Numerical simulations of imbalanced MHD turbulence

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- Current understanding of MHD turbulence belongs to the case balanced turbulence. Turbulence in the Solar Wind is imbalanced!

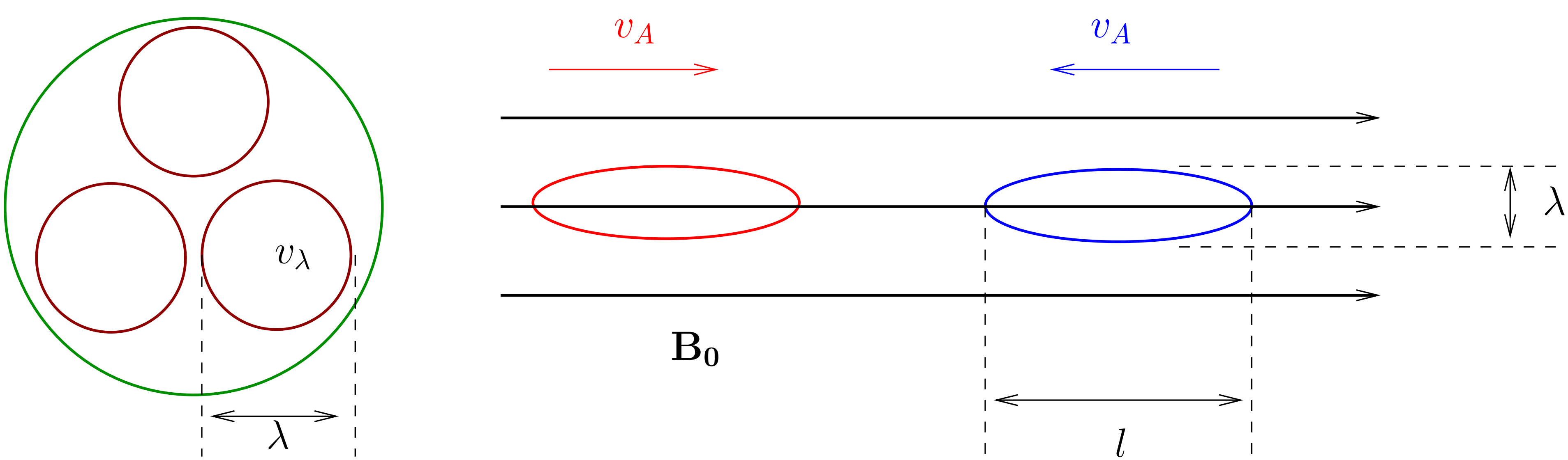
HD vs MHD turbulence pictures

Hydrodynamic turbulence:

- Interaction of eddies.
- Isotropic.
- Strong interactions.
- Energy Cascade.

Magnetohydrodynamics:

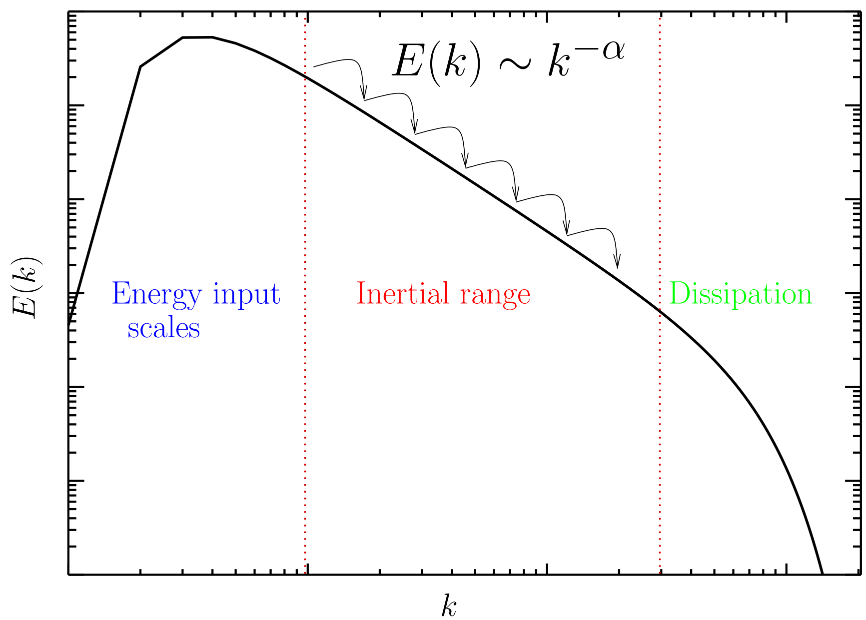
- Interaction of wave packets.
- Anisotropic.
- Weak or Strong interactions.
- Energy and Cross-Helicity cascades.
- Balanced or imbalanced.



