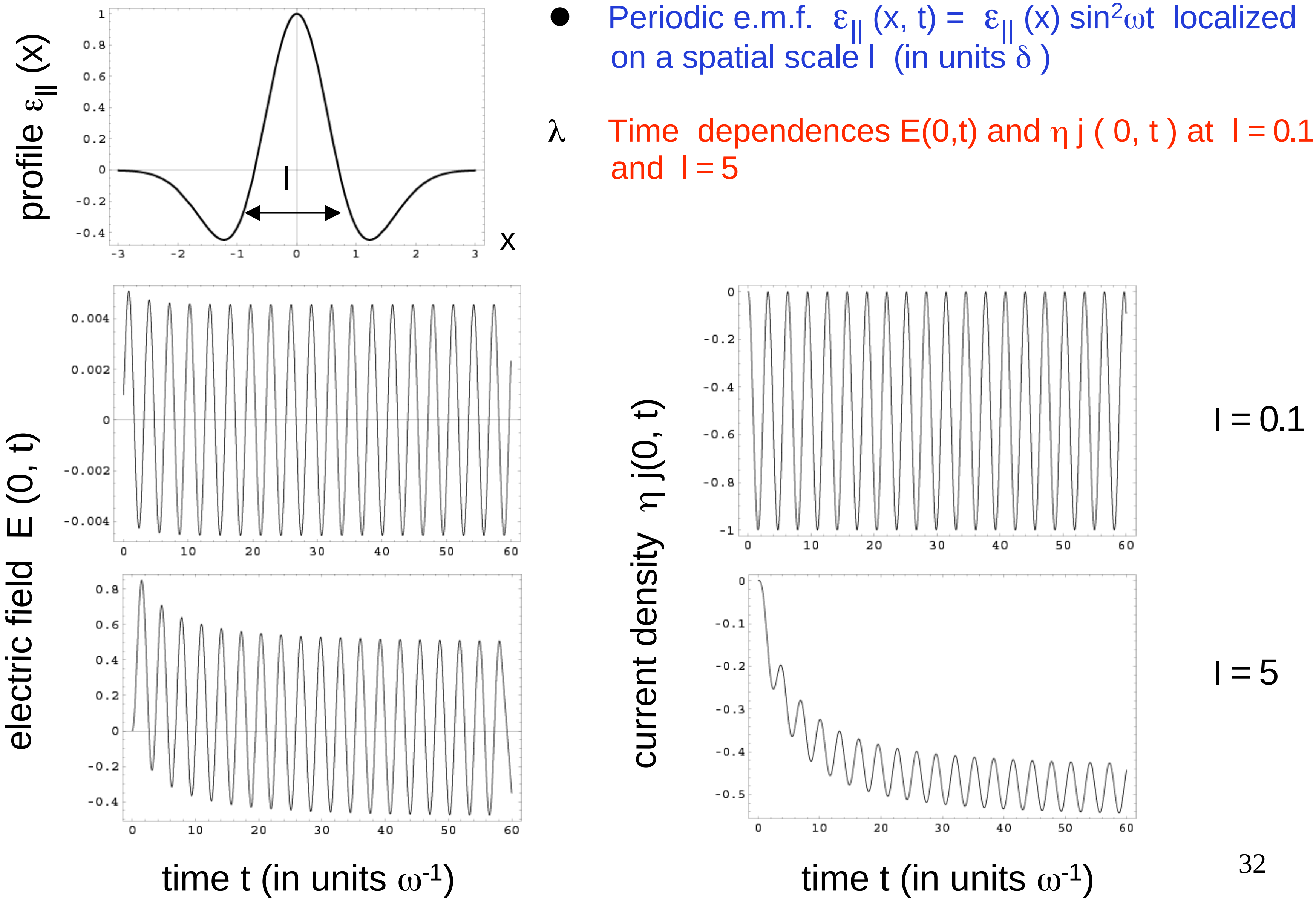
- Numerically integration: initial value problem with boundary conditions

$$E(x,0) = 0$$
$$\partial E/\partial x(0,t) = 0, \quad E(+\infty,t) = 0$$

$\varepsilon_{||}$ (x,t) is a periodic function of time with single time scale

- Periodic e.m.f. $\varepsilon_{||}(x, t) = \varepsilon_{||}(x) \sin^2\omega t$ localized on a spatial scale l (in units δ)

λ Time dependences E(0,t) and η j (0, t) at l = 0.1 and l = 5



Single time scale case (conclusion)