Homogeneous and Isotropic Hydrodynamic (HD) Turbulence

Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} + \nabla p = \frac{1}{R}\nabla^2 \mathbf{v} + \mathbf{f}$$



- For an eddy of size $\lambda \sim 1/k$ with velocity v_λ, the nonlinear time scale is dimensionally given by

$$\tau_\lambda \sim \frac{\lambda}{v_\lambda}$$

- In steady state

$$\epsilon \sim \frac{v_\lambda^2}{\tau_\lambda} = \text{const} \Rightarrow v_\lambda \sim (\epsilon\lambda)^{1/3}$$

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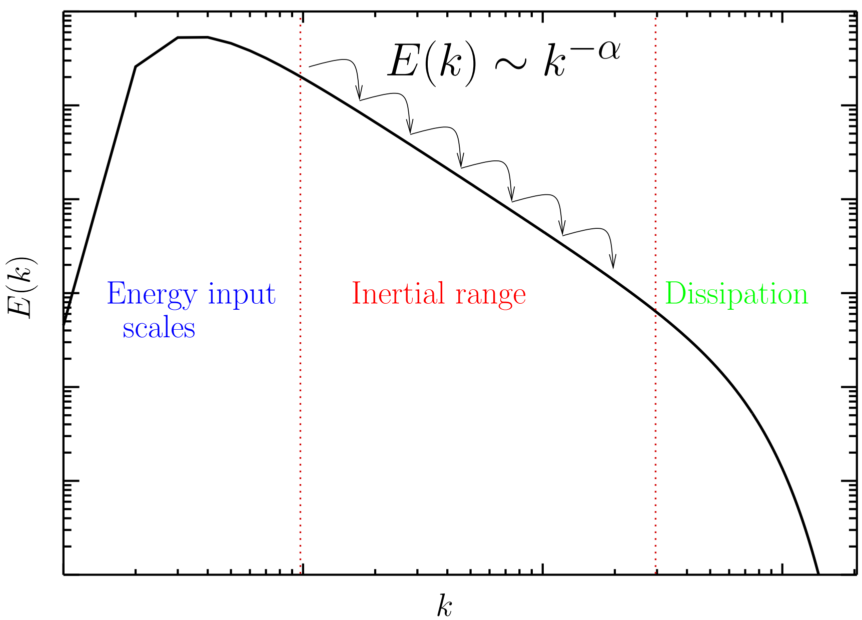

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- Kolmogorov's 5/3 law (K41)

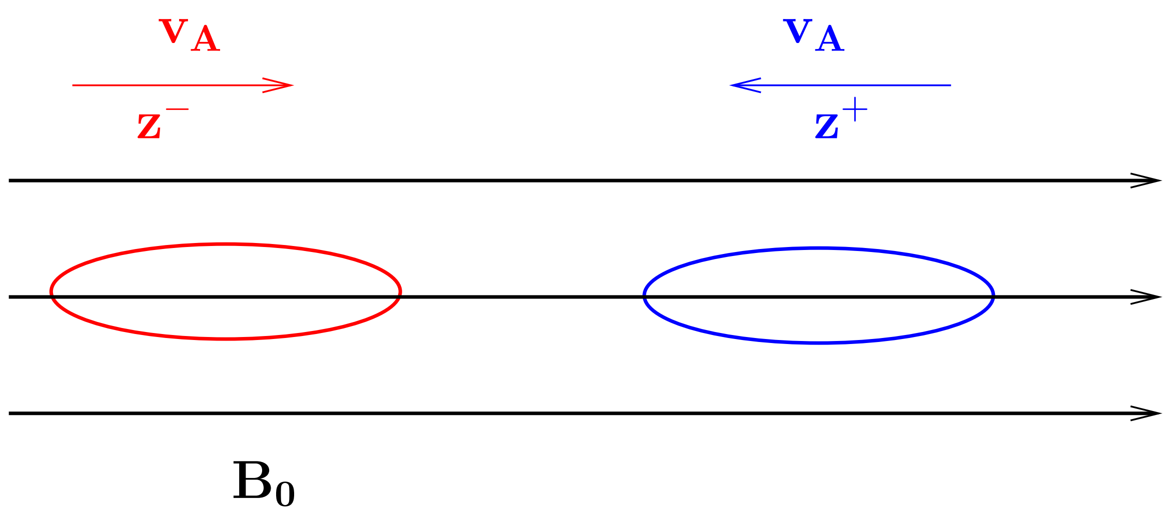
$$E(k) \sim k^{-5/3}, \quad \tau_\lambda \sim k^{-2/3}$$

Incompressible Magnetohydrodynamic (MHD) turbulence

- MHD turbulence is best studied in the Elsässer formulation

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp \underbrace{(\mathbf{v}_A \cdot \nabla)}_{\sim\ 1/\tau_A} \mathbf{z}^{\pm} + \underbrace{\left(\mathbf{z}^{\mp} \cdot \nabla\right)}_{\sim\ 1/\tau_{NL}} \mathbf{z}^{\pm} = -\nabla P + \frac{1}{R}\nabla^2 \mathbf{z}^{\pm} + \mathbf{f}^{\pm}$$