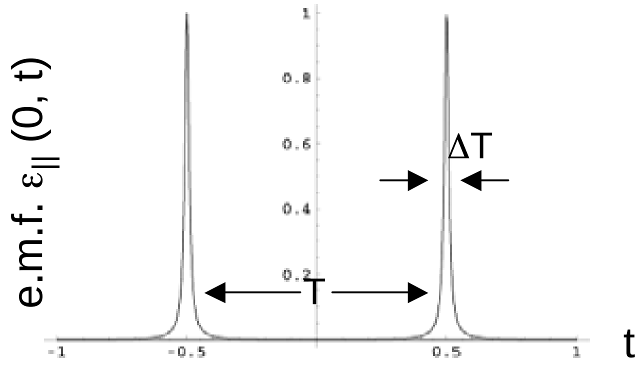
- If $\varepsilon_{\|}$ (x, t) is a periodic function of time and l << δ, it contributes mainly to the production of the current, $\eta\, j \simeq -\, \varepsilon_{\|}$ (x, t), while electric field is small.

- In this case, the amplitude of current oscillations is large, $\Delta j \simeq j$.

- If $\varepsilon_{\|}$ (x, t) is localized o~~n~~ a l~~arge sca~~le l >> δ, current oscillates near its mean value determined by $\eta\, j = -\, \varepsilon_{\|}$ (x, t).

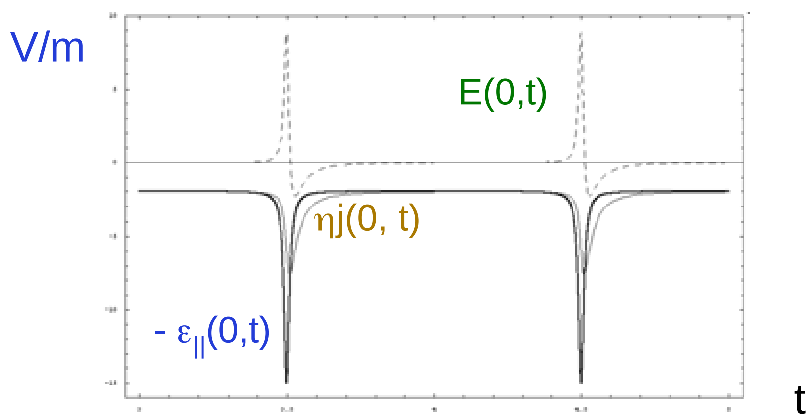
- In this case, electric field oscillations are large, $E(x,t) \simeq \varepsilon_{\|}(x, t) - \overline{\varepsilon_{\|}(x, t)}$.

Sawtooth temporal profile (the case of two strongly different time scales)

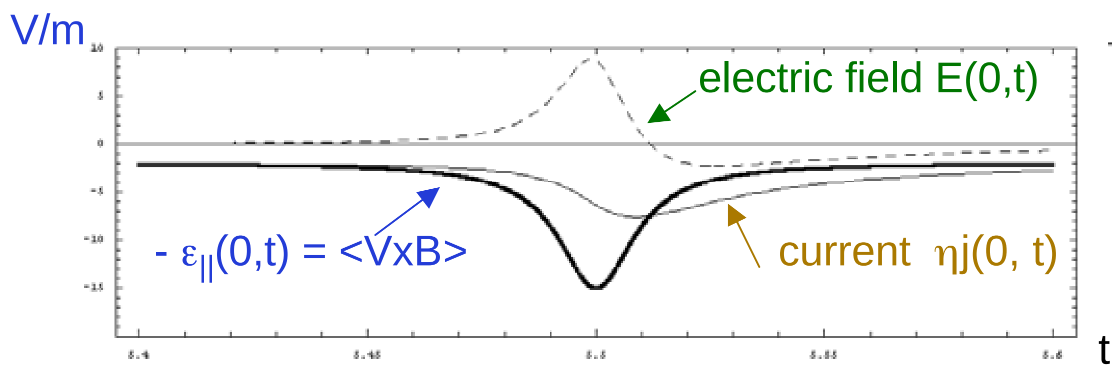


- Peak width $\Delta T \simeq 100\ \mu sec \simeq 1/50$ of the time interval between the peaks, $T \simeq 5$ msec.