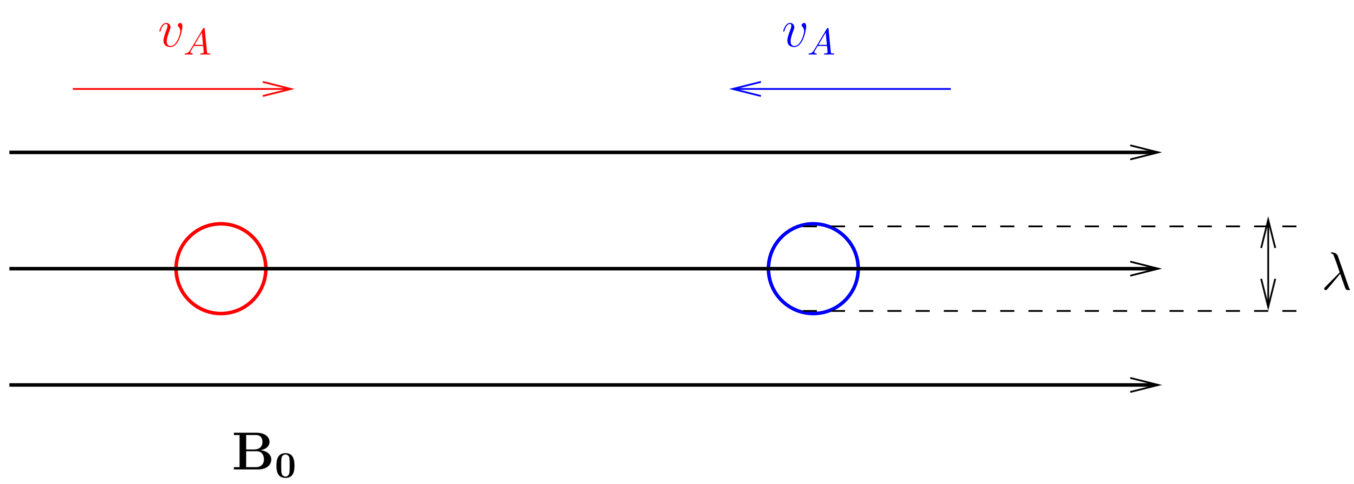
where $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$, and $\nabla \cdot \mathbf{z}^{\pm} = 0$.



- Two competing time scales in the problem: wave time scale τ_A and interaction time scale τ_{NL}.

Phenomenological models of Strong Balanced MHD turbulence

- Iroshnikov and Kraichnann (IK): Turbulence is isotropic, energy transfer results from nonlinear interaction between counter-propagating Alfvén packets:

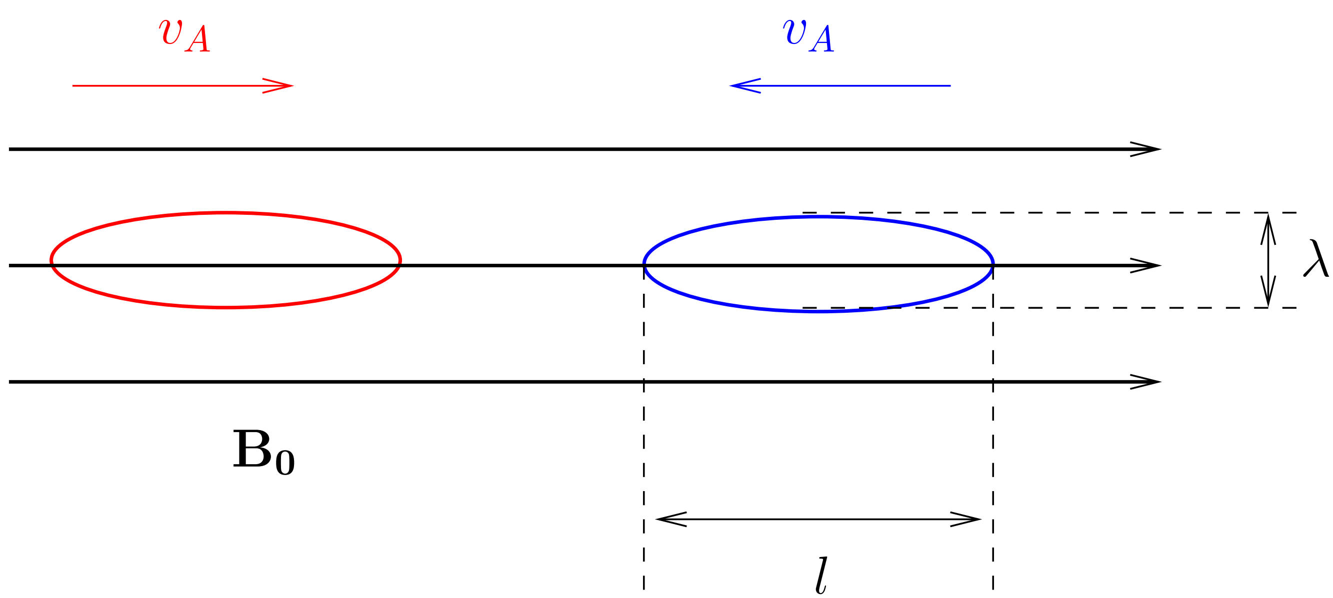


- Energy spectrum:

$$E(k) \sim k^{-3/2}$$

Phenomenological models of Strong Balanced MHD turbulence

- Goldreich & Sridhar (GS): Eddies become elongated along the local field until there is a formal balance between the linear and nonlinear time scales ($\tau_A \sim \tau_{NL}$), called *Critical Balance*

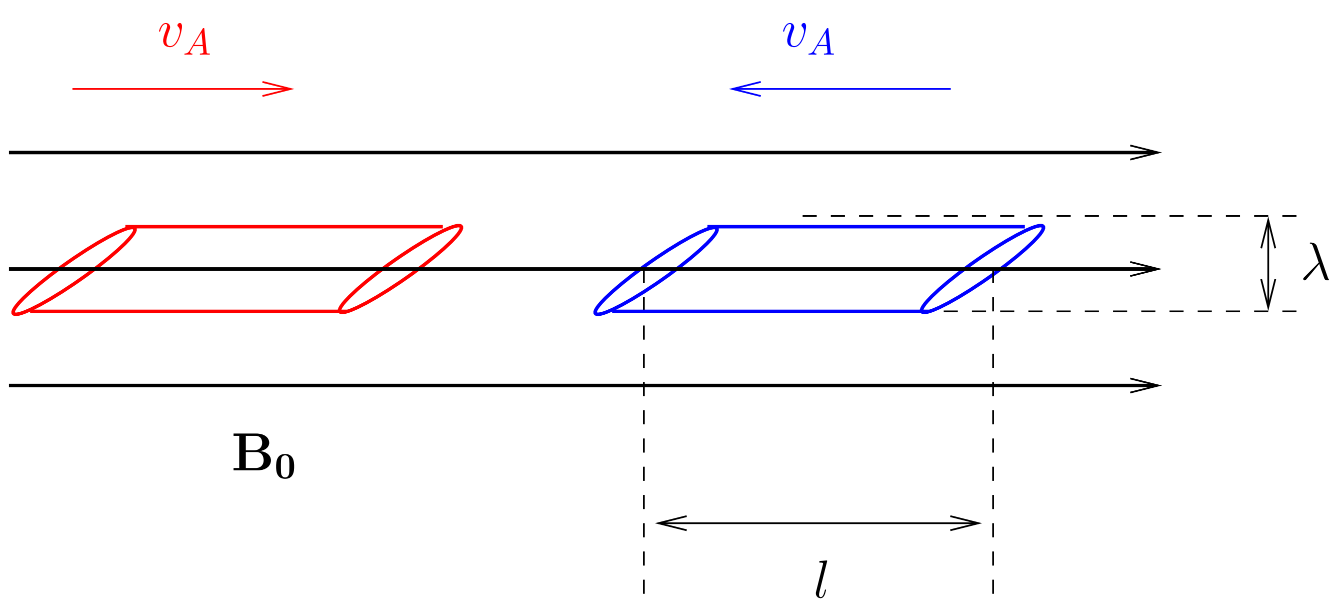


- Energy spectrum:

$$\underbrace{\frac{v_A}{l} \sim \frac{v_\lambda}{\lambda}}_{\text{Critical Balance}} \Rightarrow E(k_\perp) \sim k_\perp^{-5/3}$$

Phenomenological models of Strong Balanced MHD turbulence

- S. Boldyrev (SB), assumed the nonlinear interaction is depleted due to increasing alignment between velocity and magnetic field fluctuations, while still maintaining critical balance



- Energy spectrum:

$$\frac{v_A}{\lambda_\parallel} \sim \frac{v_\lambda}{\lambda_\perp} \underbrace{\frac{v_\lambda}{v_A}}_{\theta_\lambda} \Rightarrow E(k_\perp) \sim k_\perp^{-3/2}$$

Energy and Cross-Helicity cascades

- Energy and Cross-helicity are defined as

$$E = \frac{1}{2}\left\langle \mathbf{v}^2 \right\rangle + \frac{1}{2}\left\langle \mathbf{b}^2 \right\rangle, \quad H_c = \langle \mathbf{v} \cdot \mathbf{b} \rangle$$

- H_c measures the degree of correlation between velocity and magnetic fluctuations in the turbulent state.