- If $\delta_{\Delta T}$ (calculated for time ΔT) is large compared to l, or $\delta_T << l$, the results are similar to the previous ones of single temporal scale

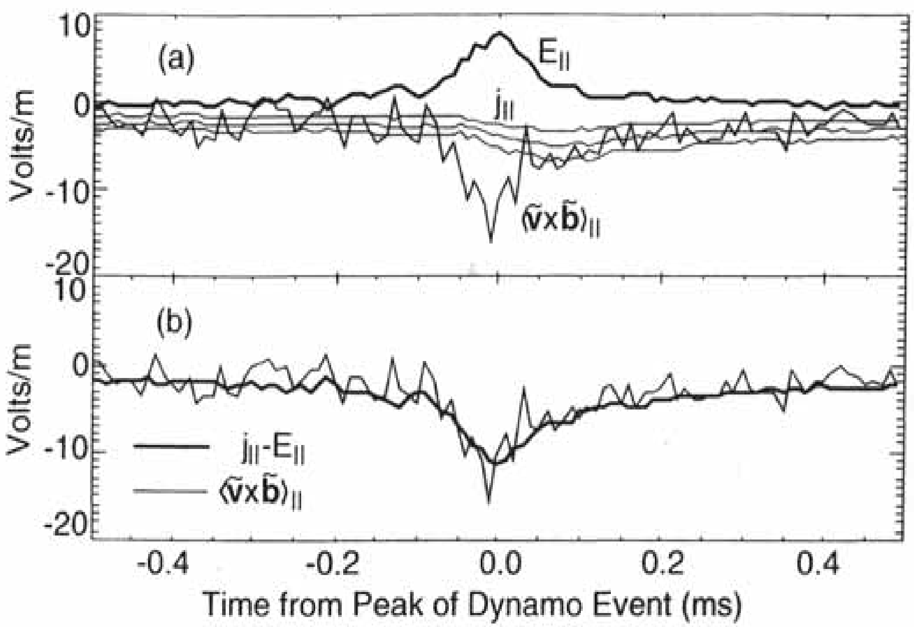


- Intermediate case, $\delta_T >> l >> \delta_{\Delta T}$, is realized at the MST reversal surface ($l = 5$ cm, $\delta_T = 13$ cm, $\delta_{\Delta T} = 1.8$ cm) ⟶ current is generated between the crashes, electric field - during the spikes



- For periodic function $\varepsilon_{||}(x,t)$, time average electric field $E(x,t) = 0$, while time average current $\eta\ j(x, t) = - \varepsilon_{||}(x, t)$

Dynamo measurements in the edge of the MST



Ohm's law during a dynamo event cycle. (a) The parallel electric field $E_\parallel$, fluctuation-induced dynamo $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_\parallel$, and $\eta j_\parallel$ with $Z_{\rm eff}$ of 1, 1.5, and 2. (b) $\eta j_\parallel - E_\parallel$ and $\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle_\parallel$ with $Z_{\rm eff} = 1.5$, showing good time-dependent agreement.

P.W.Fontana, D.J. Den Hartog, G.Fiksel, S.C.Prager, PRL 85, No 3, 2000

- The electric field is negligible during the quiescent phase while dynamo approximately accounts for the parallel current
- Contribution of parallel current is small, while the dynamo balances a large induced electric field during fast rising phase