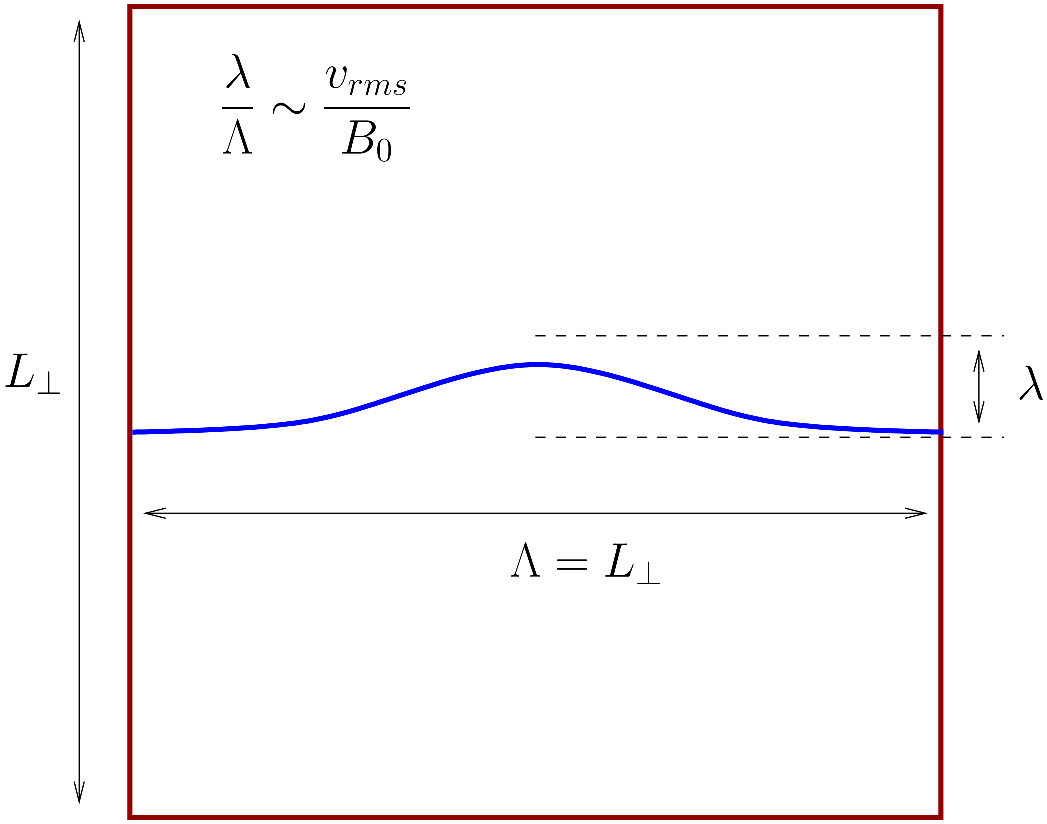
- In terms of Elsasser variables energy and cross-helicity invariants take the form

$$E = E^+ + E^-, \quad H_c = E^+ - E^-, \qquad E^\pm \equiv \frac{1}{4}\left\langle |\mathbf{z}^\pm|^2 \right\rangle$$

- Cross-helicity measures the imbalance between counter-propagating waves.

- Together with energy, cross-helicity undergoes a turbulent cascade from large to small scales. The same holds for $E^\pm$.

Anisotropy and computational domain



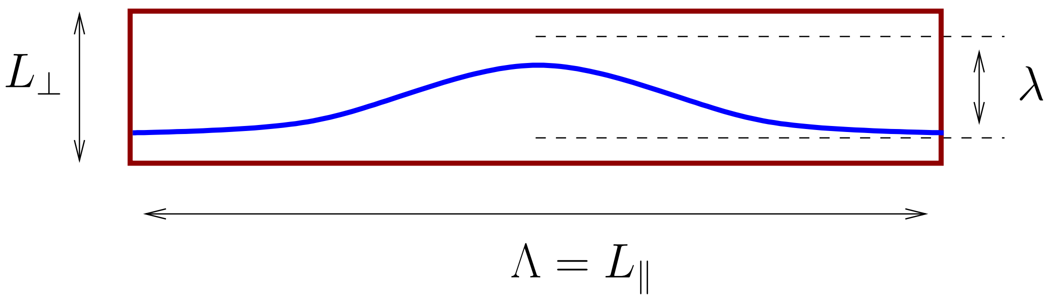
- MHD turbulence is anisotropic: eddies elongated along $\mathbf{B}_0$.
- Cubic simulation box is not optimal: a significant number of perpendicular scales wasted.
- Can lead to inaccurate measurements of spectral index.
- The box aspect ratio is crucial to maximize the range of useful perpendicular scales.

- Ratio between linear and nonlinear time scales is controlled at the forcing scale

$$\chi = \frac{k_{\|} v_A}{k_\perp v_0}$$

Anisotropy and computational domain

$$\frac{\lambda}{\Lambda} \sim \frac{v_{rms}}{B_0}$$



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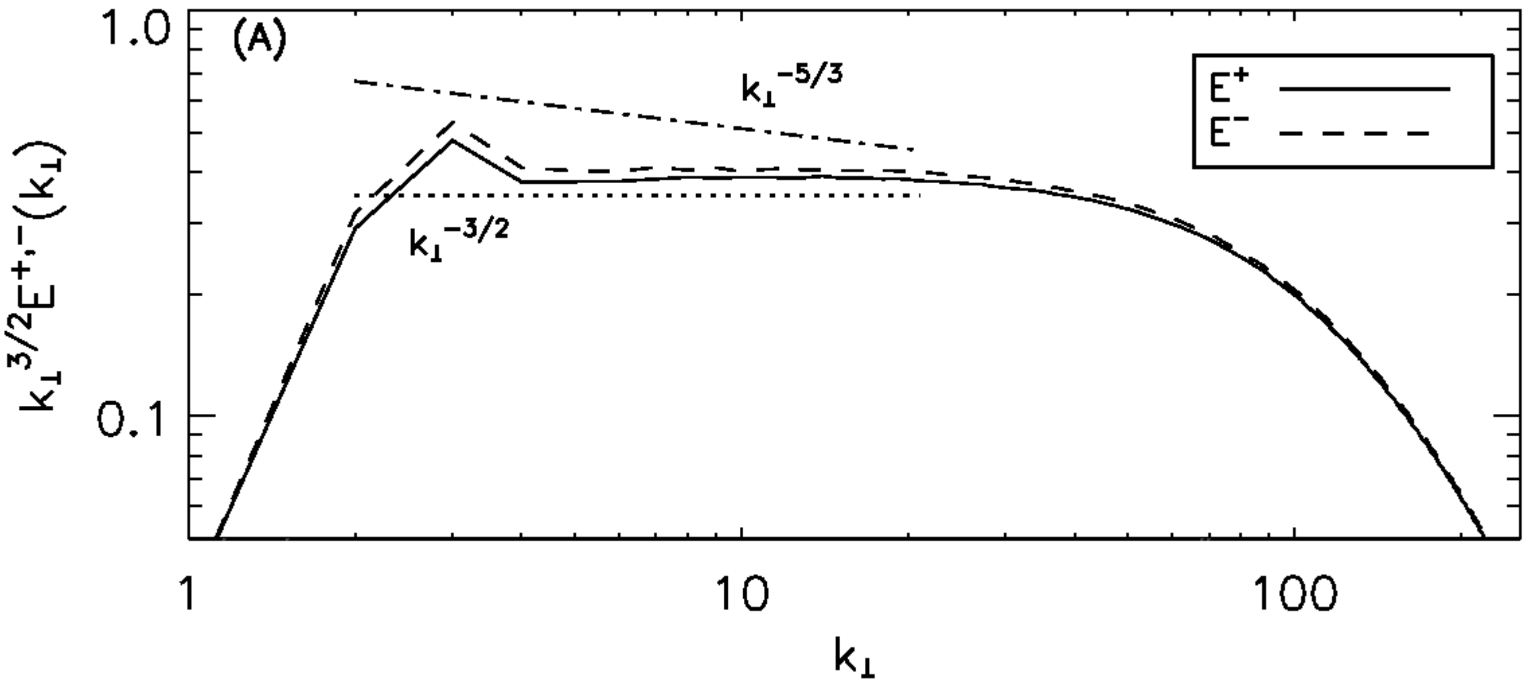
Reduced MHD turbulence code

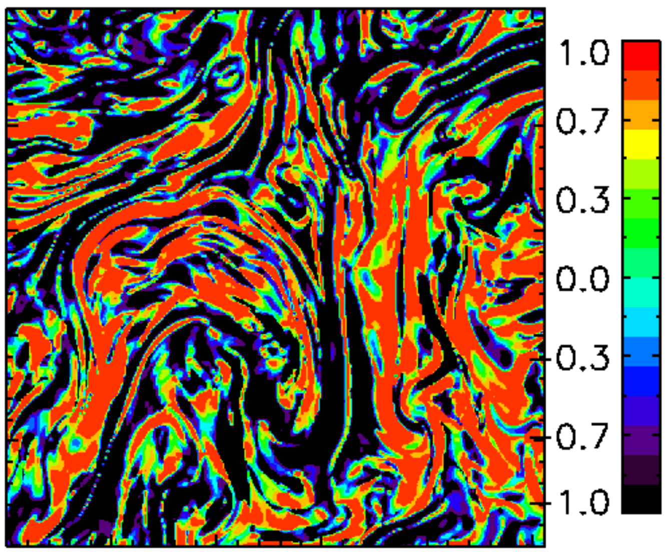
- We used a 2/3-dealiased pseudo-spectral method to solve RMHD equations.
- No artificial viscosity is used (hyper-viscosity), to minimize undesired bottleneck effects.
- Simulations are regularly performed on 1024 to 2048 processors in Ranger supercomputer at the Texas Advanced Computing Center, UT-Austin through an NSF Teragrid Allocation.
- Code has good strong and weak scaling up 16,384 cores on ranger.
- We have used over 2 million CPU hours, it would roughly take more than two centuries on a single computer.

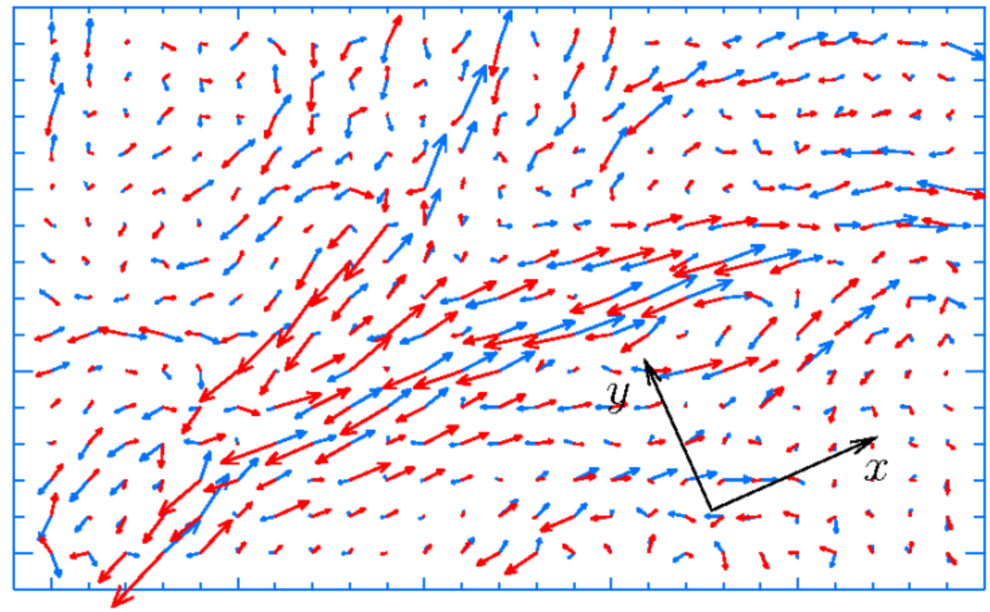
Balanced Turbulence Simulations on a $1024^2 \times 256$ grid