Hall (anti-) dynamo large near *n* = *6* resonant surface

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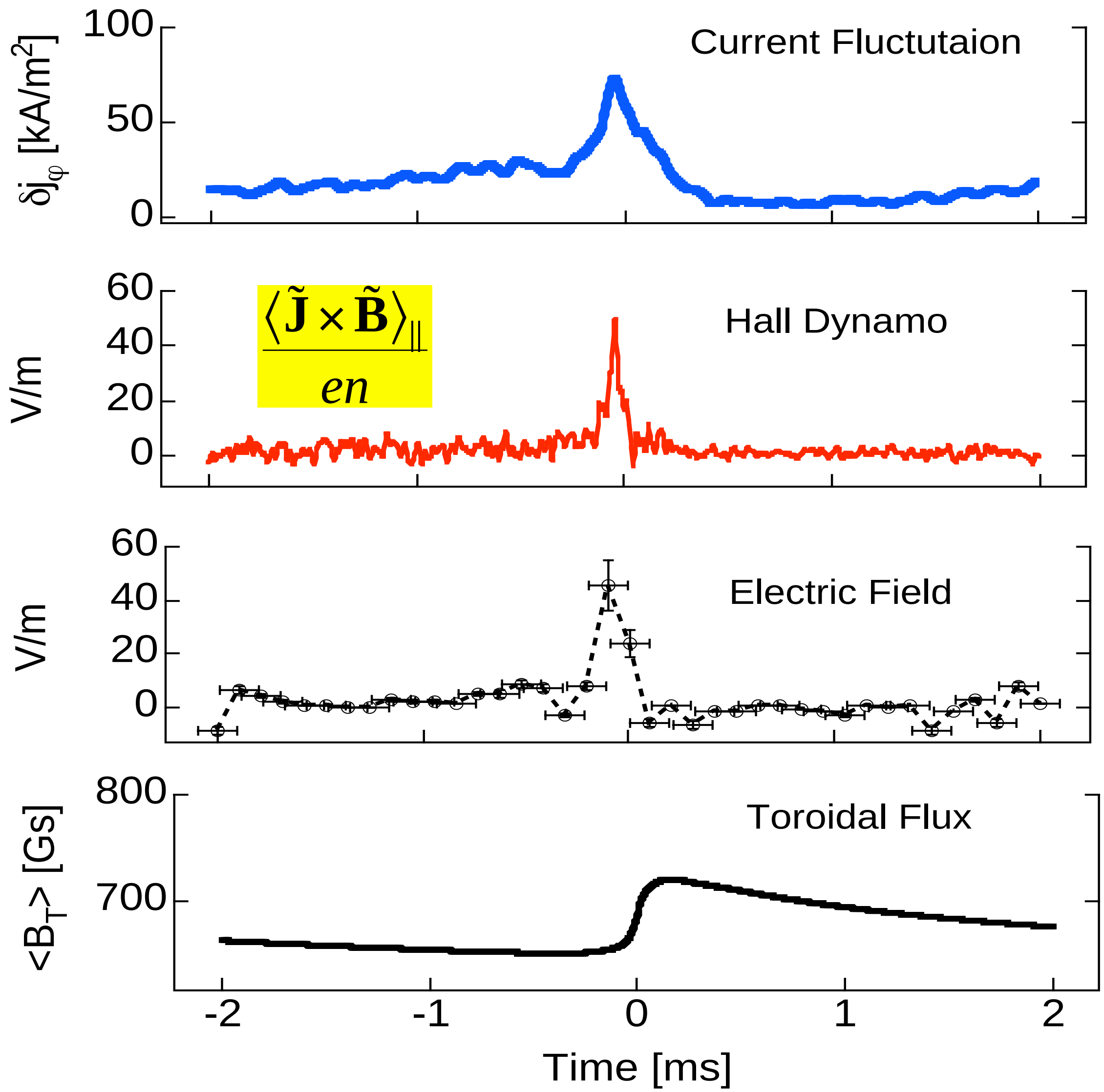
to appear in PRL



sawtooth
ensemble

Summary

- Two-fluid effects are important in laboratory plasmas with magnetic self-organization.
- Two-fluid tearing mode theory yields temporal and spatial scales of tearing instability in a wide range of plasma parameters .
- Quasi-linear theory predicts large, localized $\mathbf{J}\times\mathbf{B}$ Hall dynamo
- 1D temporal plasma slab model well describes the edge measurements
- It qualitatively agrees with the core measurements, however effects of cylindrical geometry have to be taken into account

Abstract

Current–driven tearing instabilities are believed to dominate magnetic relaxation in configurations such as the reversed field pinch (RFP) and spheromak.

In the Madison Symmetric Torus (MST) RFP experiments, the amplitudes of fluctuations in magnetic field, flow velocity and current density follow a sawtooth cycle in their time dependence. During a sawtooth crash a surge occurs in the dynamo – a fluctuation-induced mean electromotive force in the generalized Ohm's law. This e.m.f. combines both the MHD dynamo and the Hall dynamo. During a crash a substantial change occurs in the mean plasma current density profile and a very large change occurs in the mean electric field profile.

We have developed a two-fluid quasilinear dynamo theory to determine how the effect of the dynamo is divided between local current density production and local electric field production. This theory is compared with new experimental results:

(1) the Hall dynamo, measured by magnetic probe in the plasma edge

(2) in the high-temperature plasma core, where the Hall dynamo is determined by measuring both magnetic and current density fluctuations using a high-speed laser Faraday rotation diagnostic

In general, the self-inductance of the tearing layer prevents fast changes in the mean parallel current, thus, providing a "stiffness" with respect to perturbations.

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Introduction

Large-scale tearing instabilities have long been considered to underlie dynamo processes in the reversed field pinch (RFP).

The dynamo is important in RFP as a nonlinear feedback mechanism to flatten the current profile to the Taylor's state of minimum energy.

We have studied previously in quasilinear approximation the spatial (radial) structure of fluctuation induced $< \tilde{\mathrm{v}} \times \tilde{\mathrm{B}} >_{||}$ and $< \tilde{\mathrm{j}} \times \tilde{\mathrm{B}} >_{||}$ (Hall) mean electromotive force.

We have performed a two-fluid quasilinear dynamo theory to determine how the effect of the dynamo is divided between local current density production $< \delta \mathbf{j} >_{||}$ and local electric field production $< \delta \mathbf{E} >_{||}$.