- Results for $(256^2$, 512^2 and $1024^2)$ $\times 256$, are all consistent with $k_{\perp}^{-3/2}$. (No bottleneck is observed!)
- Consistent with numerics by: Maron & Goldreich 2001, Haugen, Brandenburg & Doubler 2003, Muller & Grappin 2005, Mason, Cattaneo & Boldyrev 2008, Mininni & Pouquet 2008.

The role of cross helicity $H_c = E^+ - E^-$

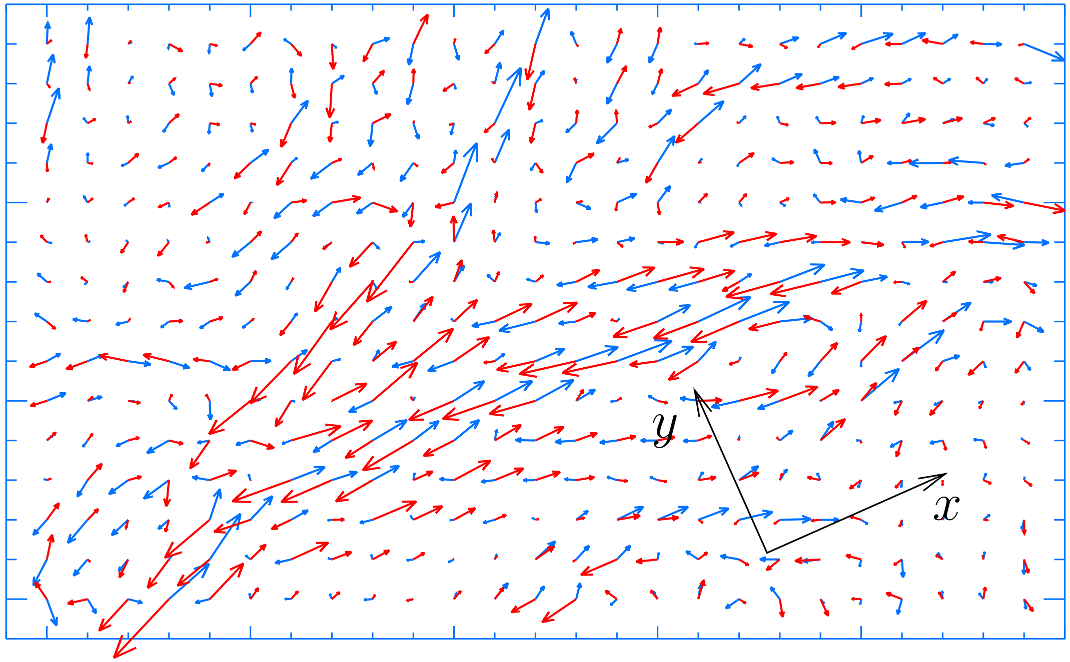




- Left panel shows cosine of the alignment angle between $\mathbf{v}_\lambda$ and $\mathbf{b}_\lambda$ fluctuations in the guide-field perpendicular plane at scales $\lambda = L_\perp/12$ for balanced turbulence. Right panel shows velocity and magnetic fields at one alignment region.

- Domains are split in patches of highly aligned and anti-aligned regions.

- A theory of imbalanced turbulence should be consistent with the fact that balanced turbulence is composed of imbalanced regions.



- Fluctuations of velocity and magnetic field develop correlated regions of alignment and anti-alignment.

- Fluctuations are aligned toward the x direction, while gradient of fluctuations is stronger in the perpendicular direction (y).

- Alignment is scale dependent. When the angle decreases with the scale as $\theta_\lambda \sim \lambda^{1/4}$ the constant energy flux constraint leads to the energy scaling $E(\mathbf{k}_\perp) \sim k_\perp^{-3/2}$.

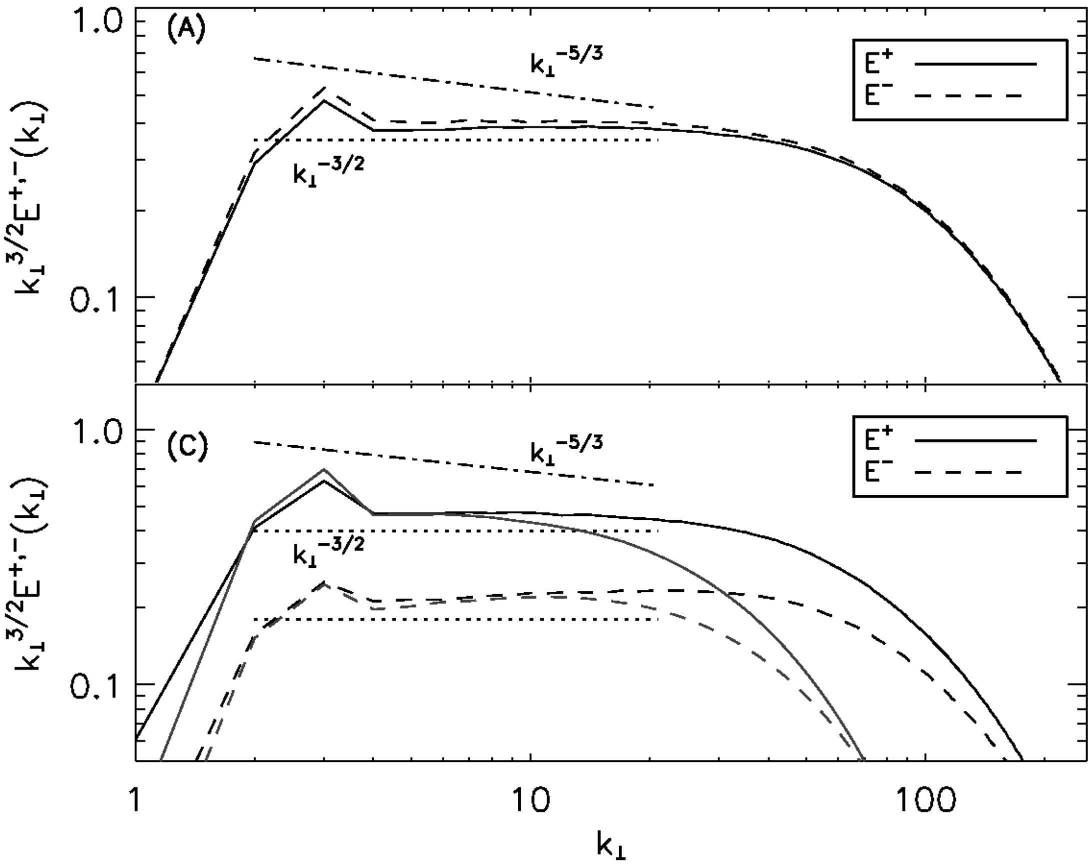
Recent work on imbalanced MHD turbulence

- Lithwick and Goldreich 2007 argue that energy cascade times are different for z^+ and z^- and lead to scalings $E^+ \sim E^- \sim k_\perp^{-5/3}$.

- Beresnyak & Lazarian 2008, and Chandran 2008 assume that the high amplitude wave, say z^+ undergoes a weak cascade while z^- undergoes a strong cascade, therefore energy spectra E^+ and E^- have different scalings.

- The fact that balanced MHD turbulence is constituted by regions of both positive and negative cross-helicity is incompatible with current models of imbalanced turbulence.

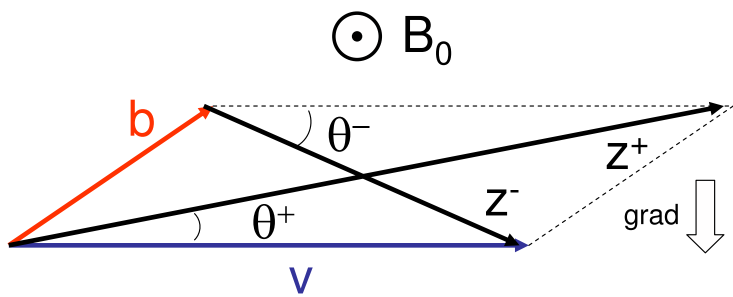
Imbalanced Turbulence Simulations on a $1024^2 \times 256$ grid



1. Strong Balanced turbulence: $E^+(\mathbf{k}_\perp) \sim E^-(\mathbf{k}_\perp) \propto k_\perp^{-3/2}$.
2. Strong Imbalanced turbulence: $E^+(\mathbf{k}_\perp) \propto E^-(\mathbf{k}_\perp) \propto k_\perp^{-3/2}$.
3. Independently observed in solar wind data by Podesta & Bhattacharjee 2008, unpublished

Dynamic alignment model for the observed spectra: imbalanced case

- We assume that alignment is also present in the imbalanced case.
- However, alignment angle is different for z^+ and z^-, with the geometric constraint $z^+\theta^+ \sim z^-\theta^-$.



- Nonlinear interaction time is the same for both, $\tau^+ = \tau^-$.

$$\frac{\partial}{\partial t}\mathbf{z}^{\pm} \mp v_A \partial_{\|}\mathbf{z}^{\pm} + \underbrace{(\mathbf{z}^{\mp}\cdot\nabla)}_{z_\lambda^{\mp}\theta_\lambda^{\mp}k_\perp \sim 1/\tau^{\pm}} \mathbf{z}^{\pm} = 0$$

- Constant flux assumption $(z^{\pm})^2/\tau_\lambda^{\pm}$ =const leads to same scaling $k_\perp^{-3/2}$, although $E^{\pm}$ have different amplitudes.

Concluding Remarks

- MHD turbulence, both balanced and imbalanced, is characterized by regions of positive and negative cross-helicity.
- Numerical simulations show that the spectra of E^+ and E^- approach the scaling $k_\perp^{-3/2}$ with different amplitudes.
- Simulations provide a coherent picture of balanced and imbalanced MHD turbulence.